Regression and inference

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I want you to understand:

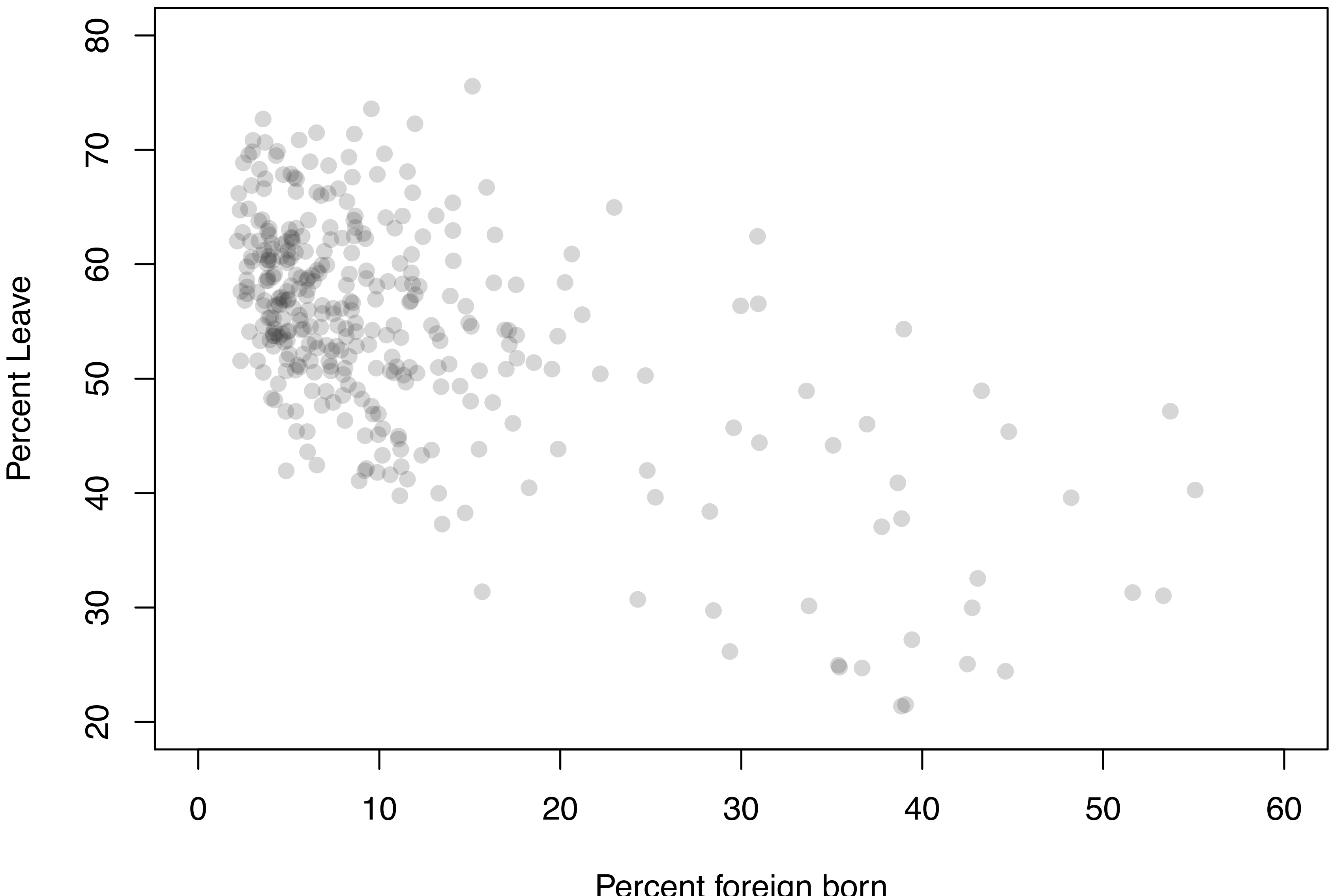
Dependent variable: Nobel Prizes awarded per capita (in log scale)

	(1)	(2)	(3)
Intercept	-1.629* (0.509)	-3.166* (0.511)	-2.982* (0.527)
Chocolate consumption per capita (log scale)	2.092* (0.298)	1.026* (0.326)	0.709 (0.415)
GDP/capita (thousands of USD)		0.105* (0.024)	0.106* (0.024)
NW Europe			0.549 (0.452)
R^2	0.70	0.85	0.86
N	34	34	34

Standard errors in parentheses. * Indicates $p<0.05$

- what a dependent variable is
- what an independent variable is
- what the coefficients mean (intercept, slopes)
- what the stars mean (i.e. what $p<0.05$ means)
- what the standard errors mean

Local authorities with more foreign-born residents were less supportive of Brexit



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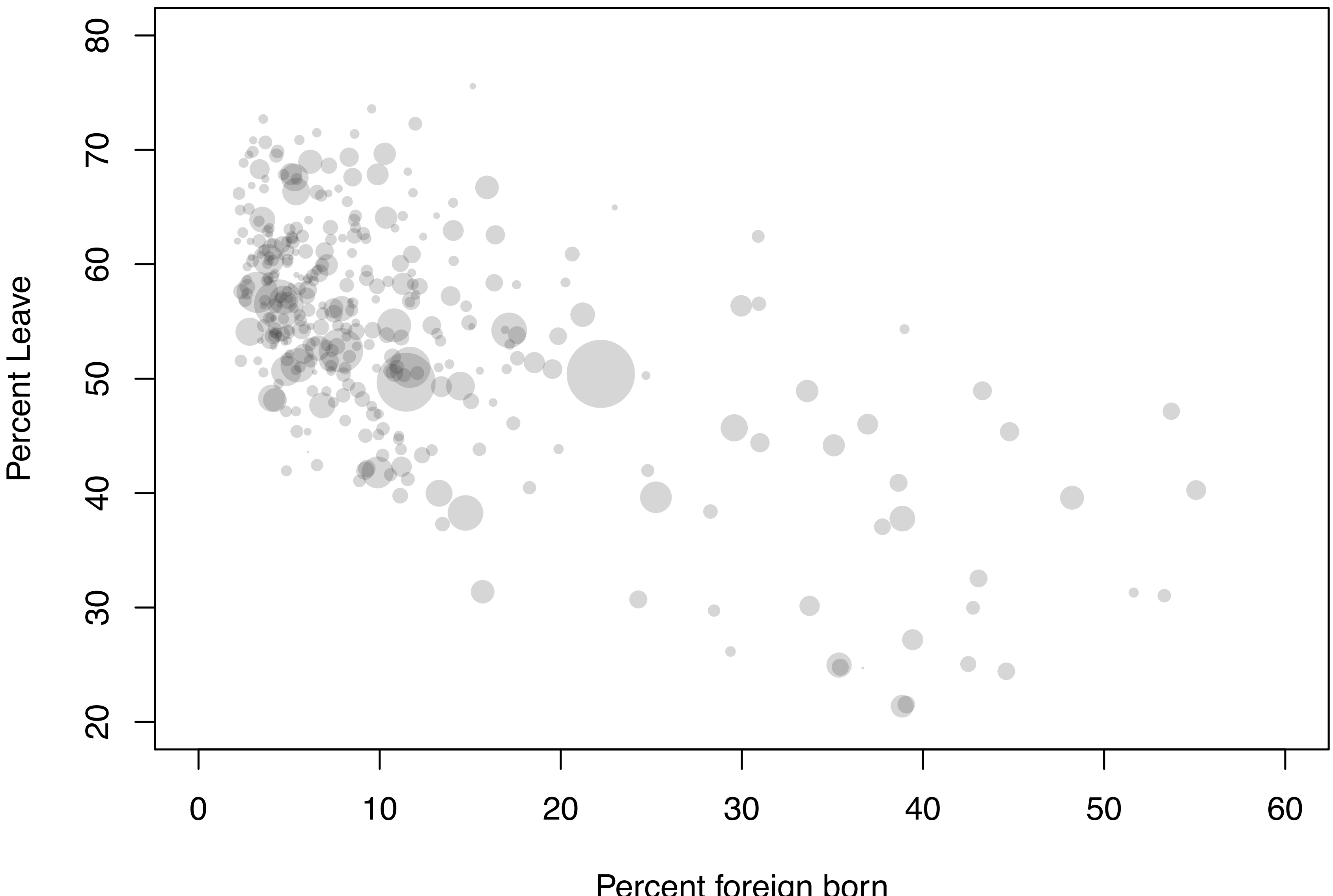

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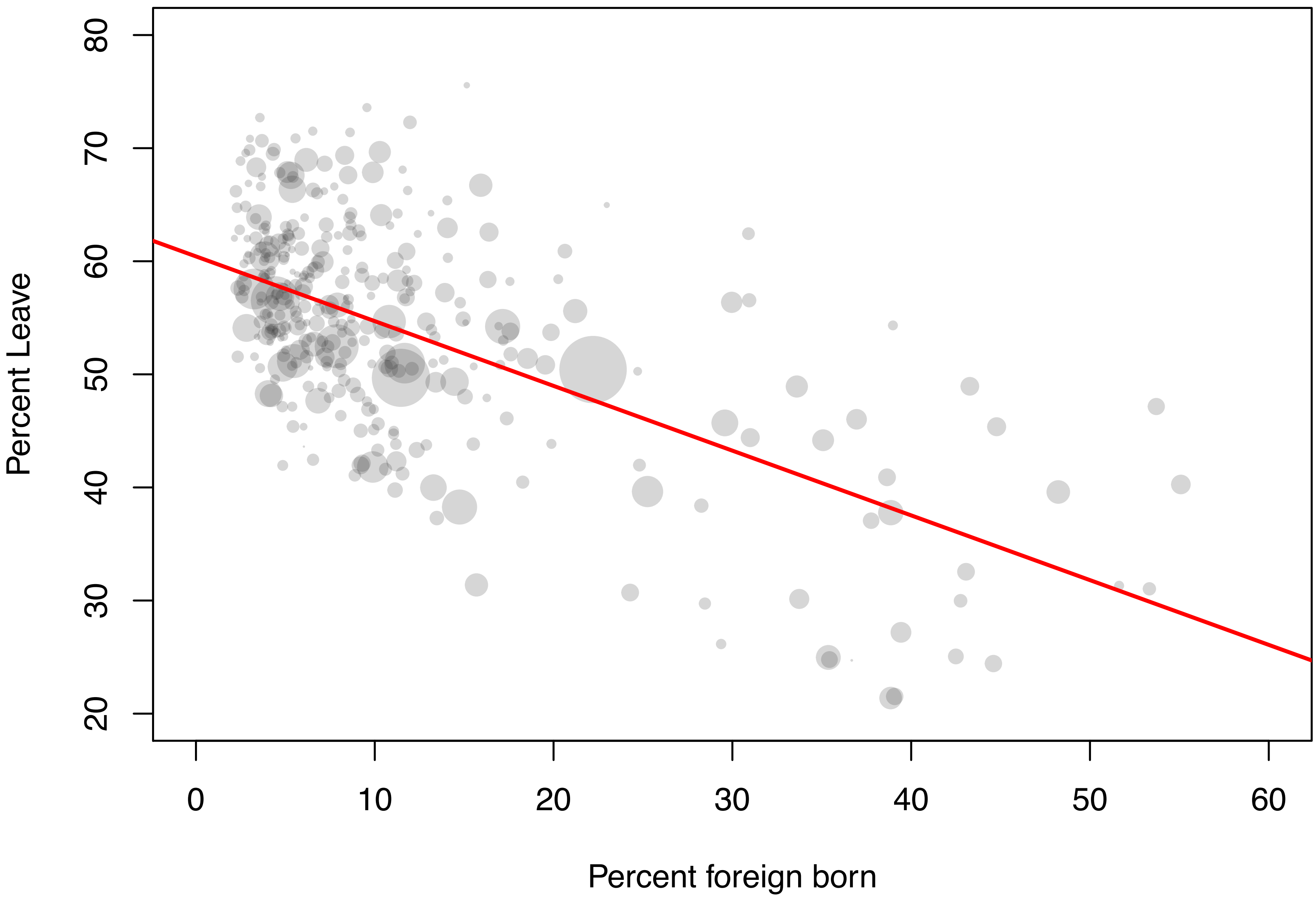

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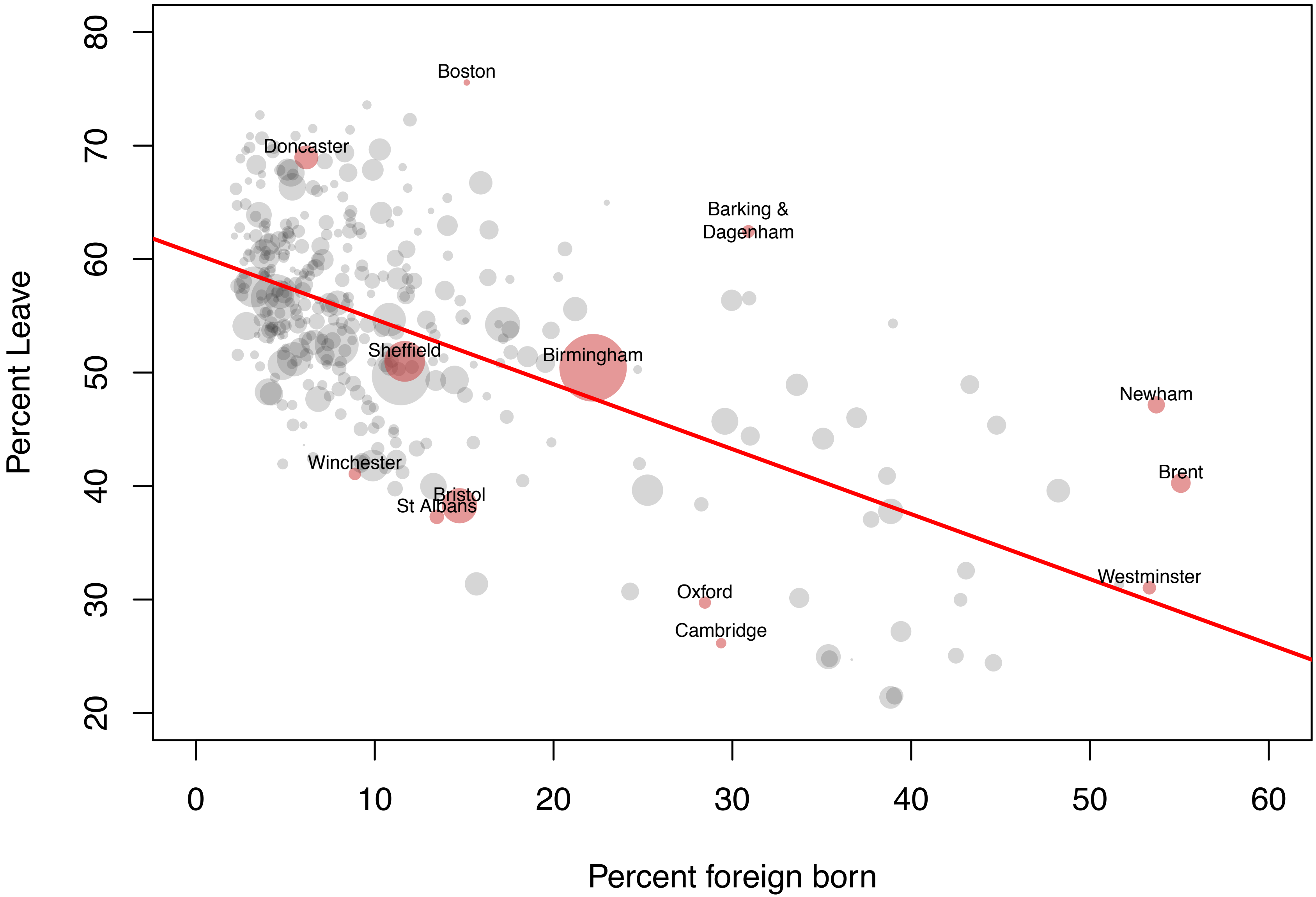

Contact hypothesis

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"Prejudice (unless deeply rooted in the character structure of the individual) may be reduced by equal status contact between majority and minority groups in the pursuit of common goals. The effect is greatly enhanced if this contact is sanctioned by institutional supports (i.e., by law, custom or local atmosphere), and provided it is of a sort that leads to the perception of common interests and common humanity between members of the two groups."

— Gordon Allport (1954) *The Nature of Prejudice*

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- How could this pattern be explained by the contact hypothesis? (easy)
- How could this pattern be explained by other factors? (harder)

Today and tomorrow

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- How do we summarize the relationship between two variables controlling for a third variable?
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- How do we summarize our uncertainty about our conclusions?
  - standard errors, p-values, confidence intervals

Summarizing bivariate relationships: non-OLS options

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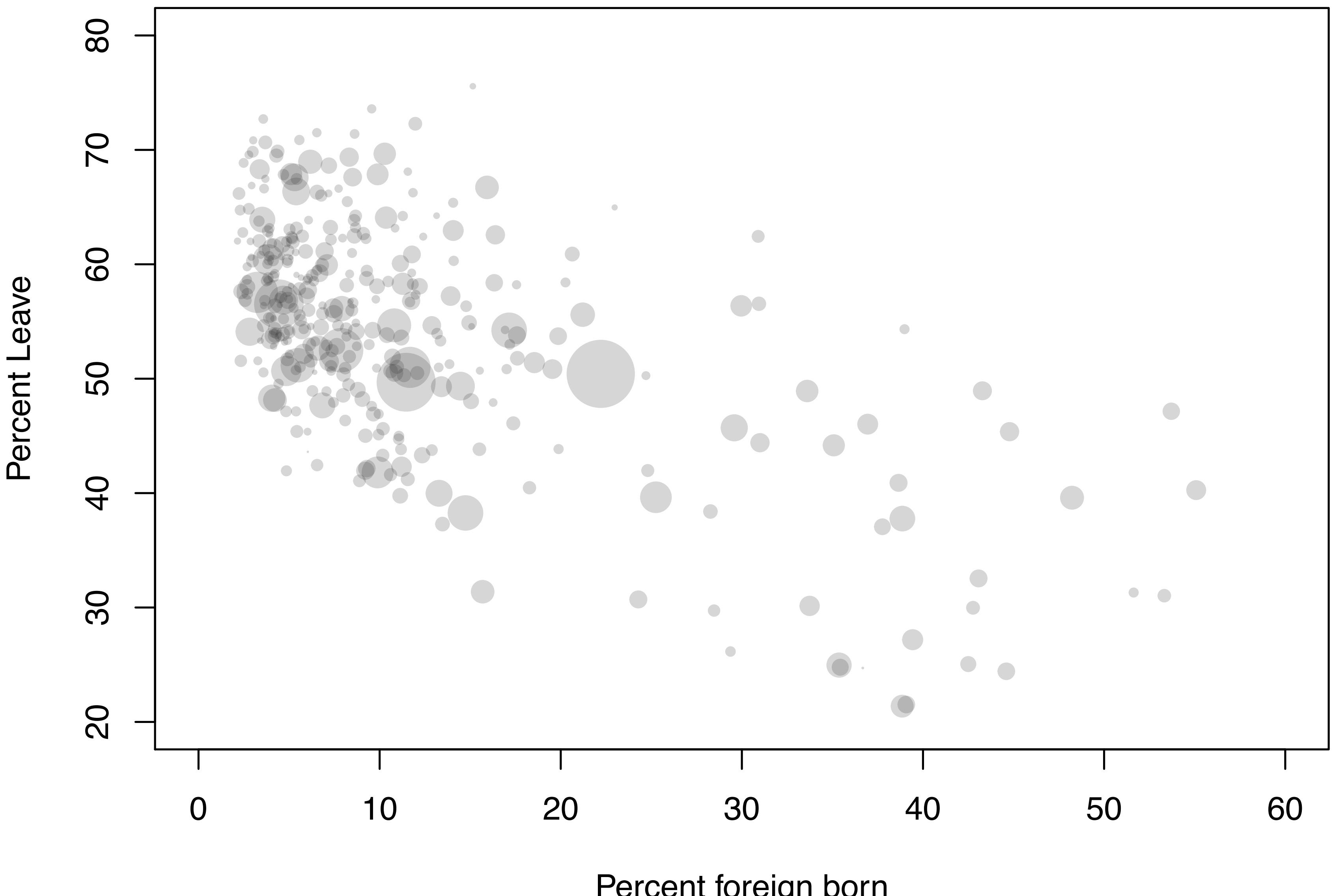
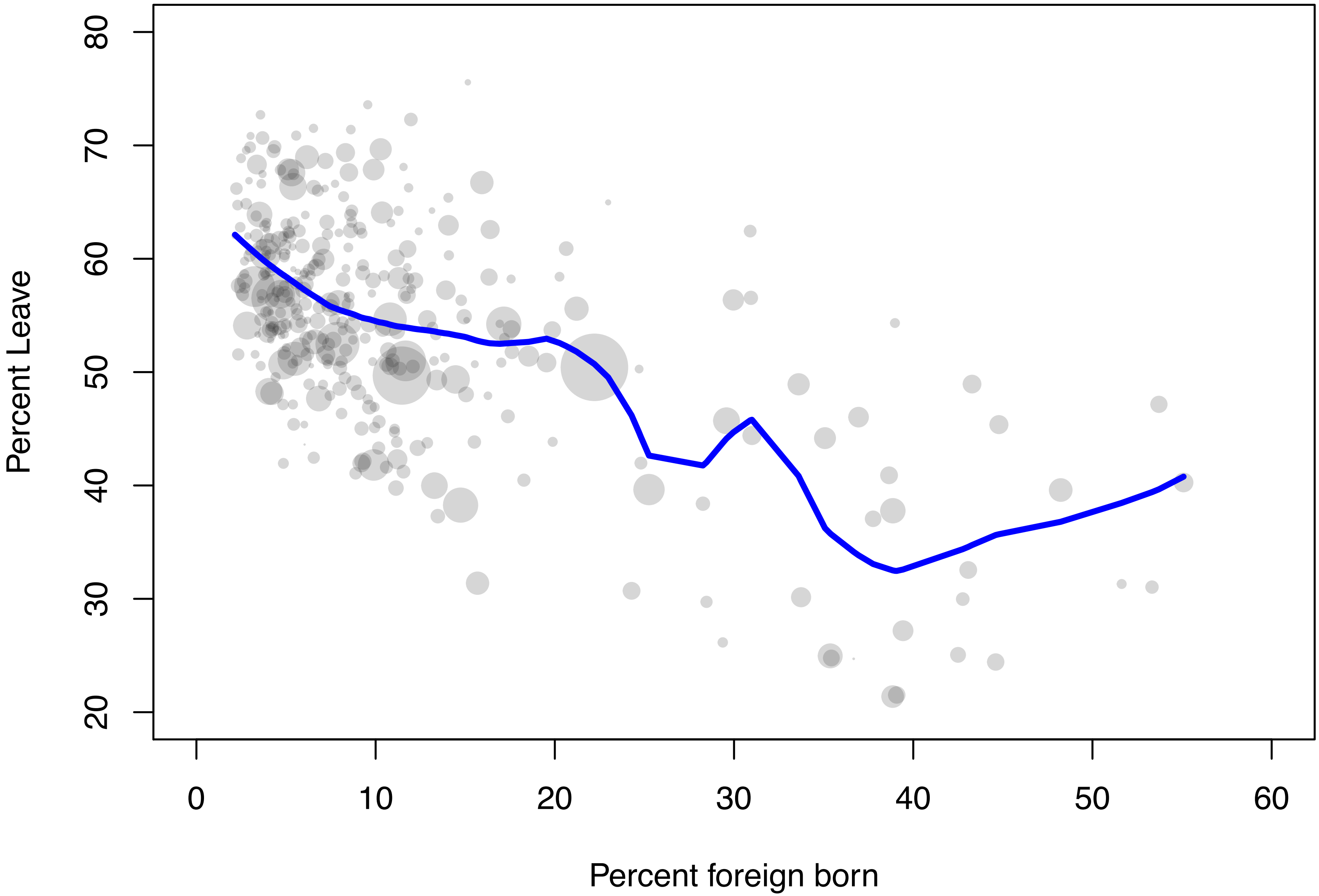


Kernel smoother (lokern function in R)



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```
> cov(d$Percent_foreign_born, d$Percent_Leave, use = "complete")
[1] -62.17755
```

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> cor(d$Percent_foreign_born, d$Percent_Leave, use = "complete")
[1] -0.6125353
```

Correlation examples

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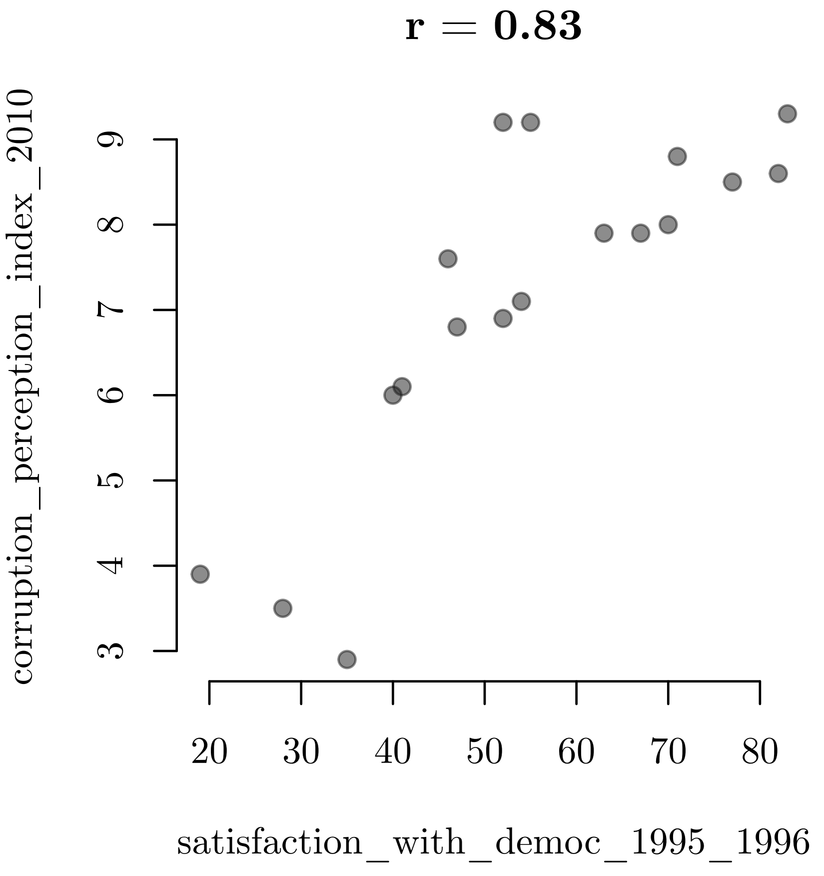

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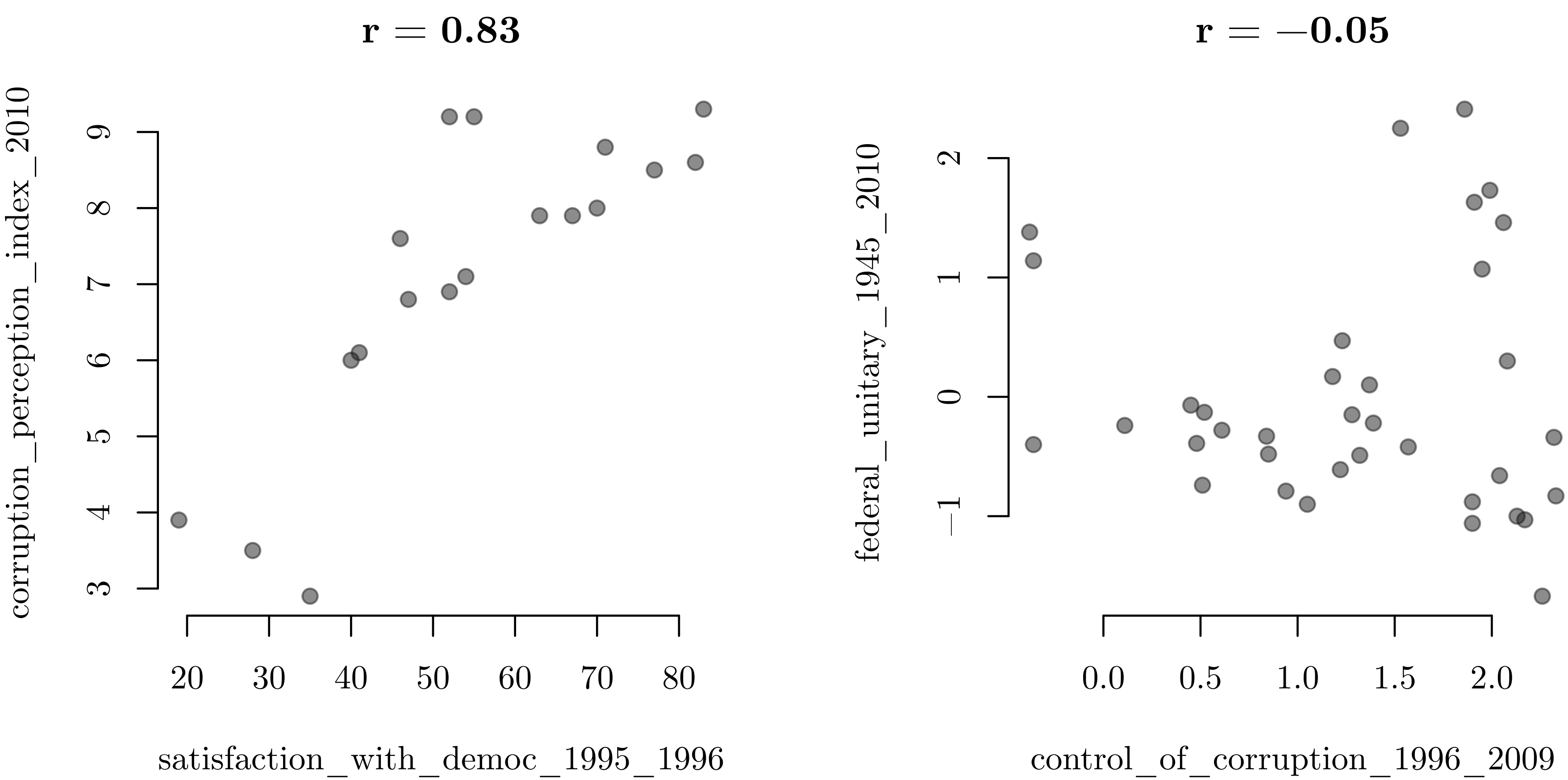
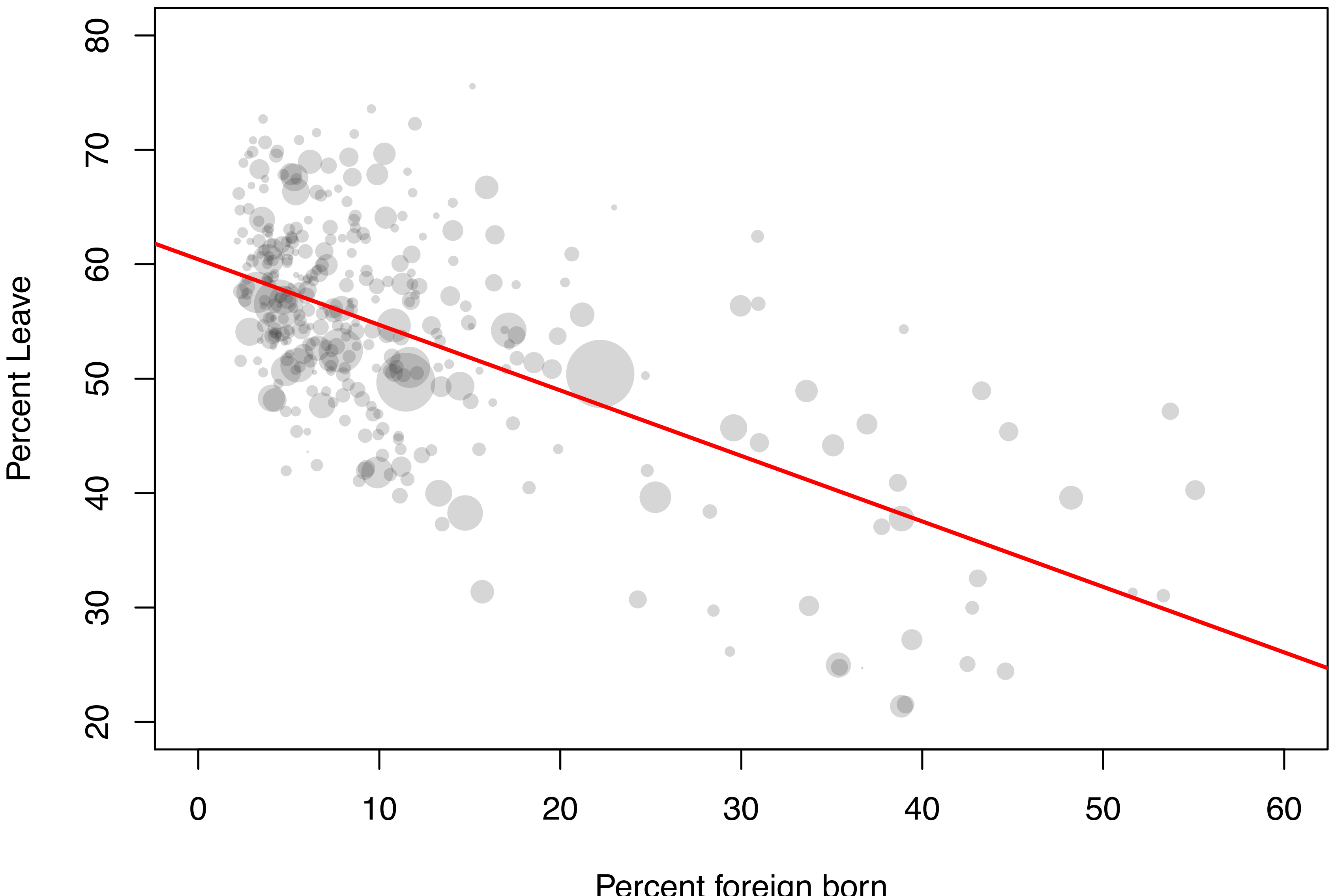


The most important summary: OLS regression



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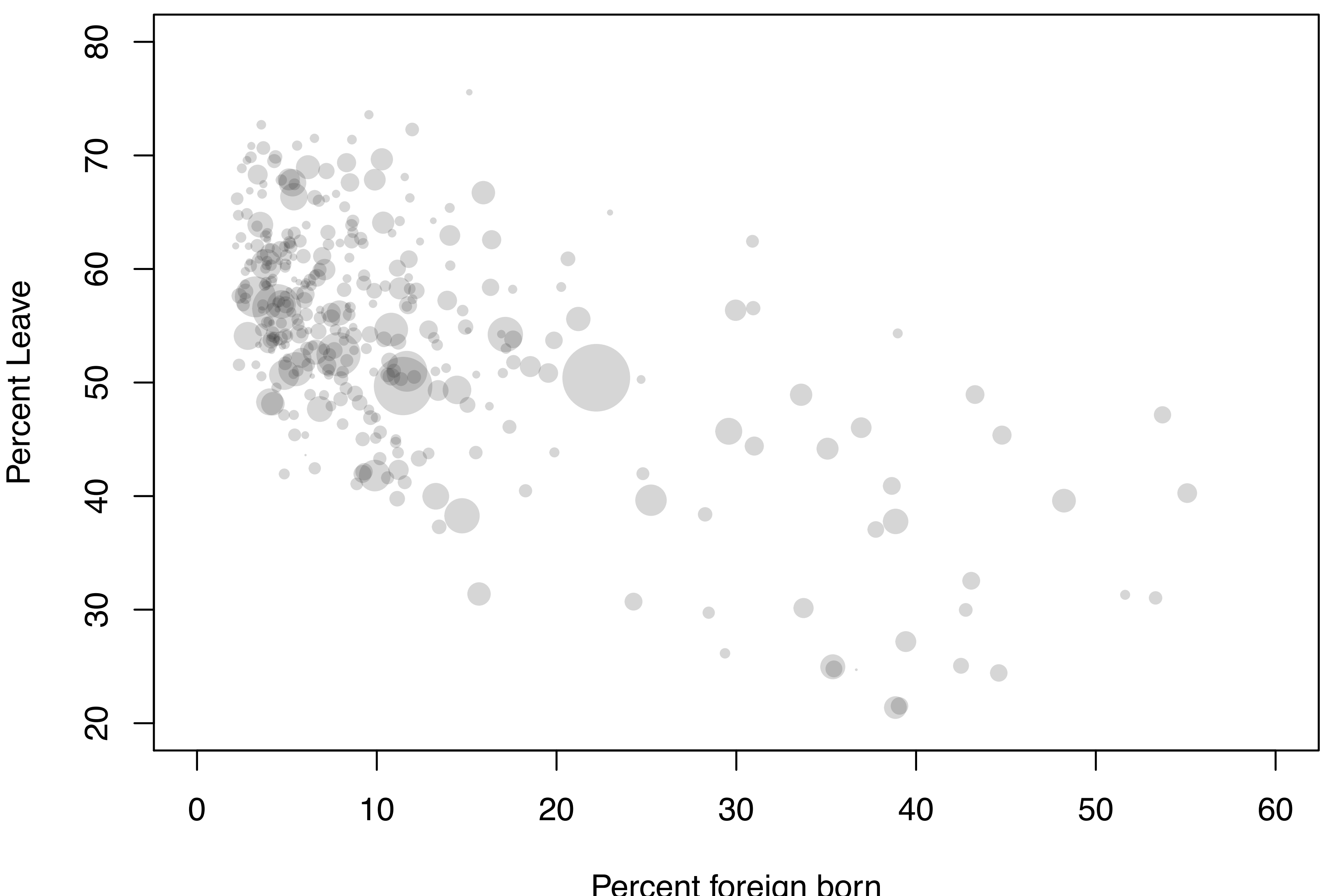
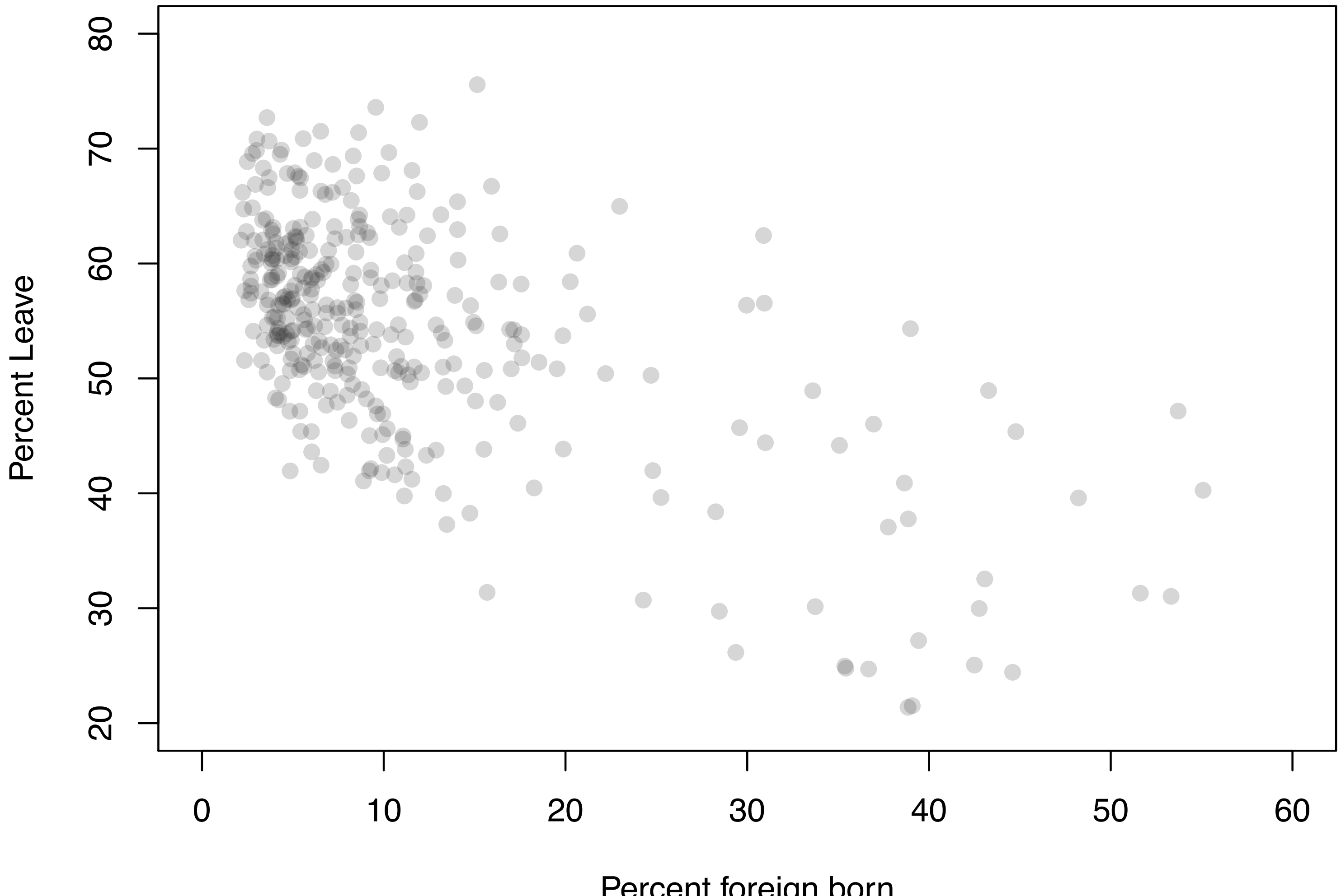


Step 1 for understanding OLS: residuals

A **residual** is the difference between the *actual* y-value and the *predicted* y-value (i.e. vertical diff. btw point and line).



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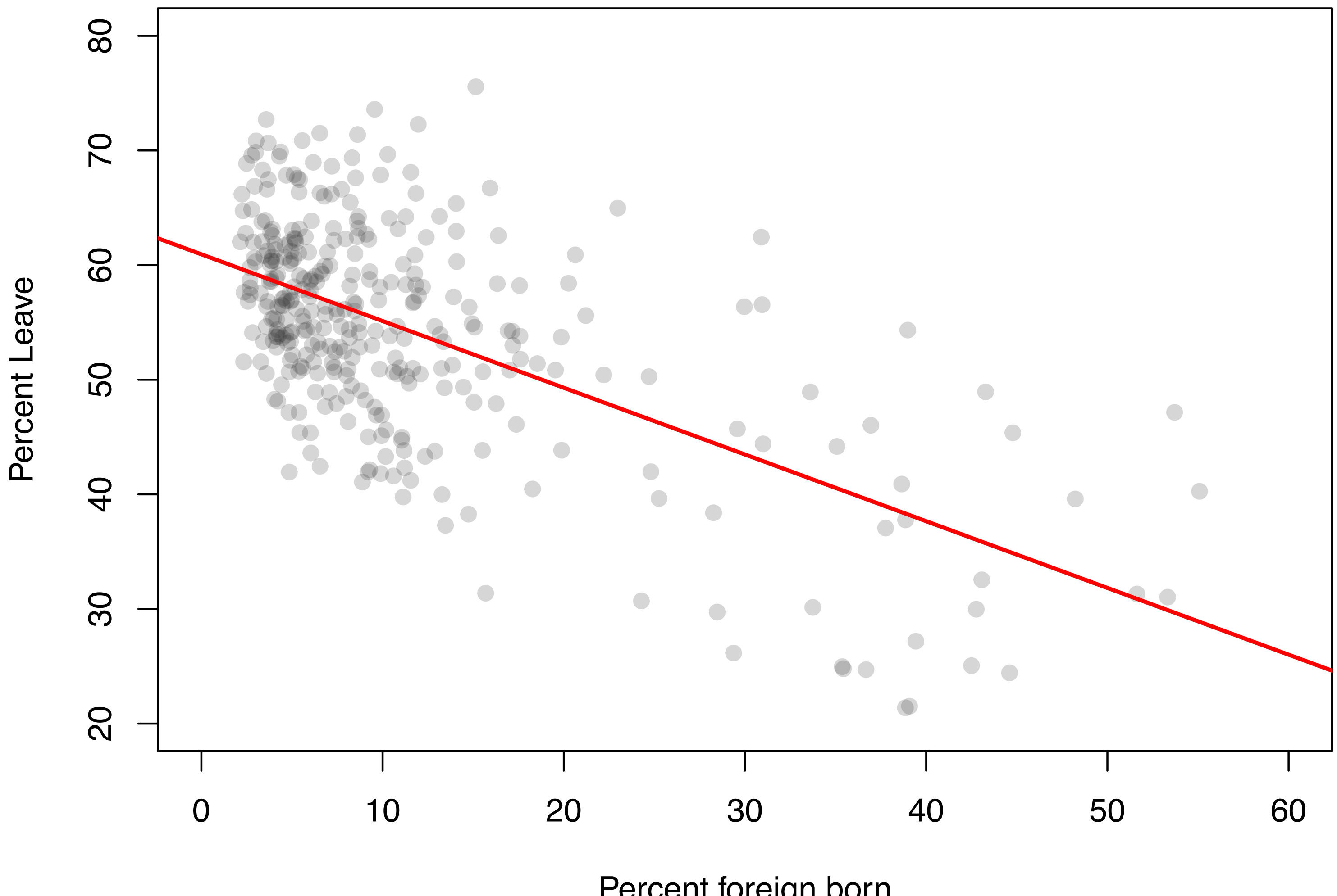

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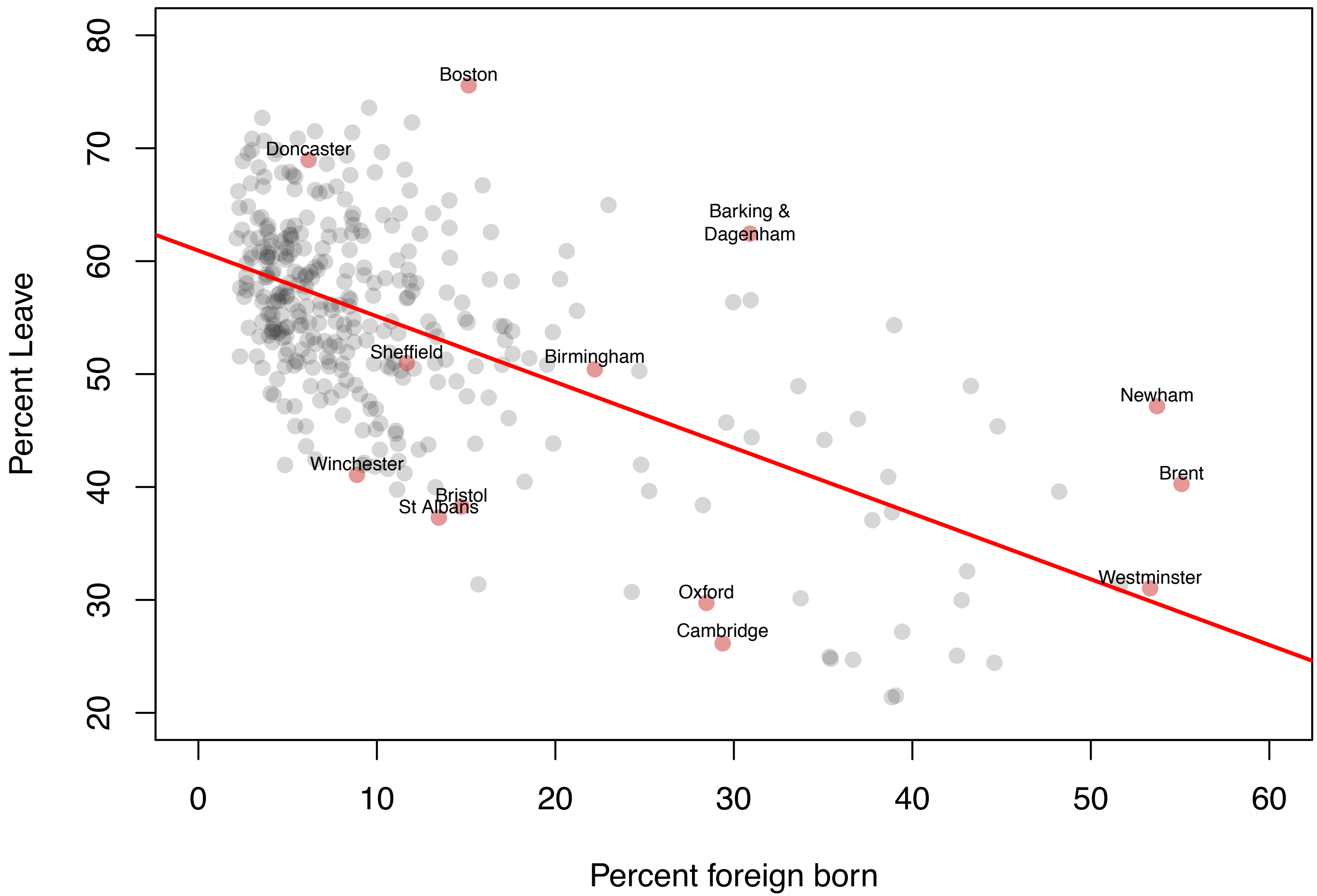

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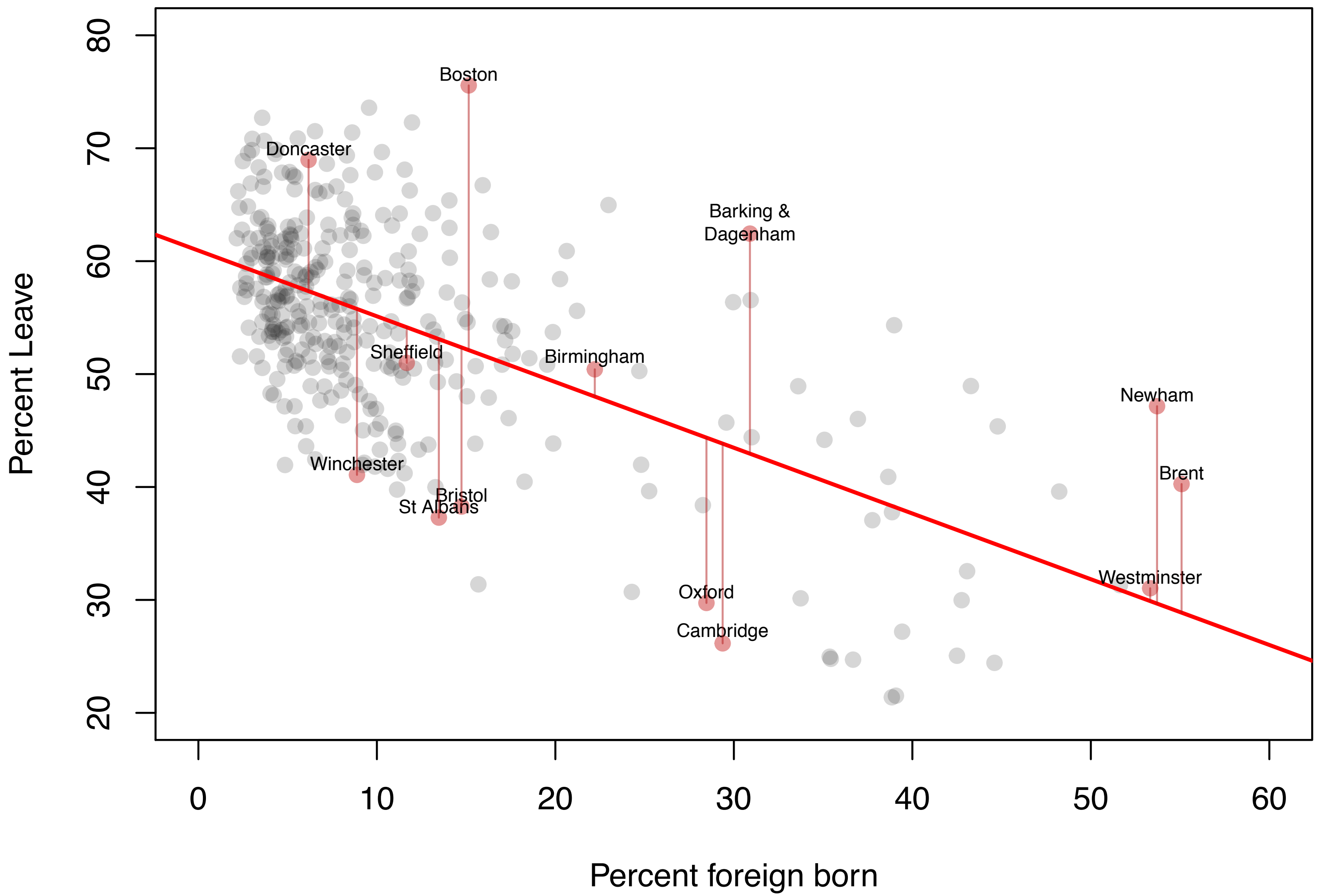
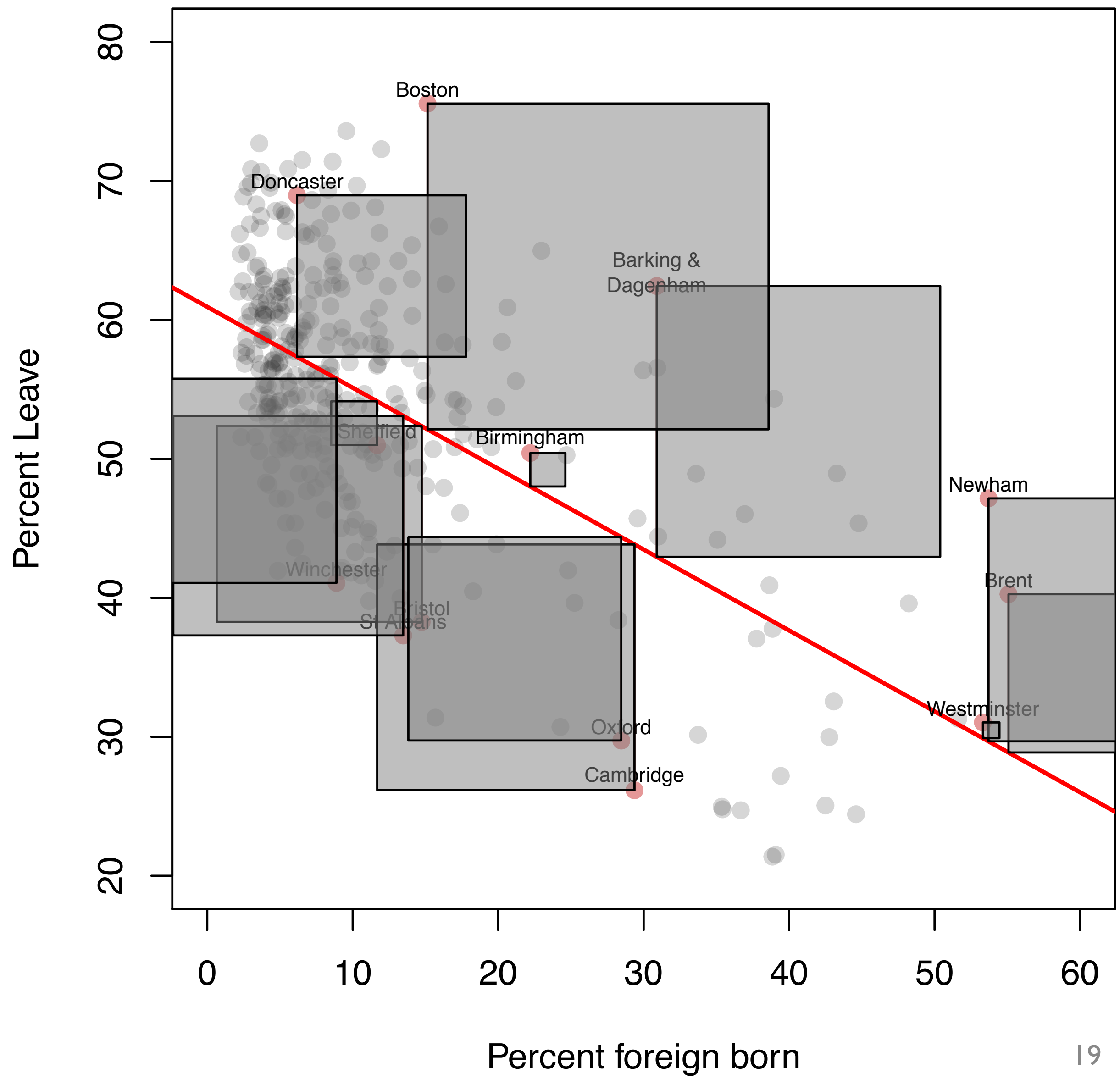


Step 2 for understanding OLS: sum of squared residuals

For any prediction line you draw, you can calculate residuals, square them, and sum them.



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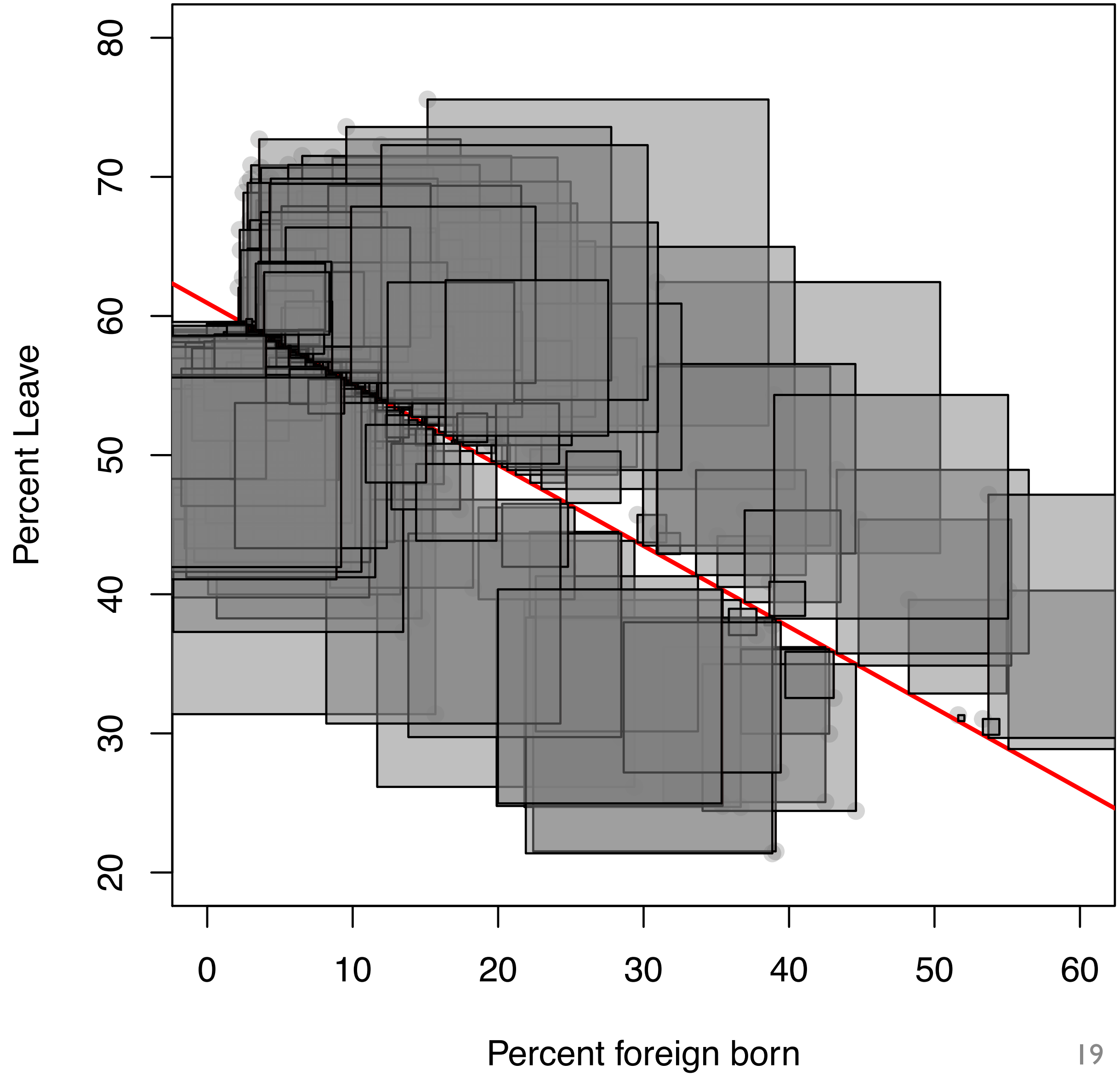
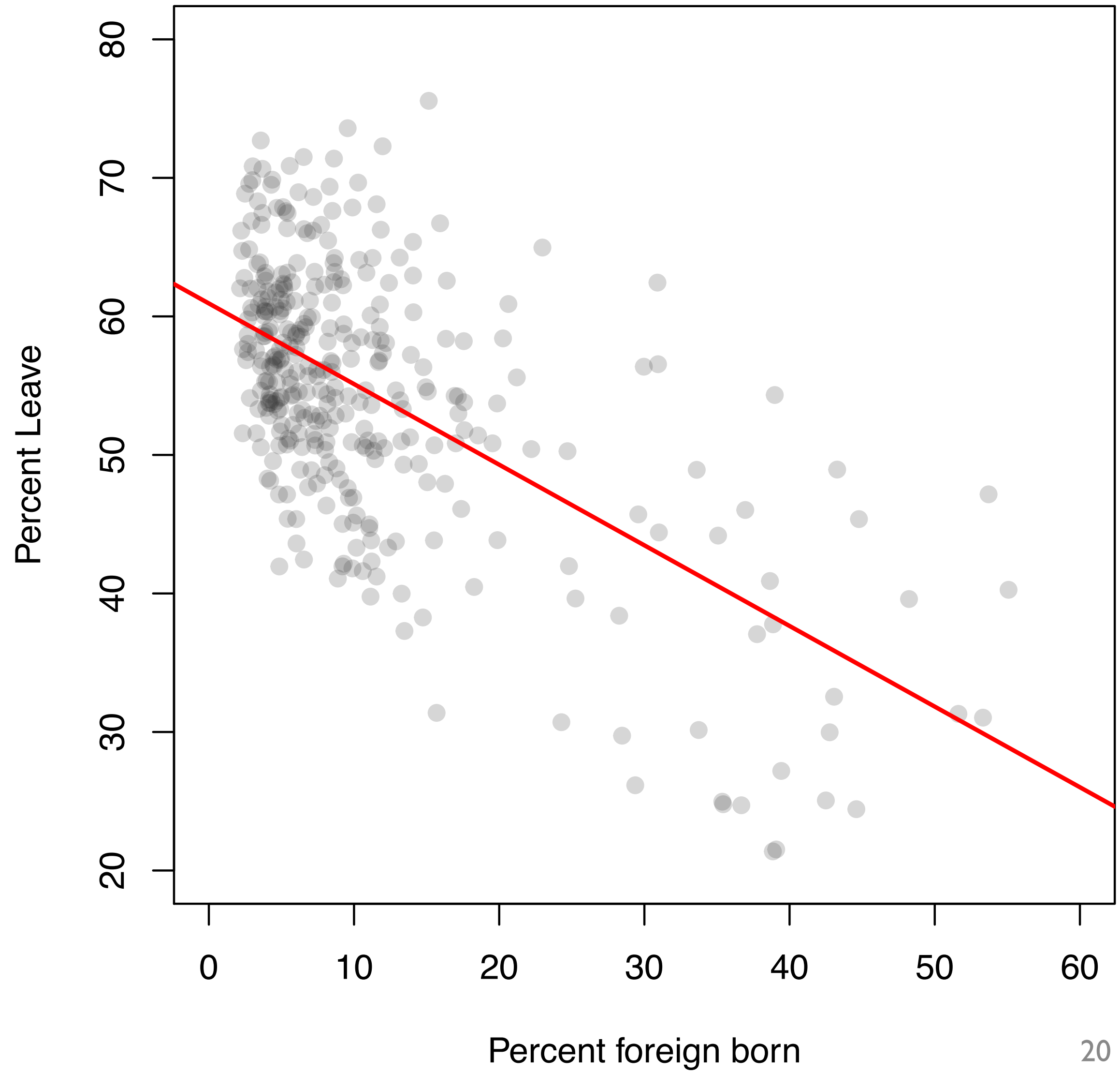


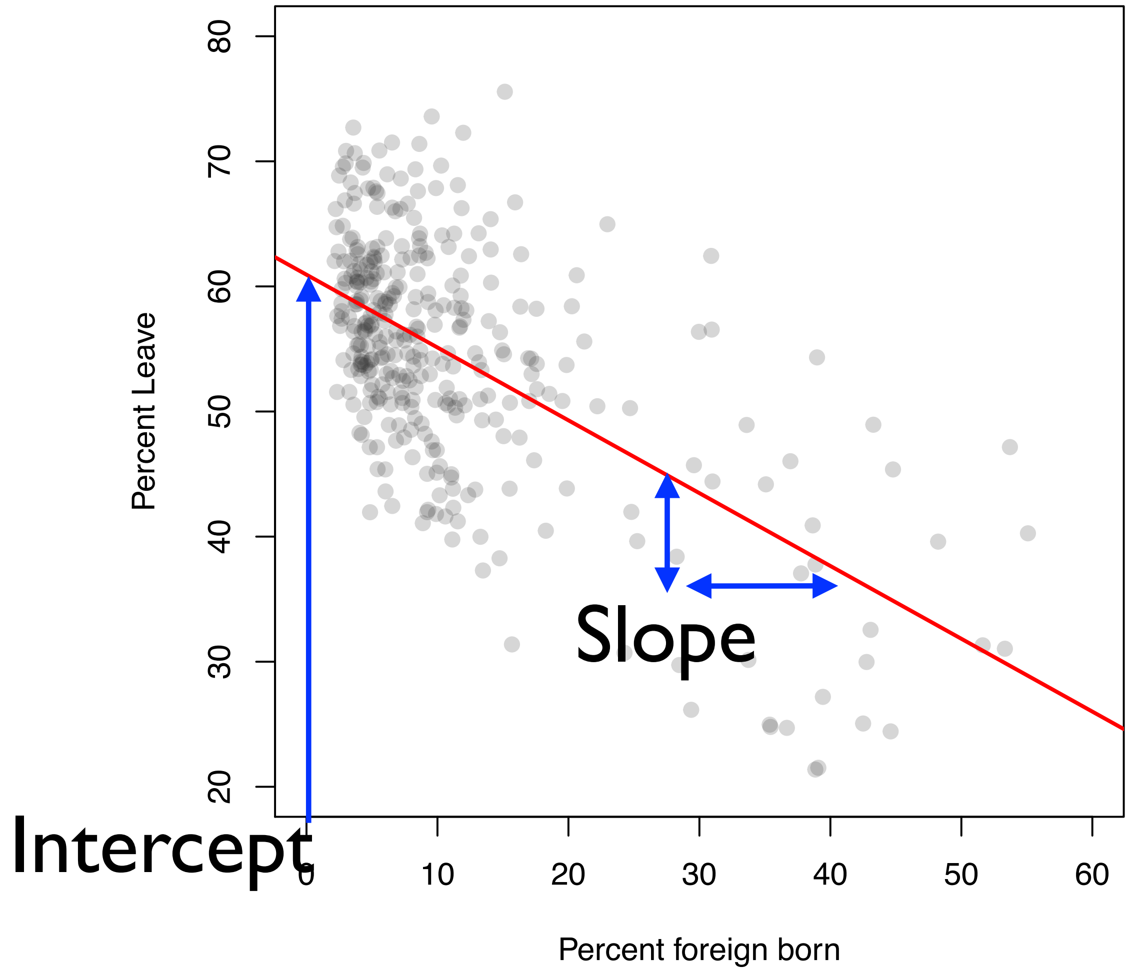
Step 3 for understanding OLS: minimizing the sum of squared residuals

The OLS regression line minimizes the sum of squared residuals (SSR).

Hence ordinary **least squares.**



Any line can be summarized by an **intercept** and a **slope**.



The intercept and slope of a bivariate regression are called the regression **coefficients.**

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3. Use lm() function in R:

```
> lm(d$Percent_Leave ~ d$Percent_foreign_born)

Call:
lm(formula = d$Percent_Leave ~ d$Percent_foreign_born)

Coefficients:
           (Intercept)  d$Percent_foreign_born
               60.9373                 -0.5821
```

A (surprising?) fact about the slope coefficient

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Covariance of x and y:

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Slope from OLS regression of y on x:

$$\hat{\beta} = \frac{\text{Cov}(x,y)}{\text{Var}(x)}$$

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> cov(d$Percent_Leave, d$Percent_foreign_born, use = "complete")/var(d
$Percent_foreign_born, na.rm = T)
[1] -0.582101
> lm(d$Percent_Leave ~ d$Percent_foreign_born)

Call:
lm(formula = d$Percent_Leave ~ d$Percent_foreign_born)

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How well does our regression line predict the outcome? R^2

```
> summary(lm(d$Percent_Leave ~ d$Percent_foreign_born))

Call:
lm(formula = d$Percent_Leave ~ d$Percent_foreign_born)

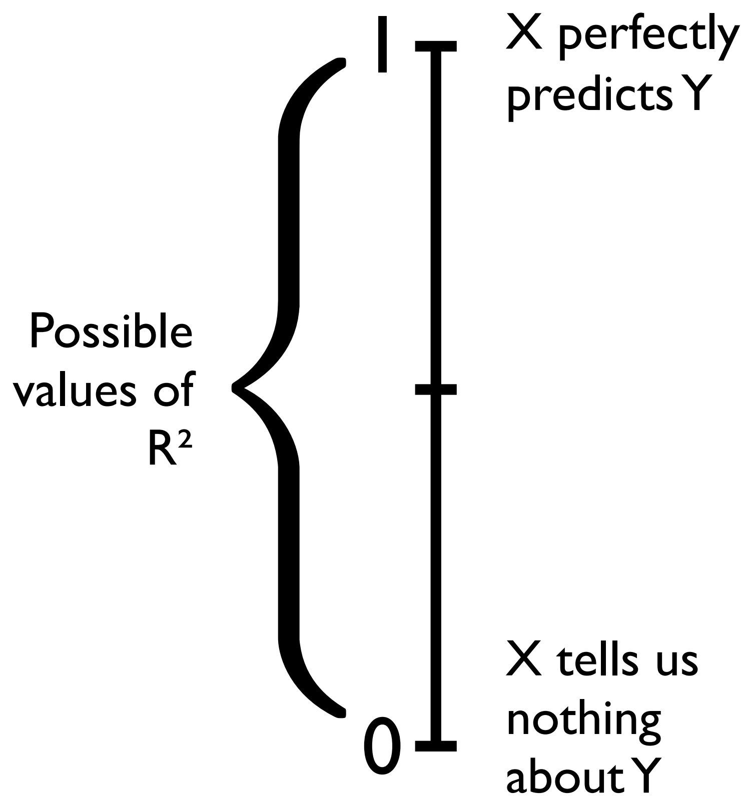
Residuals:
     Min       1Q   Median       3Q      Max 
-20.4253  -4.7247  -0.0025   4.4336  23.4417 

Coefficients:
                       Estimate Std. Error t value Pr(>|t|)    
(Intercept)            60.93732    0.61845   98.53   <2e-16 ***
d$Percent_foreign_born -0.58210    0.04062  -14.33   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 7.775 on 342 degrees of freedom
  (38 observations deleted due to missingness)
Multiple R-squared:  0.3752,Adjusted R-squared:  0.3734 
F-statistic: 205.4 on 1 and 342 DF,  p-value: < 2.2e-16
```

R^2: intuition

How much better are the predictions from our OLS regression line than the predictions from a flat line (i.e. not using X at all)?



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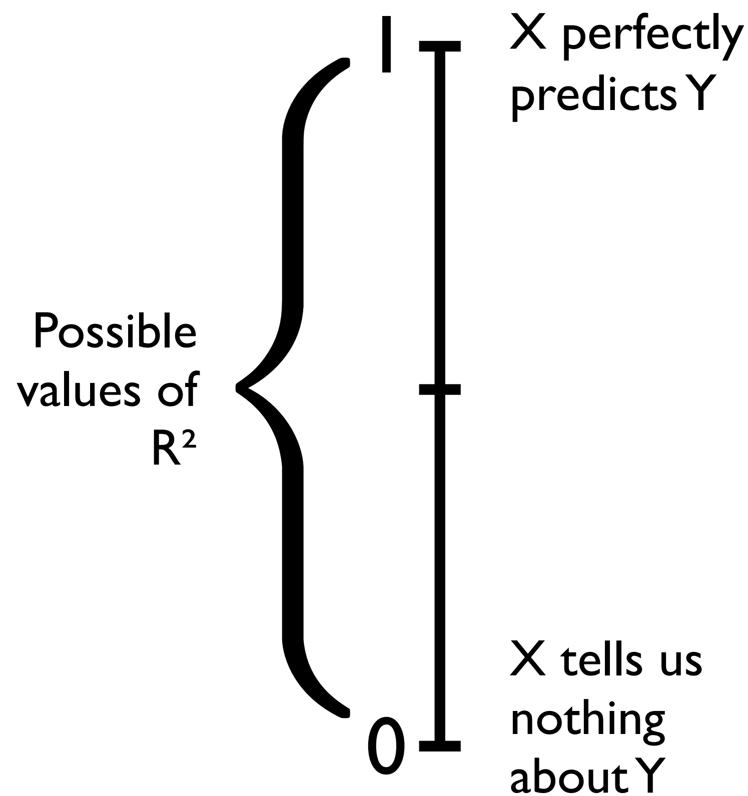
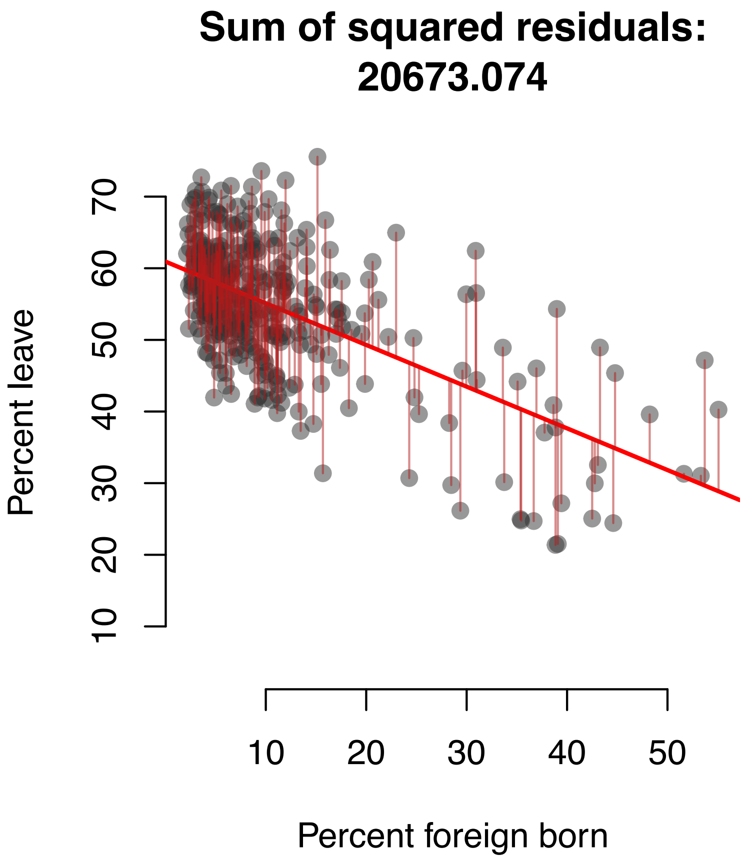
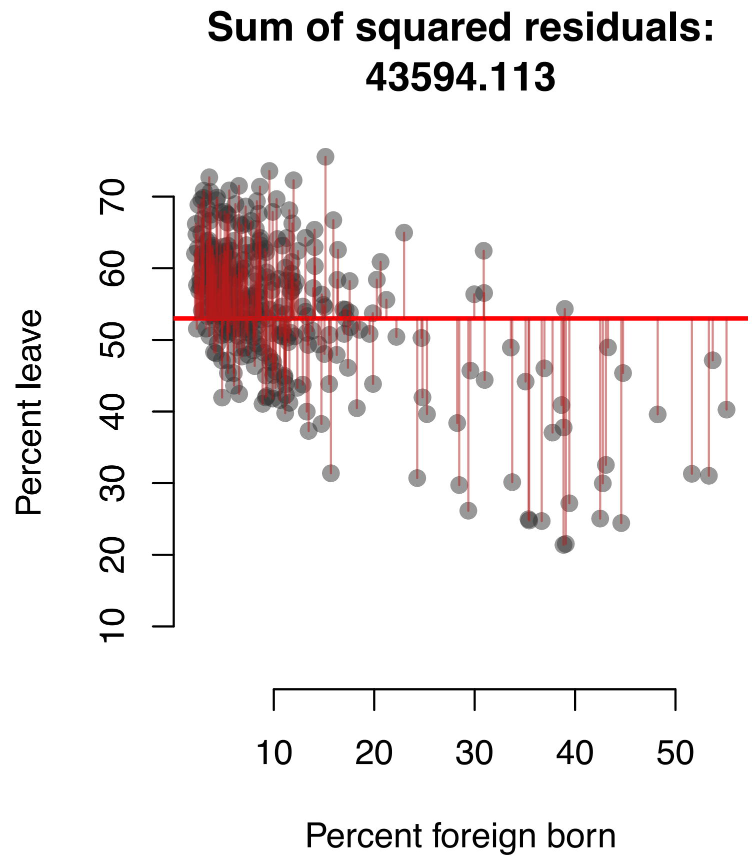


R^2: calculation






```
> 1 - (20673.074/33087.482)
[1] 0.3751995
```

Connections between measures of bivariate relationships

Key measures:

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- OLS regression output:
  - intercept
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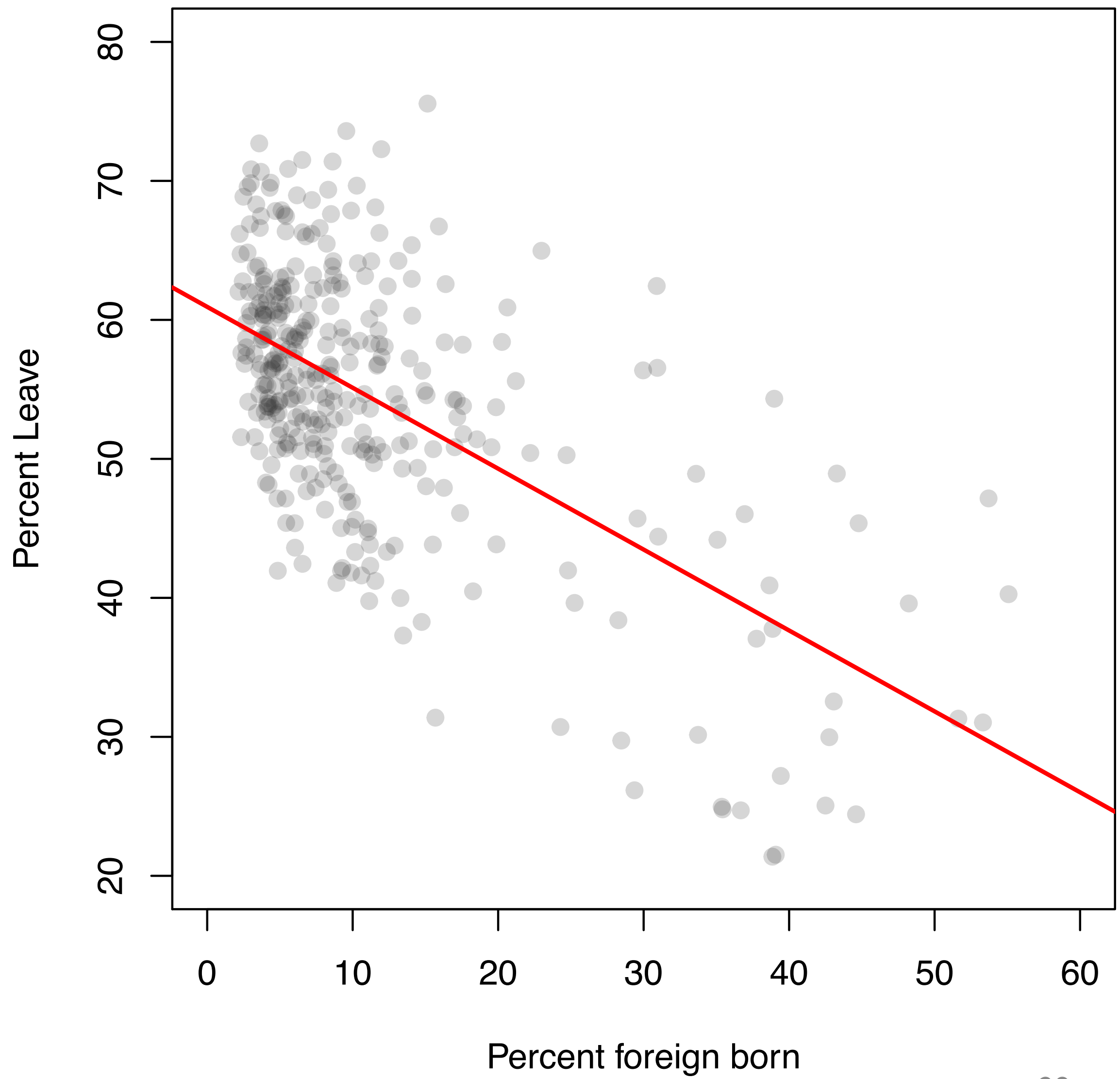
Regression slope (but not covariance or correlation) depends on which is Y and which is X

Covariance and regression slope (but not correlation) depend on the units

To discuss

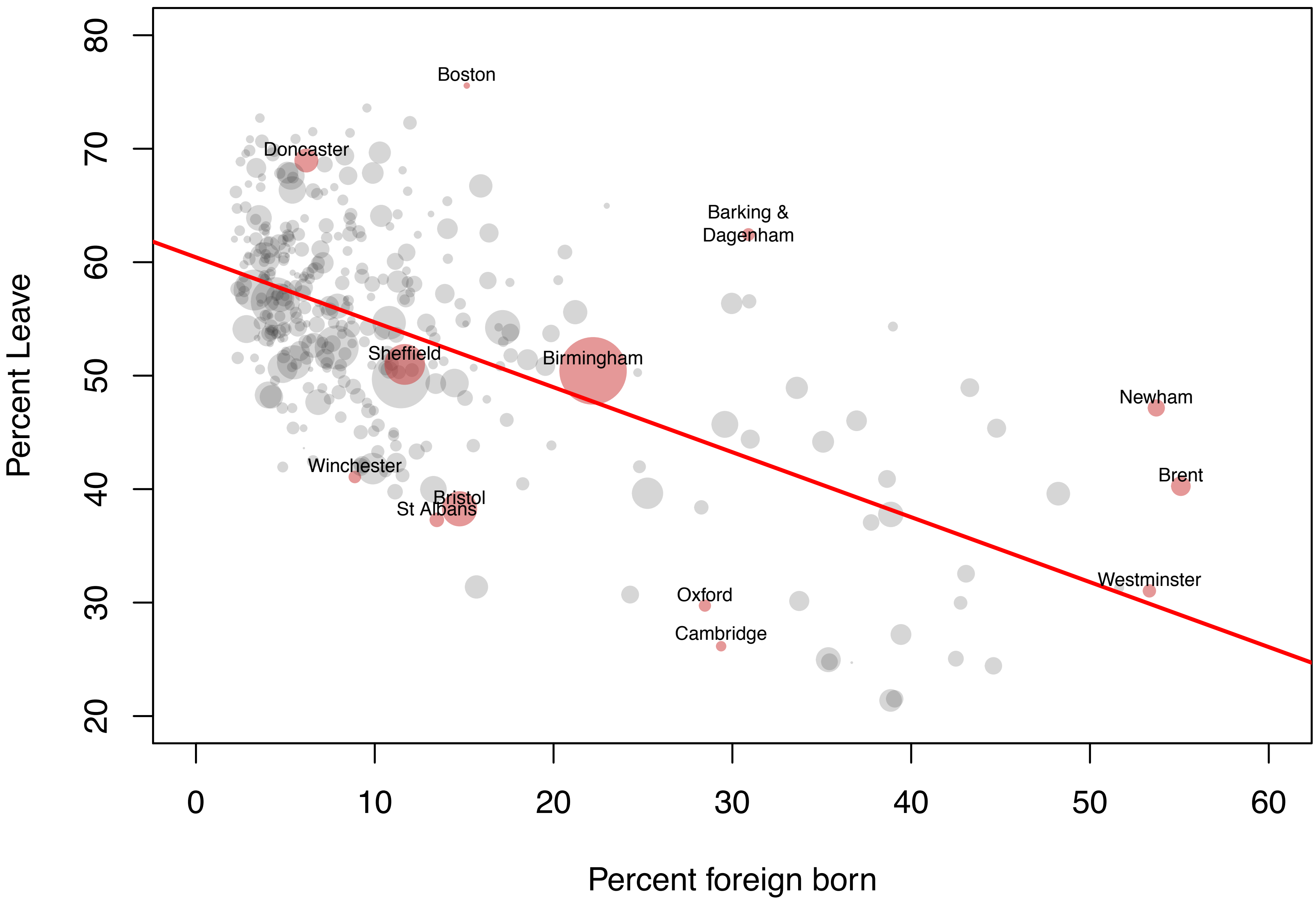
What other options could you imagine for deciding on a predictive line?

What are the advantages of OLS?



Summarizing multivariate relationships: motivation and one non-OLS solution

Did this pattern arise because contact with immigrants makes people less opposed to immigration?



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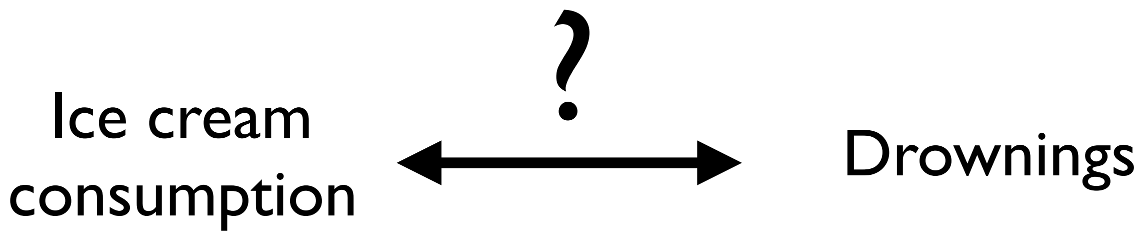

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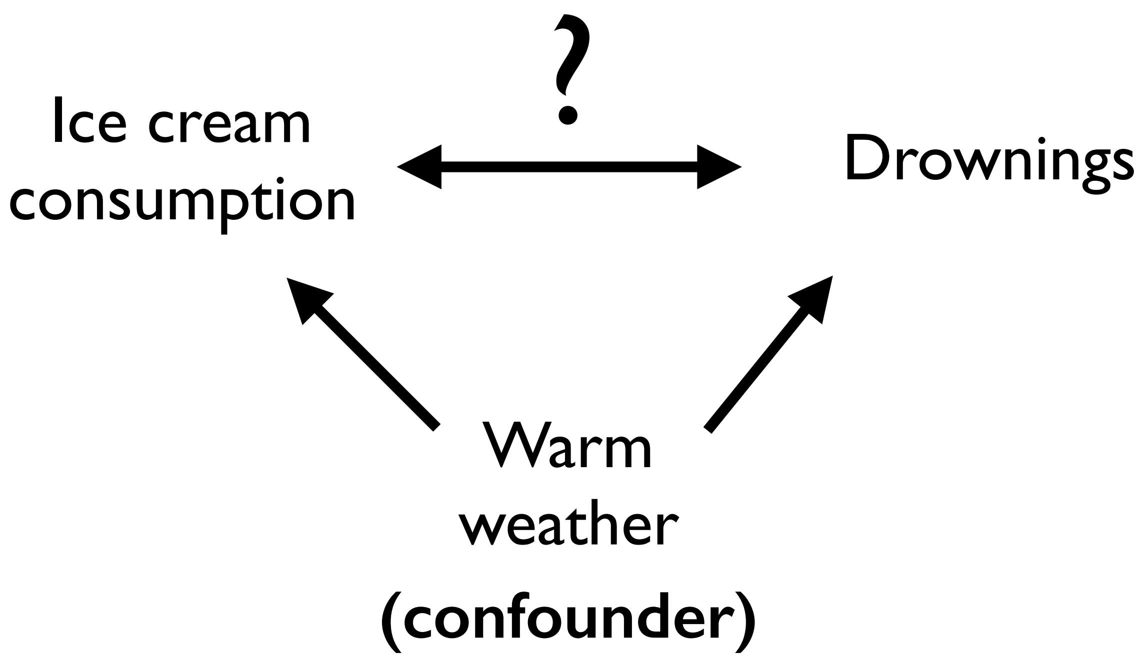

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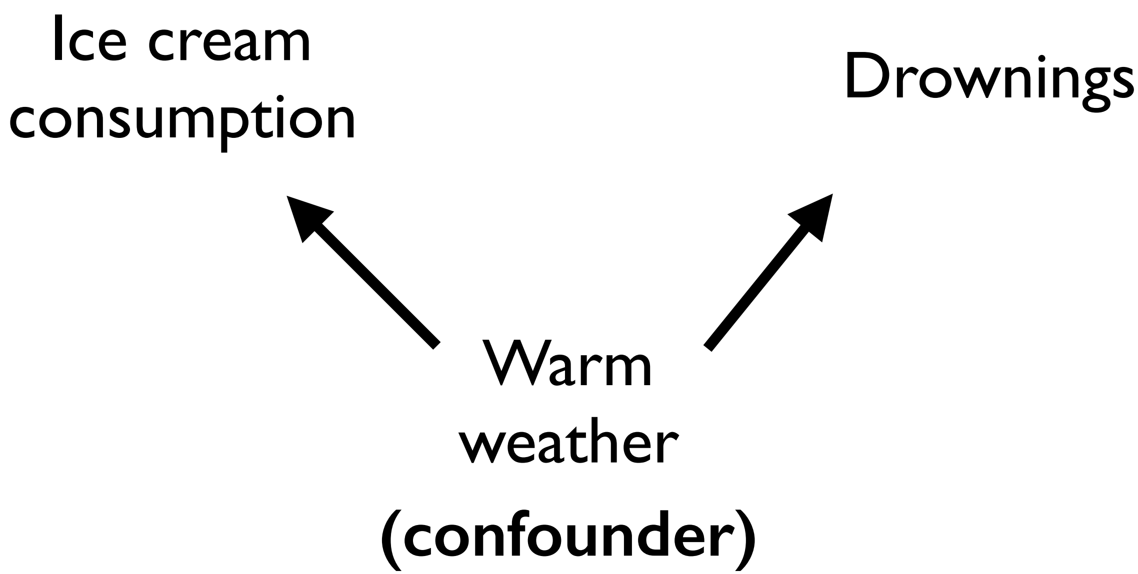

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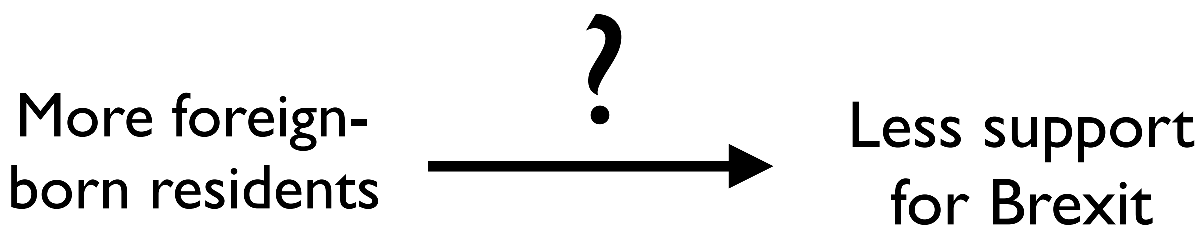

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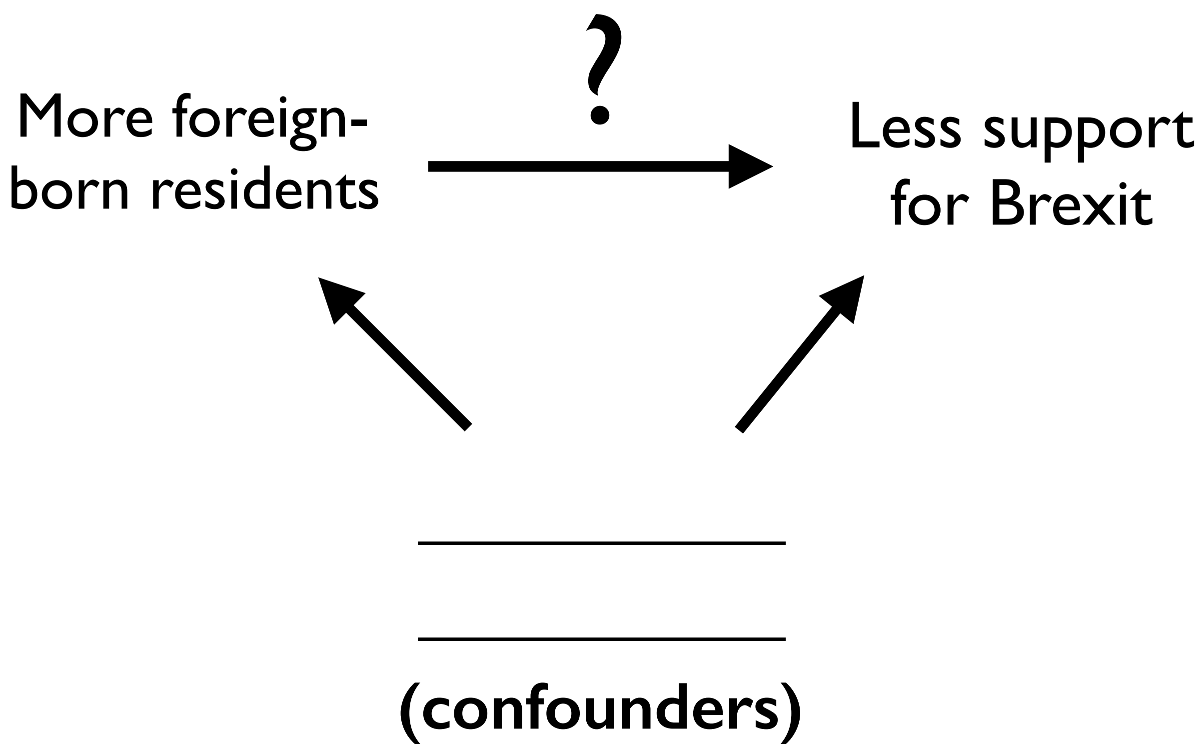

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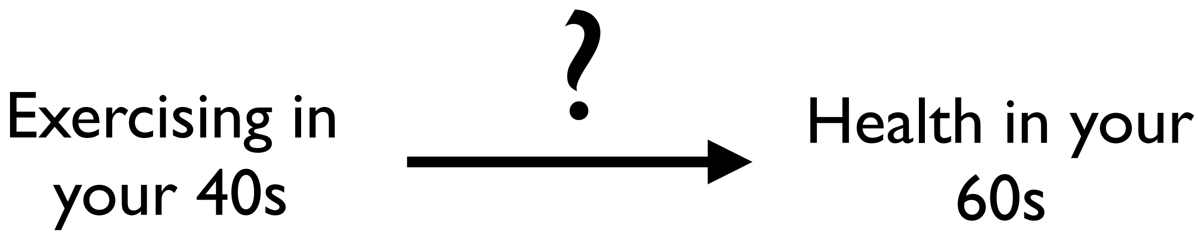

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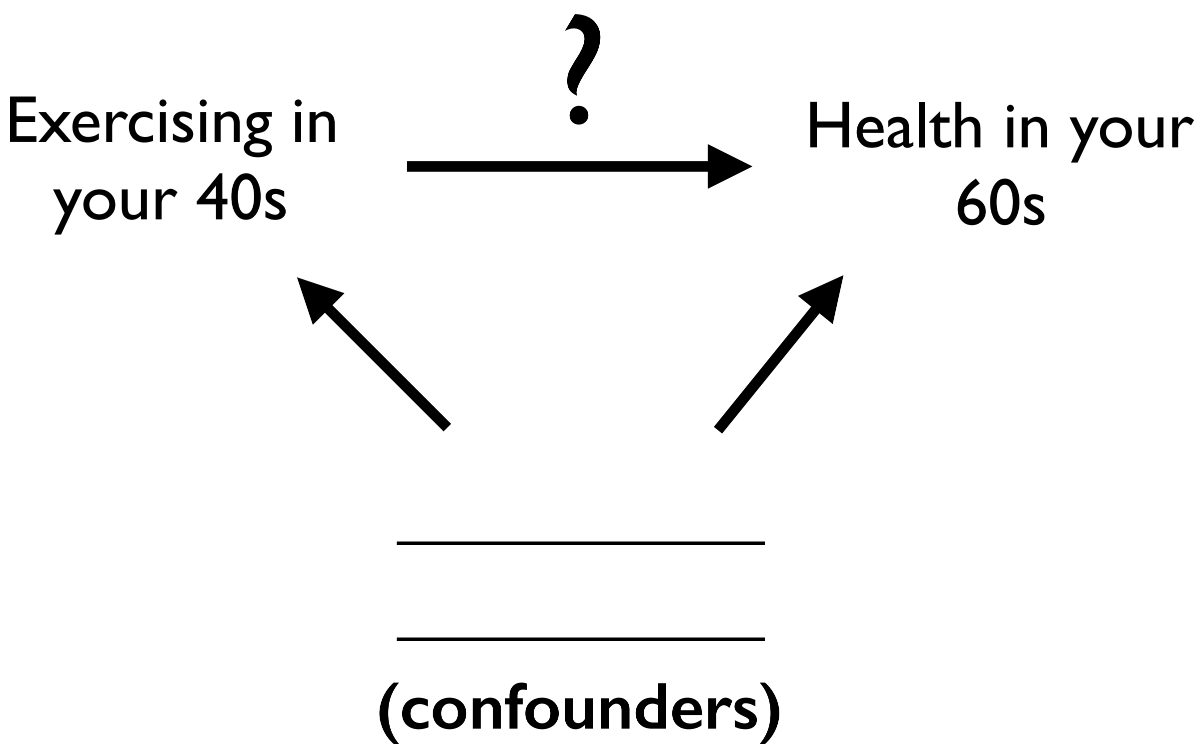

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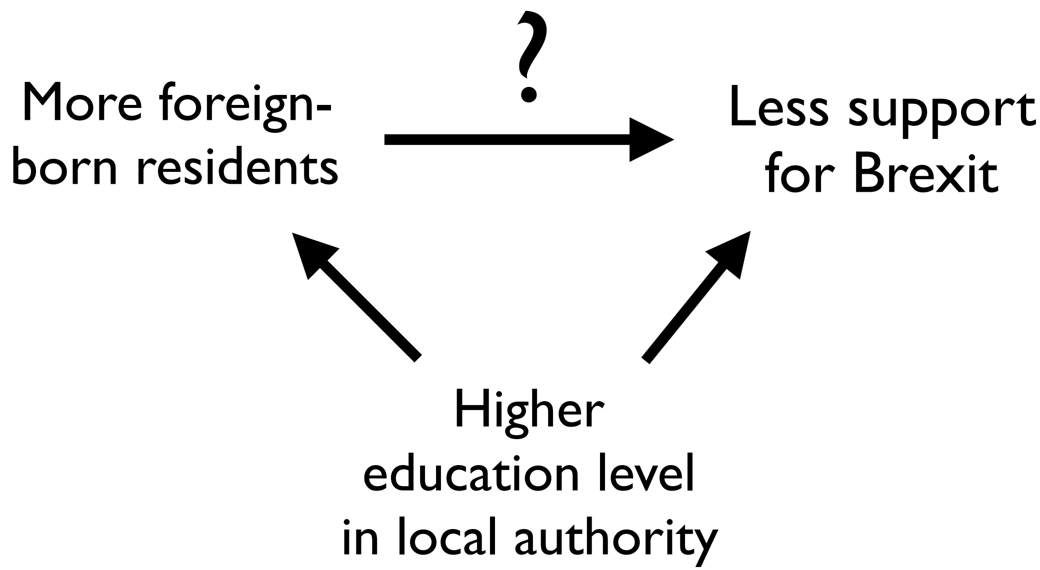
- Are countries with more inclusive political systems less likely to experience violence, controlling for economic development and the number of ethnic groups?
- Are local authorities with more foreign-born residents less likely to support Brexit, controlling for __________?

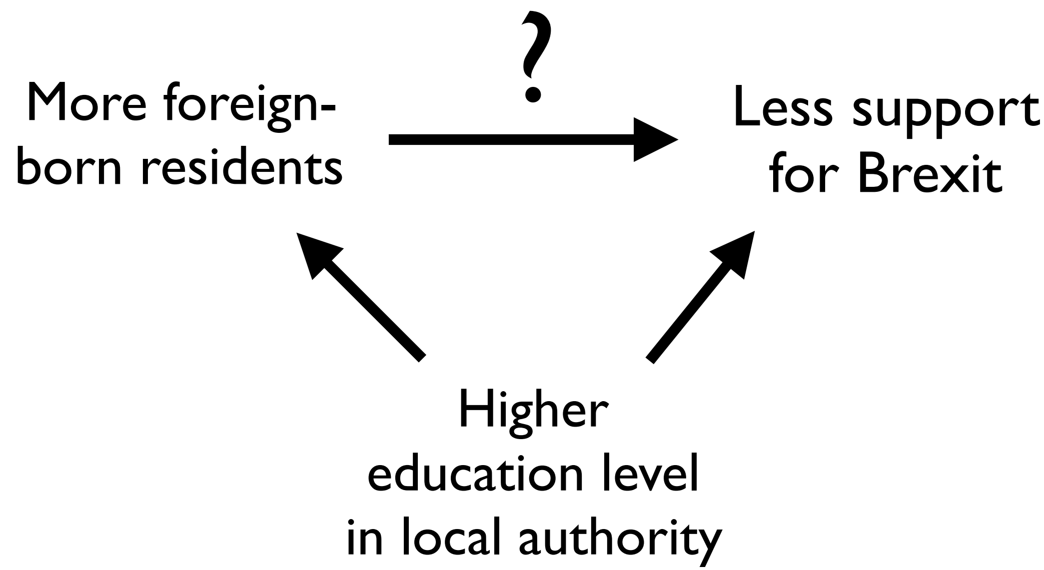
How do we control for confounders?

Let’s focus on education as a confounder in our Brexit example:



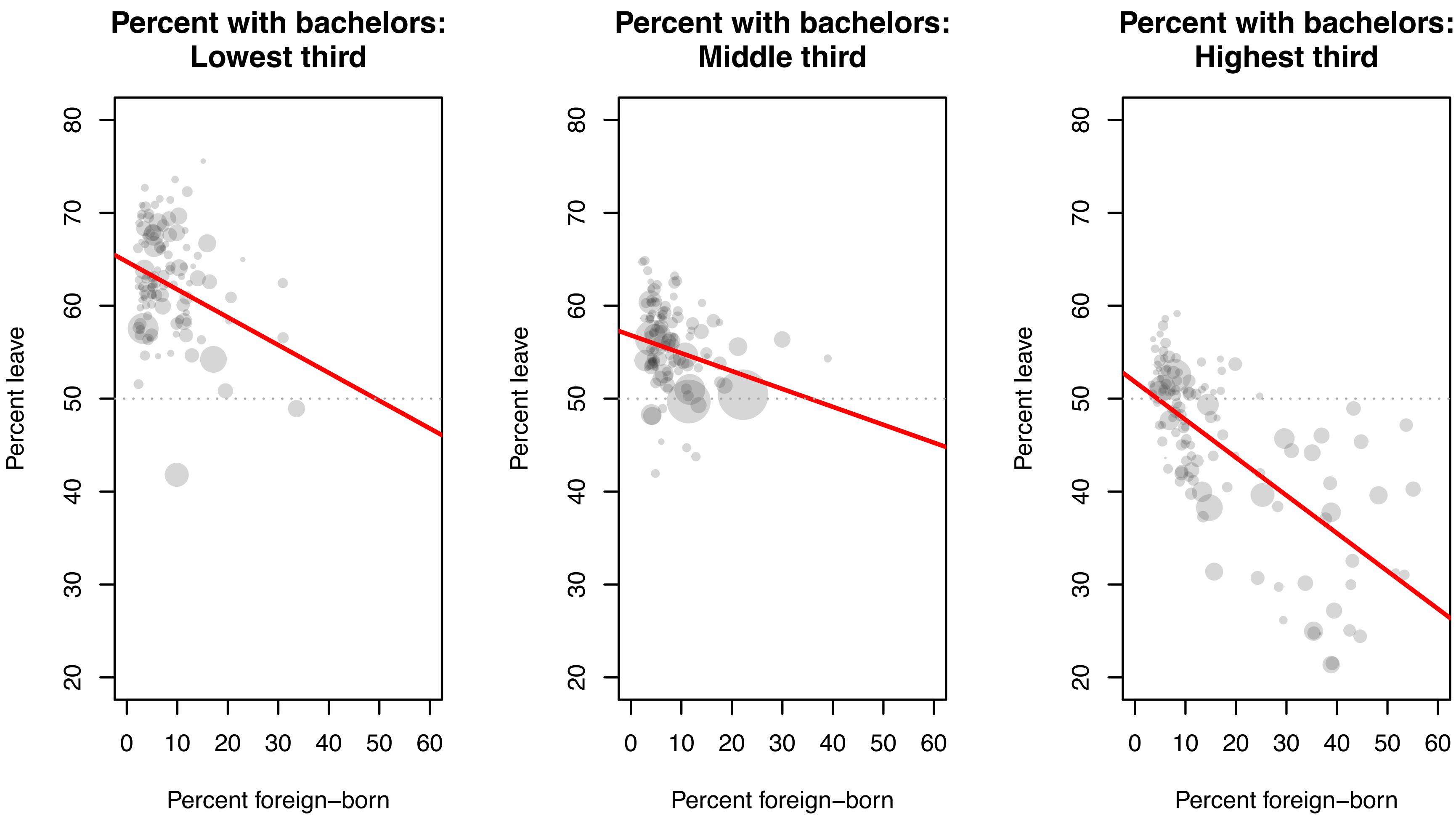
How do we control for confounders?

Let's focus on education as a confounder in our Brexit example:

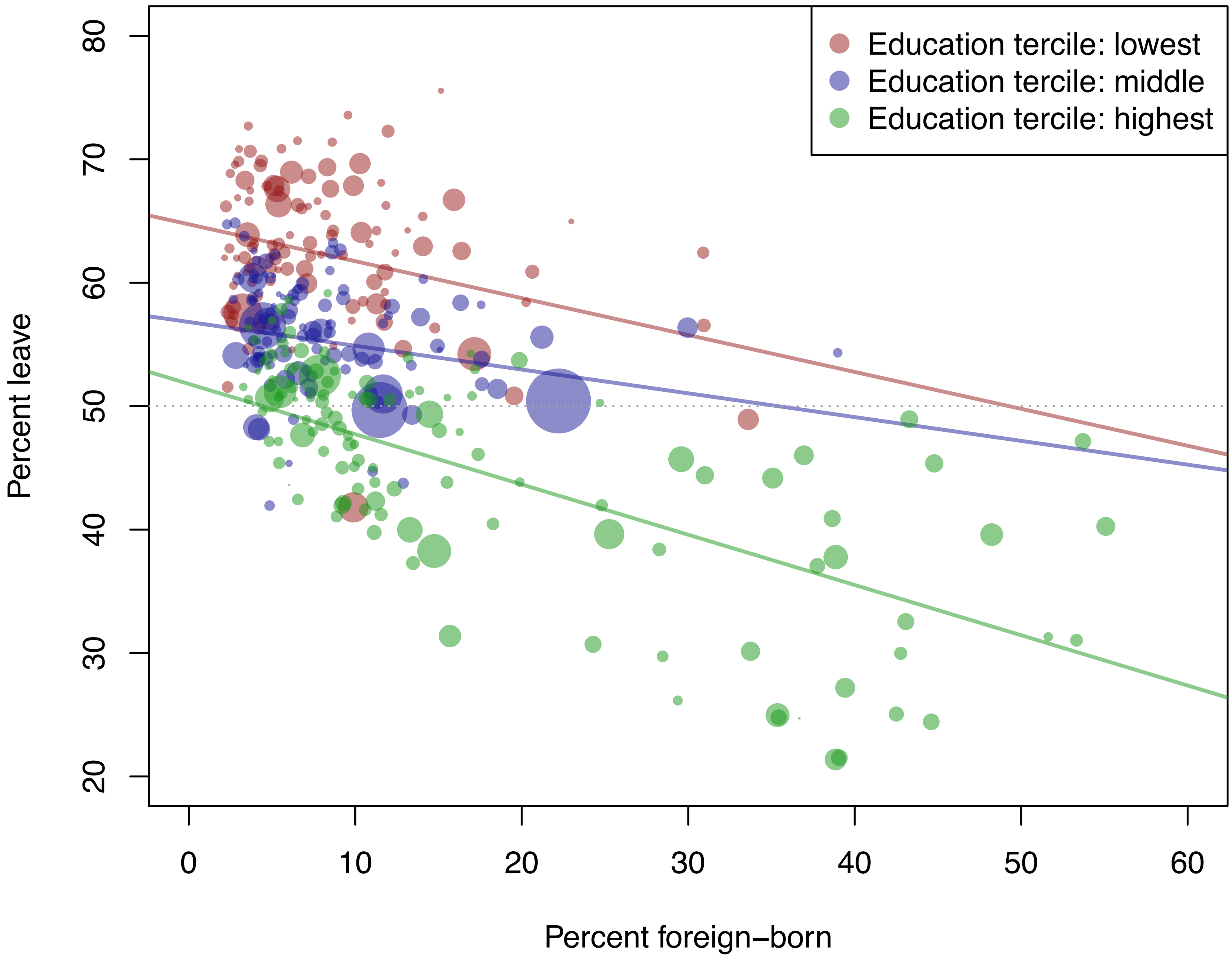


How can we measure the relationship between a local authority's proportion of foreign-born residents and its support for Brexit, controlling for its education level?

One idea: stratify by education level



Same thing in one plot



Summarizing multivariate relationships:
multivariate regression

A more general approach: multivariate regression

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Goal: measure relationship between
- “support for Leave” and
- “% foreign-born”

controlling for “% bachelors degree”.

A more general approach: multivariate regression

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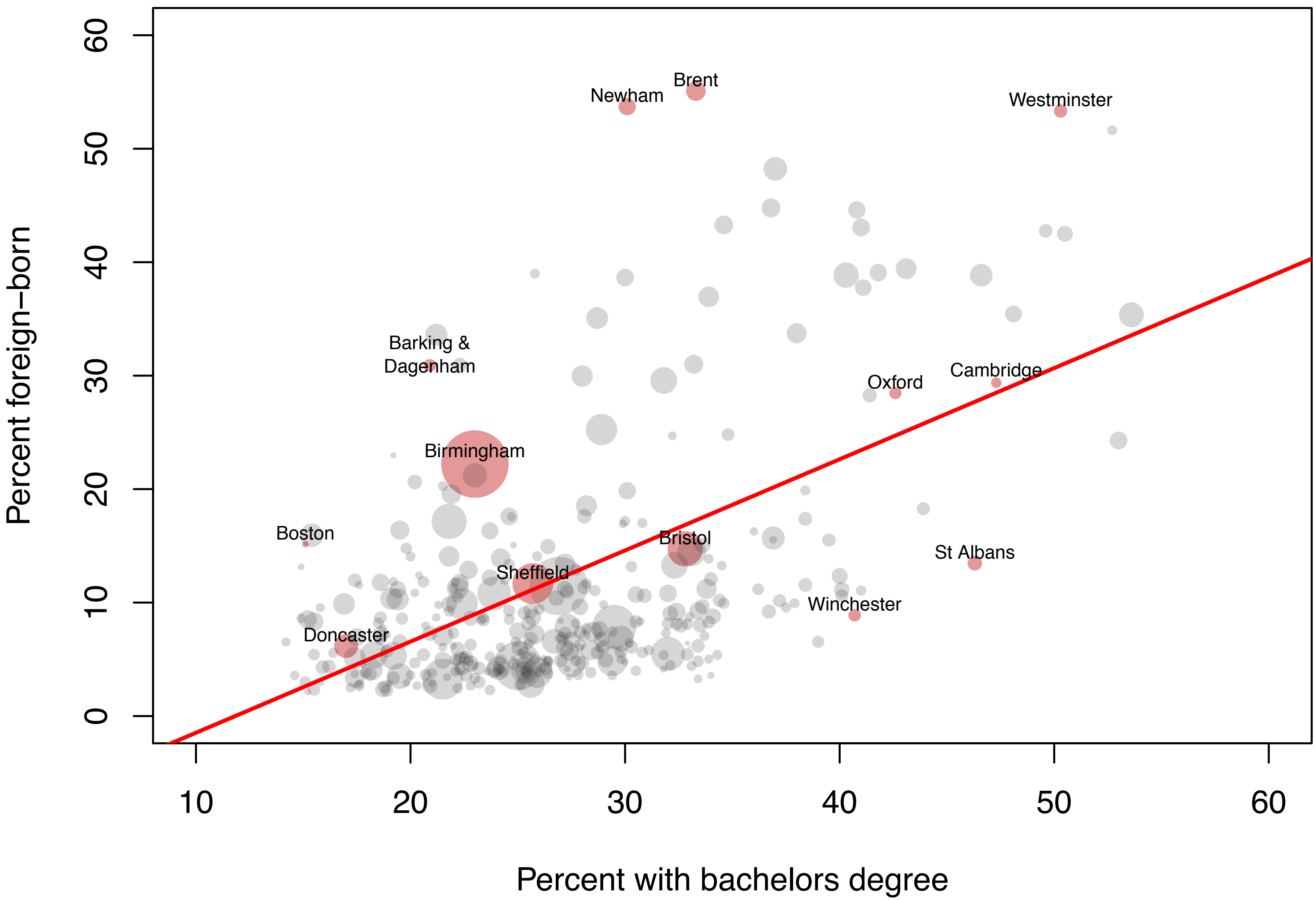
- "support for Leave" and
- "% foreign-born"

controlling for "% bachelors degree".

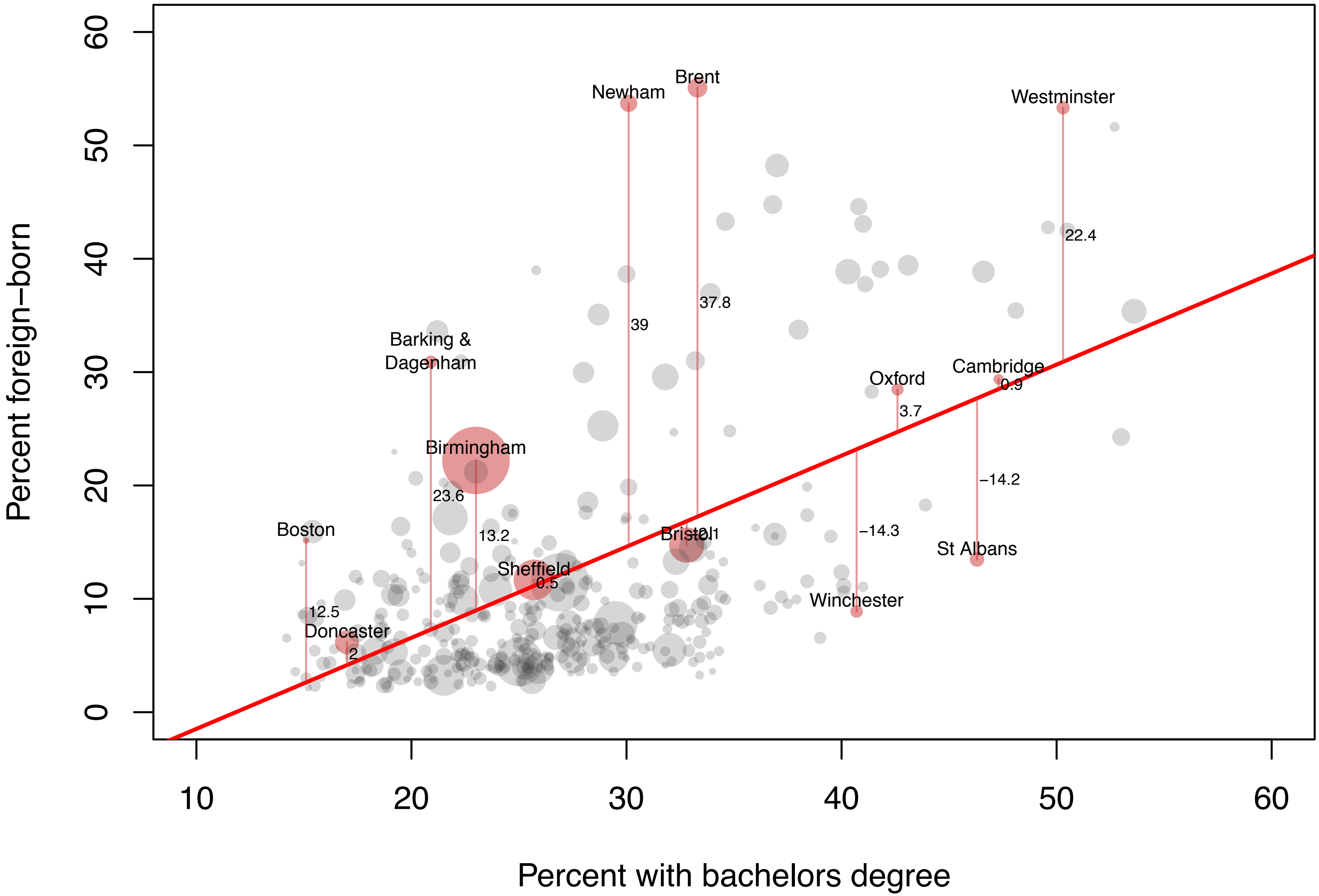
Basic idea: measure relationship between

- "support for Leave" and
- the part of "% foreign-born" that is not explained by "% bachelors degree"

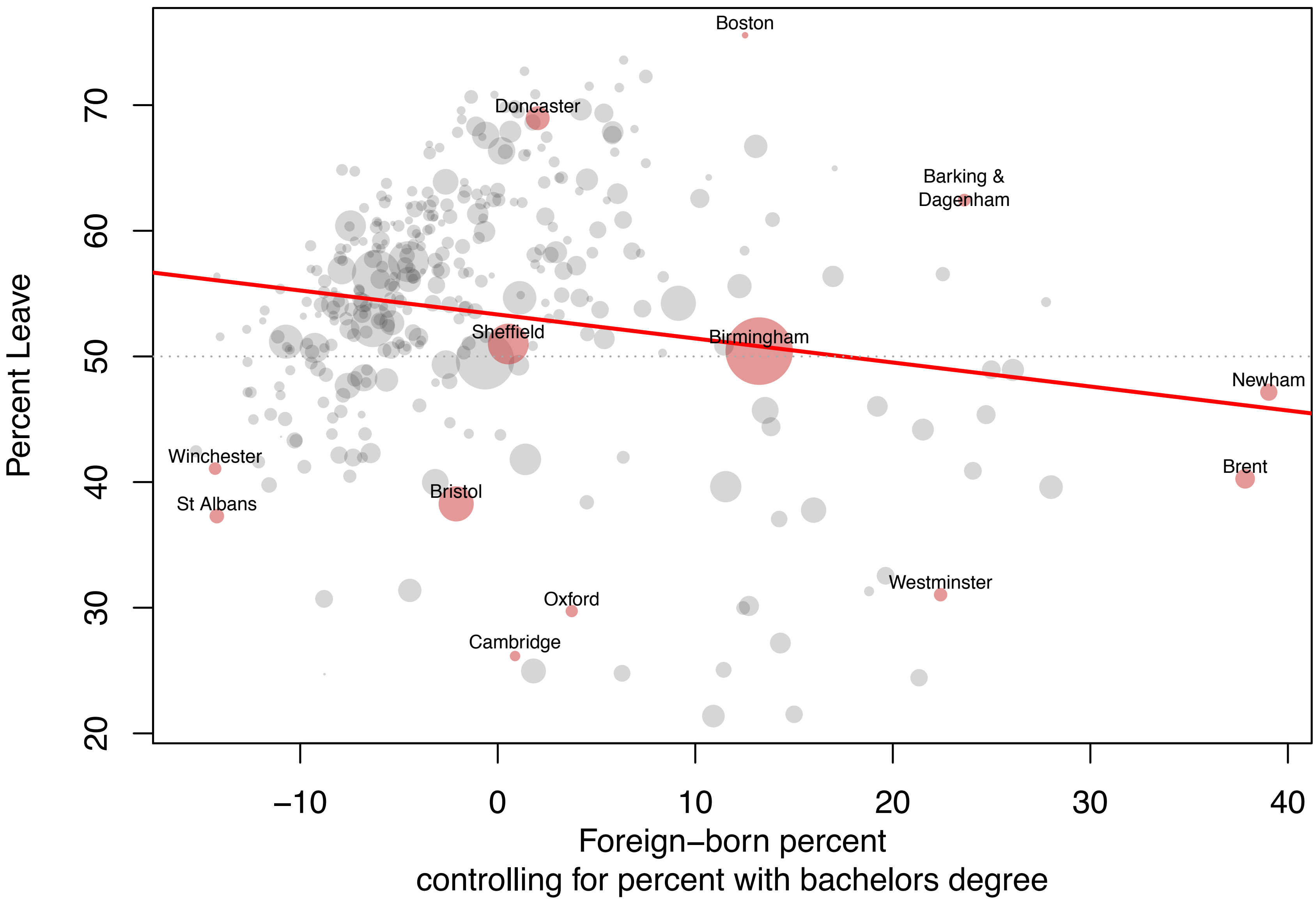
Step 1: regress explanatory variable (%foreign-born) on confounder (education)



Step 2: calculate residuals, i.e. the part of %foreign-born not "explained" by education



Step 3: regress outcome (%leave) on those residuals



Group activity

Group activity

Using the data sheet on the handout, find each group member's local authority (or another if we've already highlighted yours!) on Figures 1, 2, and 3.

1. Was your local authority more supportive of Brexit than would be expected given its % foreign born? or less?
2. Does your local authority have a higher % foreign born than would be expected given its % with bachelors? or lower?
3. Was your local authority more supportive of Brexit than would be expected given its % foreign born, controlling for its % with bachelors? or less?

Two ways to get the same answer

```
> lm(Percent_Leave ~ Percent_foreign_born + Bachelors_deg_percent, data = d)

Call:
lm(formula = Percent_Leave ~ Percent_foreign_born + Bachelors_deg_percent,
    data = d)

Coefficients:
          (Intercept)   Percent_foreign_born  Bachelors_deg_percent
              83.1386                -0.1742                -0.9875
```

```
> d$resids = resid(lm(Percent_foreign_born ~ Bachelors_deg_percent, data = d,
na.action = "na.exclude"))
> lm(Percent_Leave ~ resids, data = d)

Call:
lm(formula = Percent_Leave ~ resids, data = d)

Coefficients:
(Intercept)       resids
    54.4209      -0.1742
```

The usual way to think about multivariate regression

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Bivariate:

$\text{PercentLeave} = \beta_0 + \beta_1 \text{PercentForeignBorn}$

The usual way to think about multivariate regression

Above we showed how to interpret and estimate the regression coefficient on one variable controlling for the other.

Can also just think about minimizing sum of squared residuals for a different prediction equation.

Bivariate:

PercentLeave = β_0 + β_1PercentForeignBorn

Multivariate:

PercentLeave = β_0 + β_1PercentForeignBorn + β_2Education

Implementing multivariate regression

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Some options:

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Implementing multivariate regression

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```
> lm(d$Percent_Leave ~ d$Percent_foreign_born + d$Bachelors_deg_percent)

Call:
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Coefficients:
            (Intercept)   d$Percent_foreign_born  d$Bachelors_deg_percent
                83.1386                  -0.1742                  -0.9875
```

Presenting and interpreting results

	Dependent variable:		
	Percent supporting Leave		
	(1)	(2)	(3)
Percent foreign born	−0.582***	−0.174***	0.013
	(0.041)	(0.027)	(0.038)
Percent w. bachelors degree		−0.988***	−1.066***
		(0.035)	(0.035)
Mean age			0.755***
			(0.115)
Constant	60.937***	83.139***	52.808***
	(0.618)	(0.857)	(4.701)
Observations	344	344	344
R^2	0.375	0.813	0.834
Note:			*p<0.1; **p<0.05; ***p<0.01

Using “stargazer” package to present results

```
reg.1 = lm(Percent_Leave ~ Percent_foreign_born, data = d)
reg.2 = lm(Percent_Leave ~ Percent_foreign_born + Bachelors_deg_percent, data = d)
reg.3 = lm(Percent_Leave ~ Percent_foreign_born + Bachelors_deg_percent + age_mean, data = d)
```

```
stargazer(reg.1, reg.2, reg.3, dep.var.labels = "Percent supporting Leave", covariate.labels =
c("Percent foreign born", "Percent w. bachelors degree", "Mean age"), out = "brexit.html")
```

Using "stargazer" package to present results

To include table in Word document:

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4. Open html file in browser (explorer/safari, not chrome), copy and paste table into word document

Inference

i.e. making claims beyond your sample

Sample vs population (1)

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Say you have the data from a May 2016 survey asking how respondents plan to vote in the referendum and whether they went to university or not.

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1. How much support is there for Brexit?

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Say you have the data from a May 2016 survey asking how respondents plan to vote in the referendum and whether they went to university or not.

Your research questions are:

1. How much support is there for Brexit?
2. How is support for Brexit related to education?

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How would you answer these questions?

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1. How much support is there for Brexit?
2. How is support for Brexit related to education?

How would you answer these questions?

Is there any **uncertainty** in your answers?

Sample vs population (2)

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Is there uncertainty? Depends on who you are asking about.

Sample vs population (2)

Is there uncertainty? Depends on who you are asking about.

1. How much support is there for Brexit *among respondents to this survey*?
2. How is support for Brexit related to education *among respondents to this survey*?

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(About the sample)	(About the population)
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(About the sample)

(About the population)

No real uncertainty.

(Maybe about measurement.)

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(About the sample)

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(Maybe about measurement.)

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(About the population)

Uncertainty due to **sampling variation**.

Sample vs population (3)

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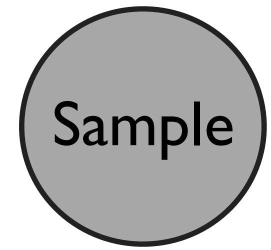

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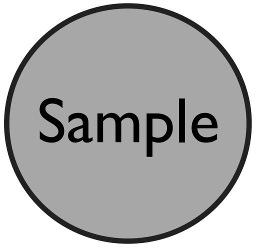

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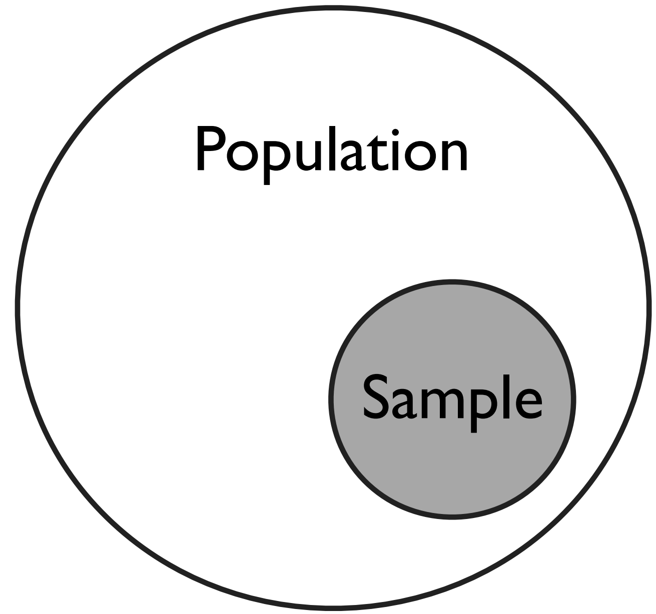

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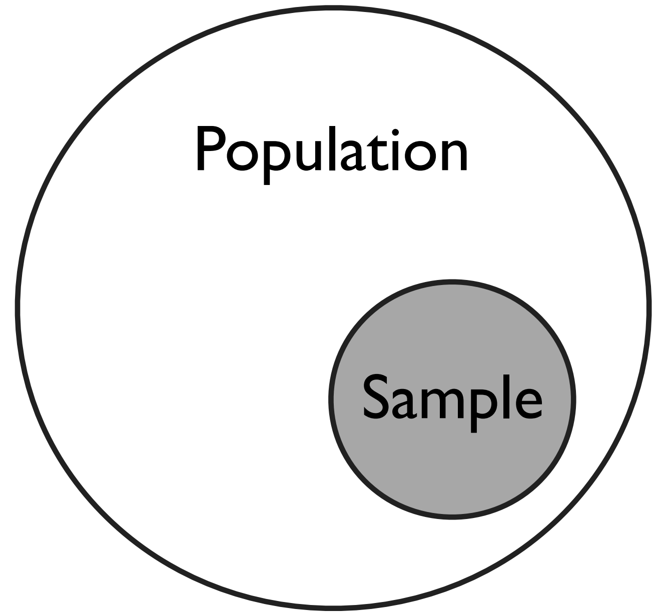

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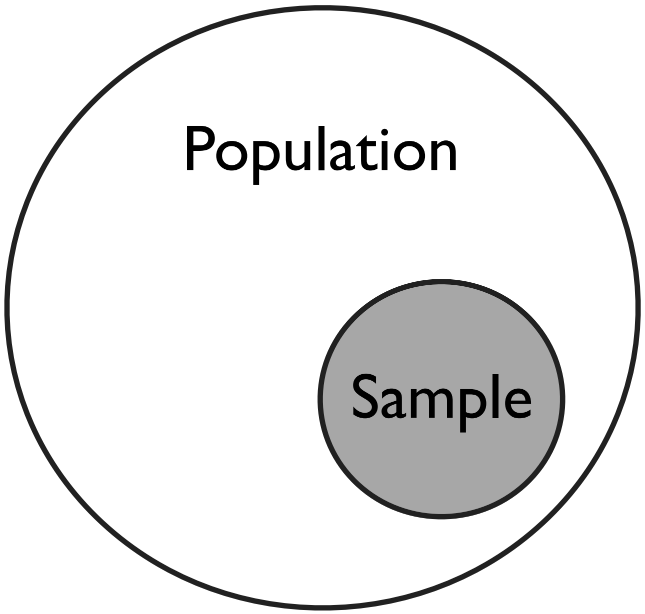

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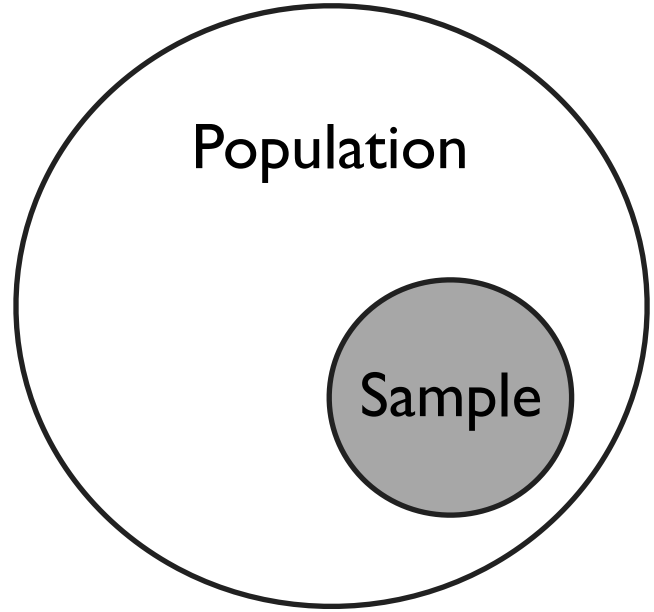

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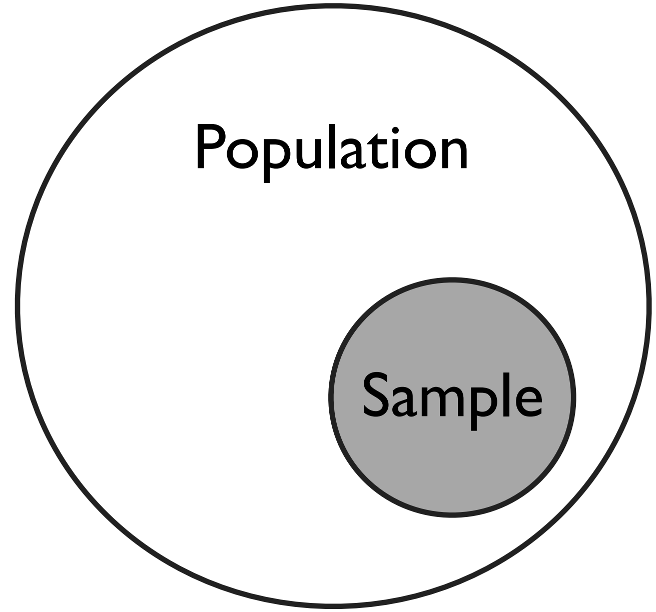

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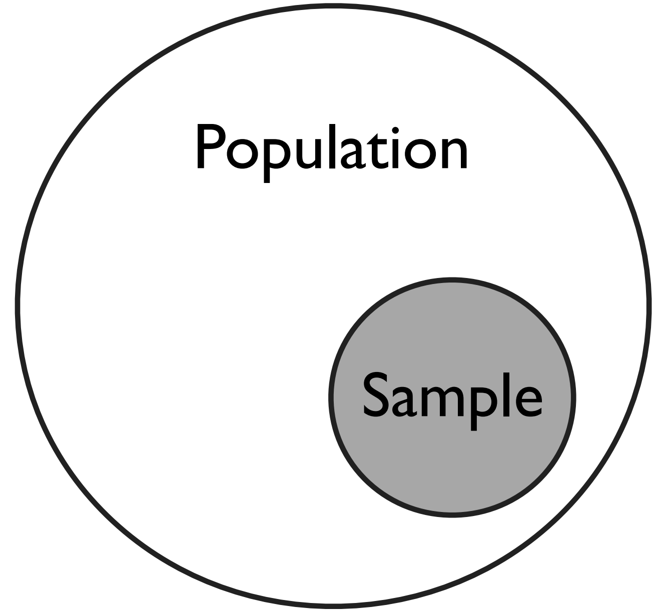

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- regression coefficients have **standard errors**
- our conclusions in hypothesis testing are guesses, with confidence summarized by **p-values**

Thought experiment

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Why would the level of support in your sample differ from the true value? By how much?

What would the magnitude of this **random error** depend on?

- size of sample (1,006 GB adults vs. 10,000,000)
- true level of support (what if 100% supported remaining in EU?)

Simulating the thought experiment in R

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I can increase the number of "respondents" to 1,006:

```
> sample(x = c(0,1), size = 1006, replace = T, prob = c(.48, .52))
   [1] 0 0 0 0 0 0 0 1 1 1 1 1 0 1 1 1 1 1 1 1 1 0 0 1 1 0 0 0 0 0 0 0 1
  [39] 0 0 0 1 1 1 1 0 0 0 1 0 0 1 0 1 1 1 0 0 0 1 1 0 0 0 0 0 0 0 0 0 1
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Simulating the thought experiment (2)

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I can store the sample and take the mean:

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> samp = sample(x = c(0,1), size = 1006, replace = T, prob = c(.48, .52))
> mean(samp)
[1] 0.5318091
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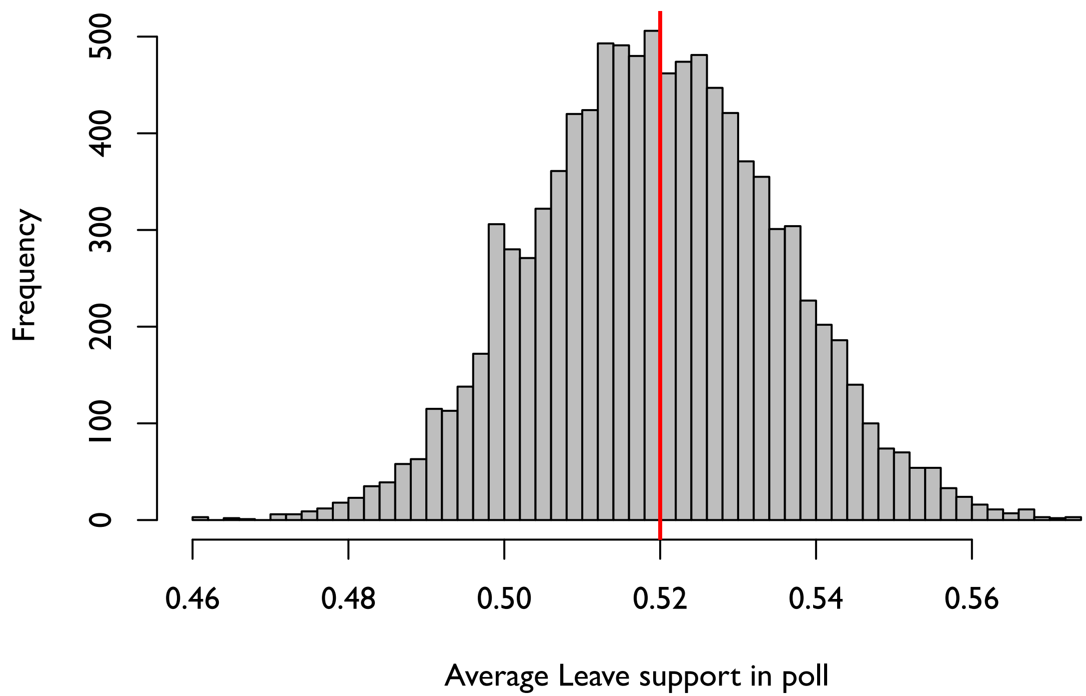
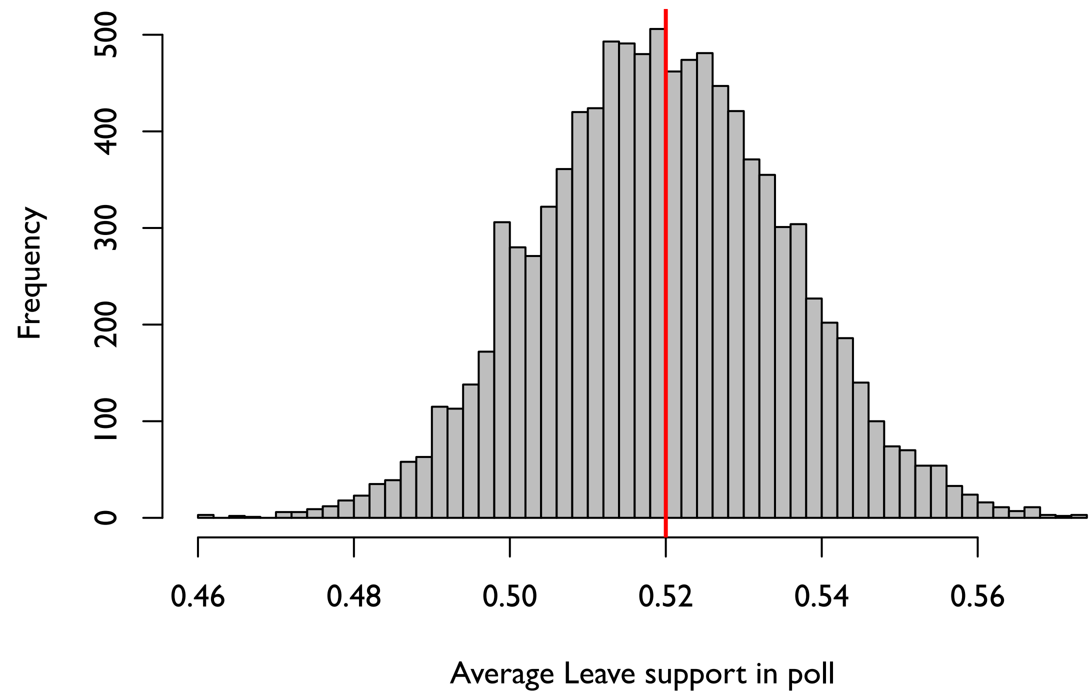
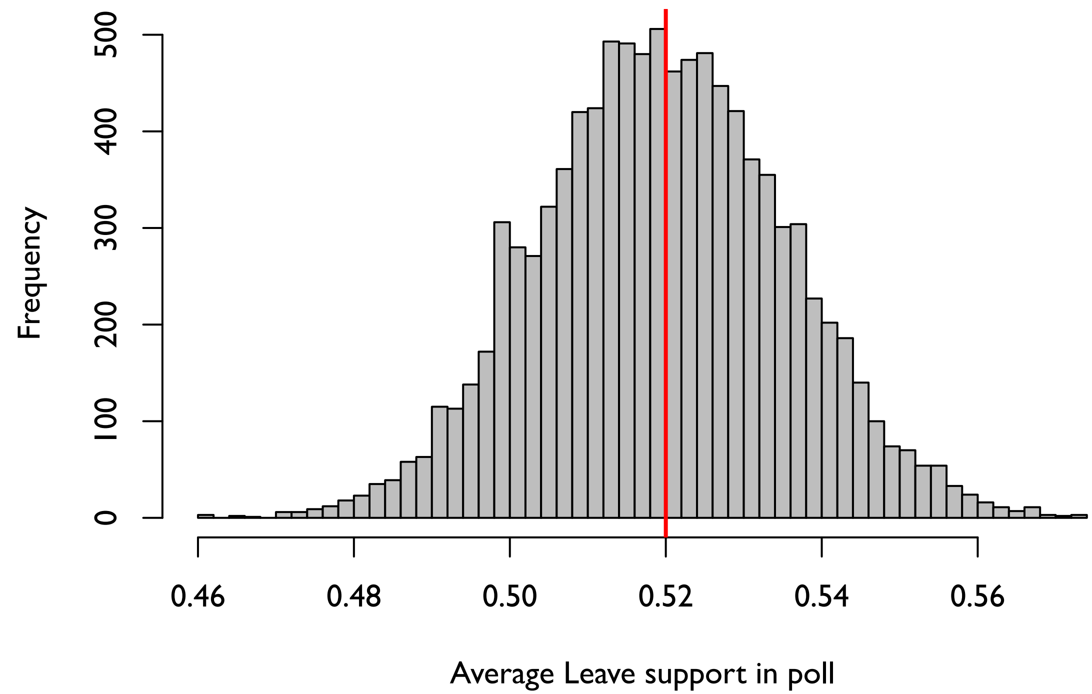


Simulating the thought experiment (3)



Simulating the thought experiment (3)

The results vary across our 10,000 "surveys" because of **sampling error**.



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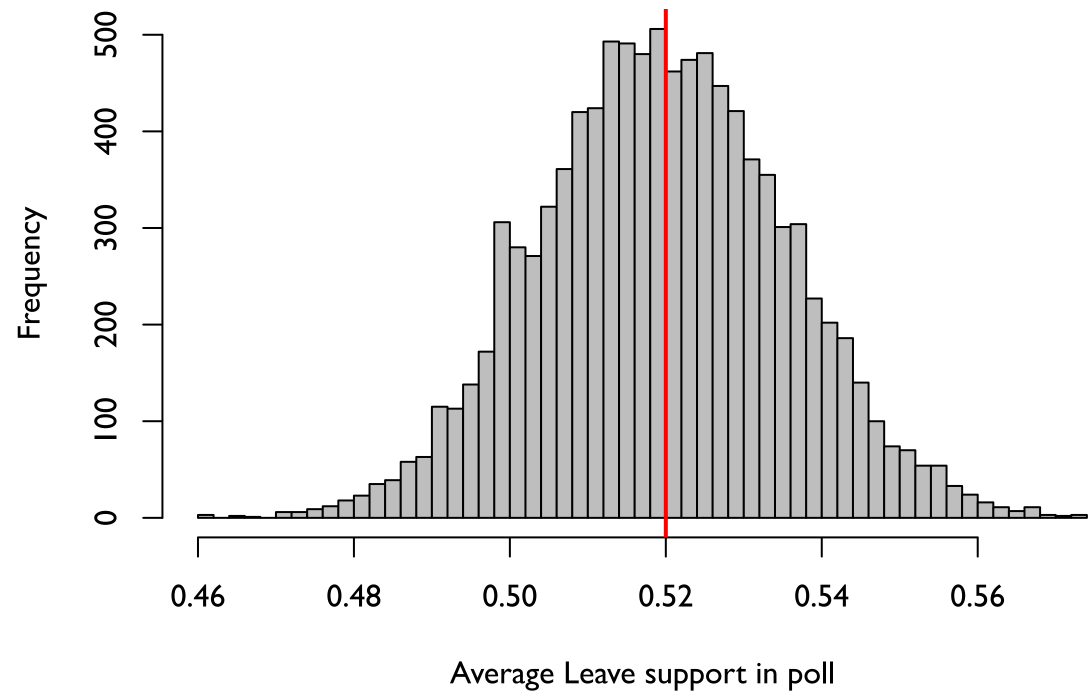
How much sampling error is there in our simulation?



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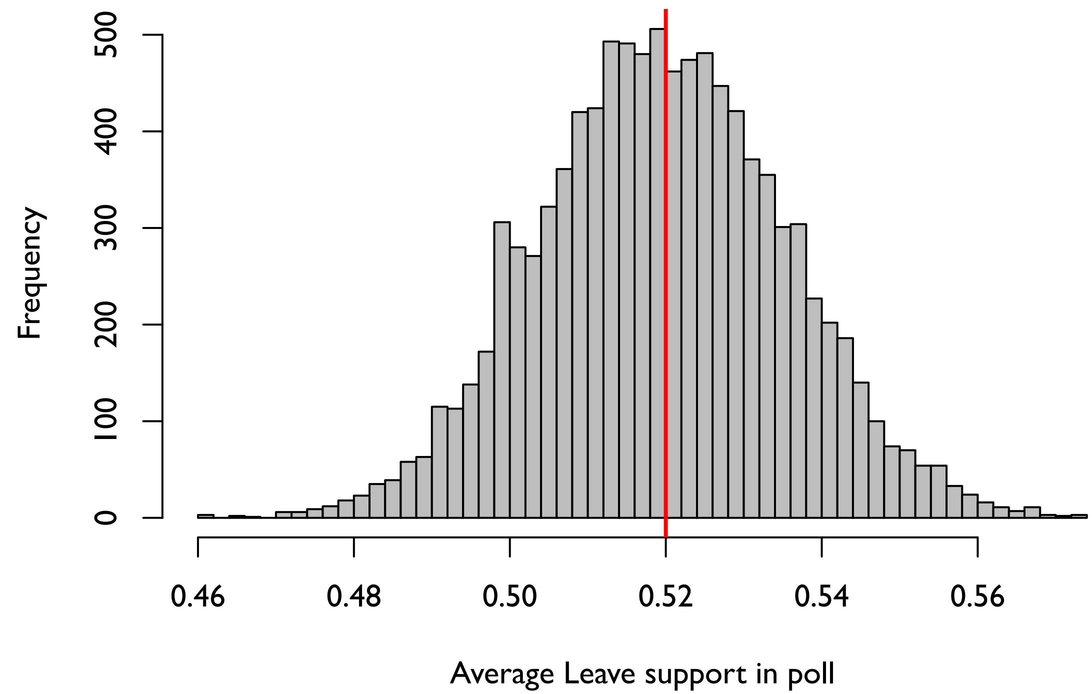

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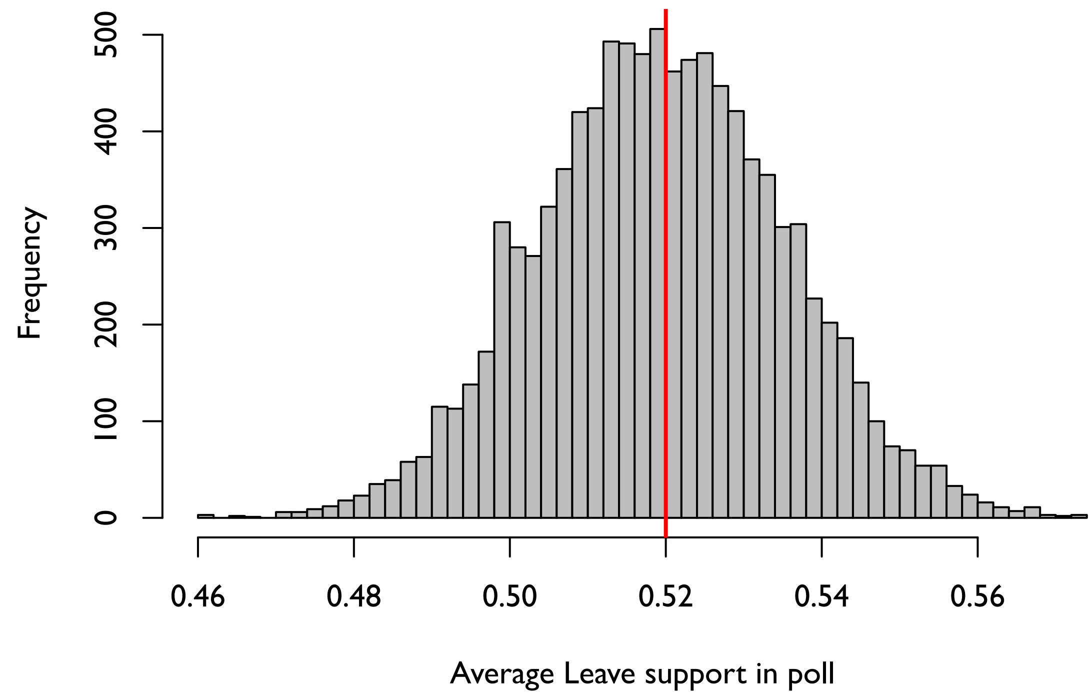

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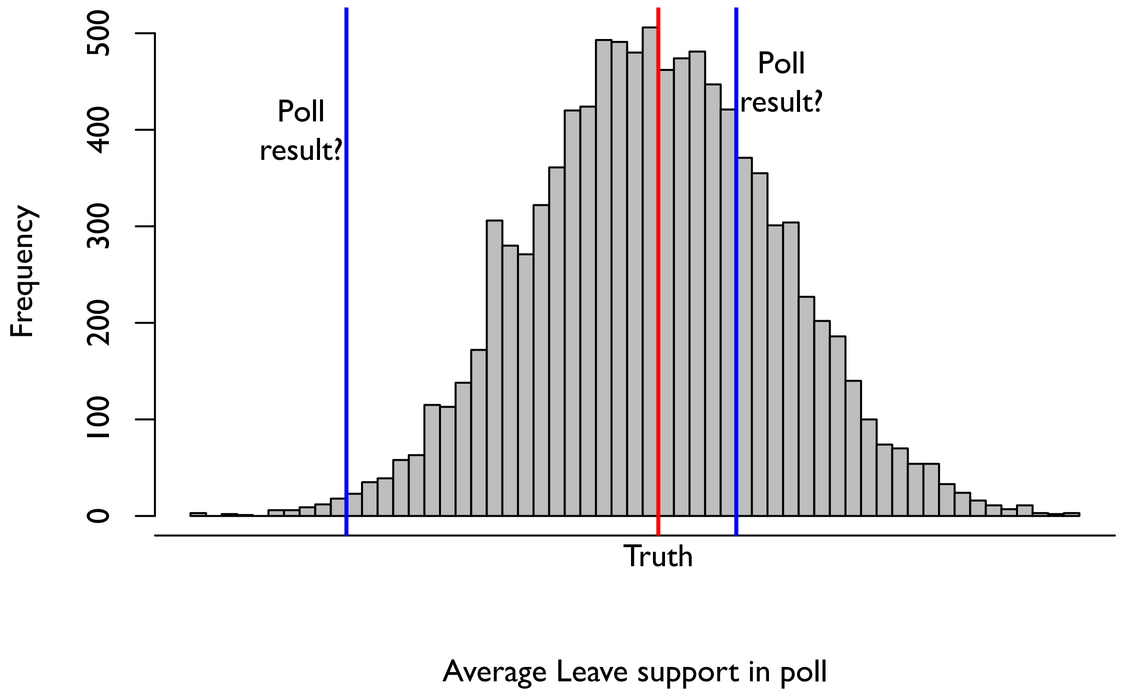
95% of the samples had a mean between 0.49 and 0.55:

```
> quantile(poll.results, c(.025, .975))
     2.5%     97.5%
0.4890656 0.5497018
```

From thought experiment to margin of error



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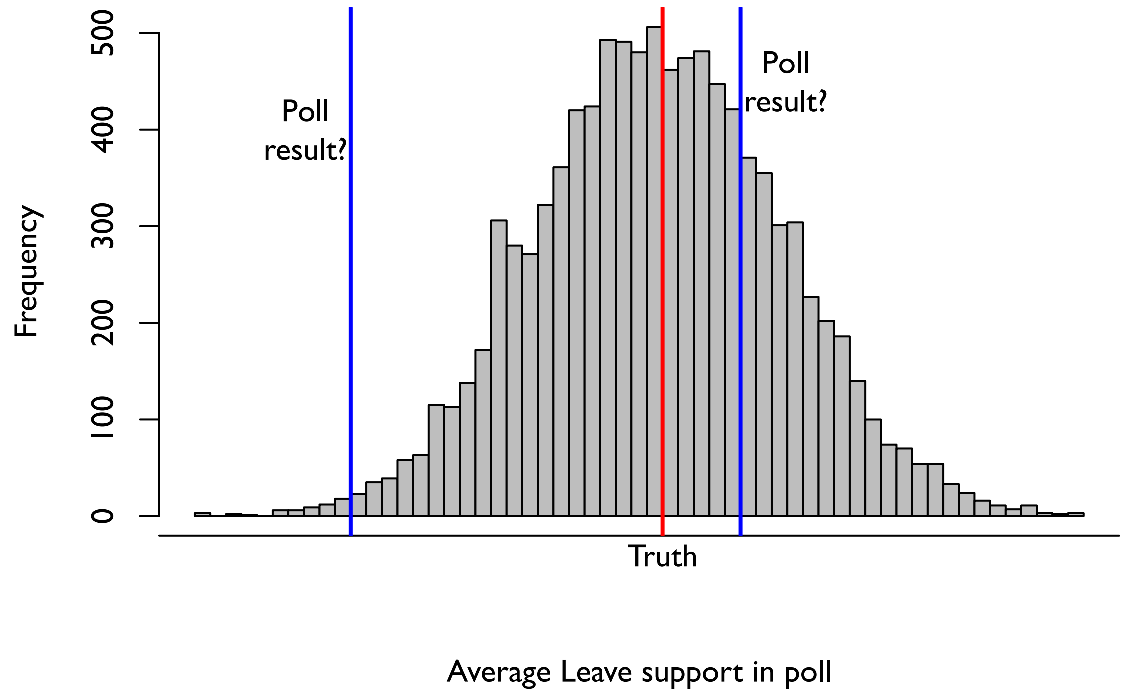

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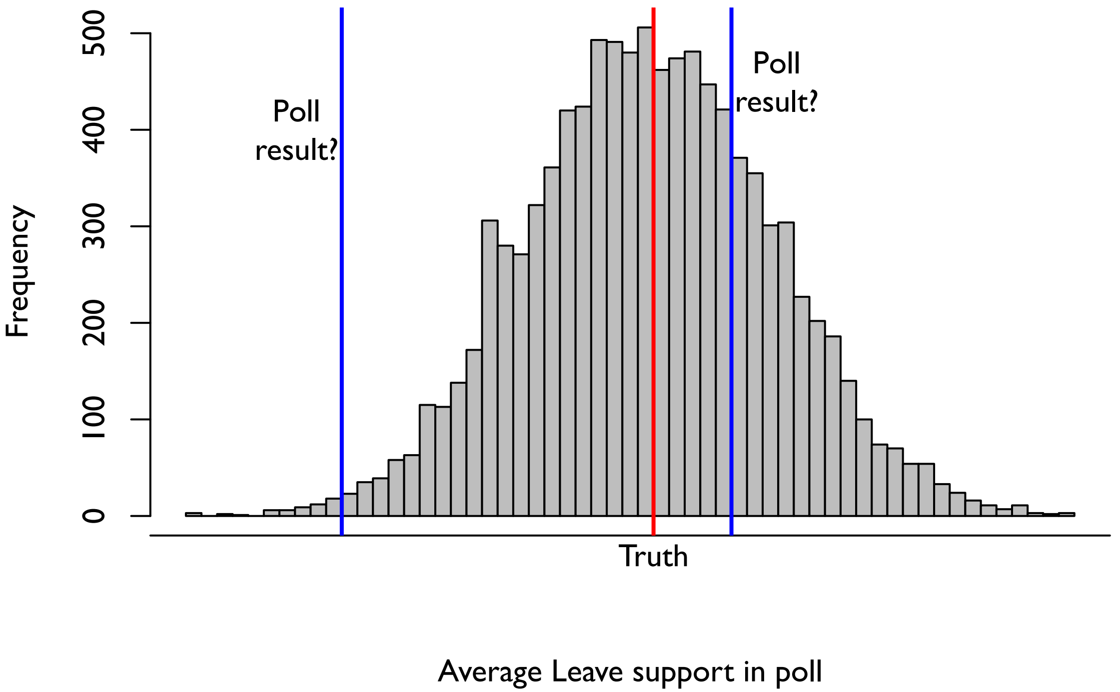

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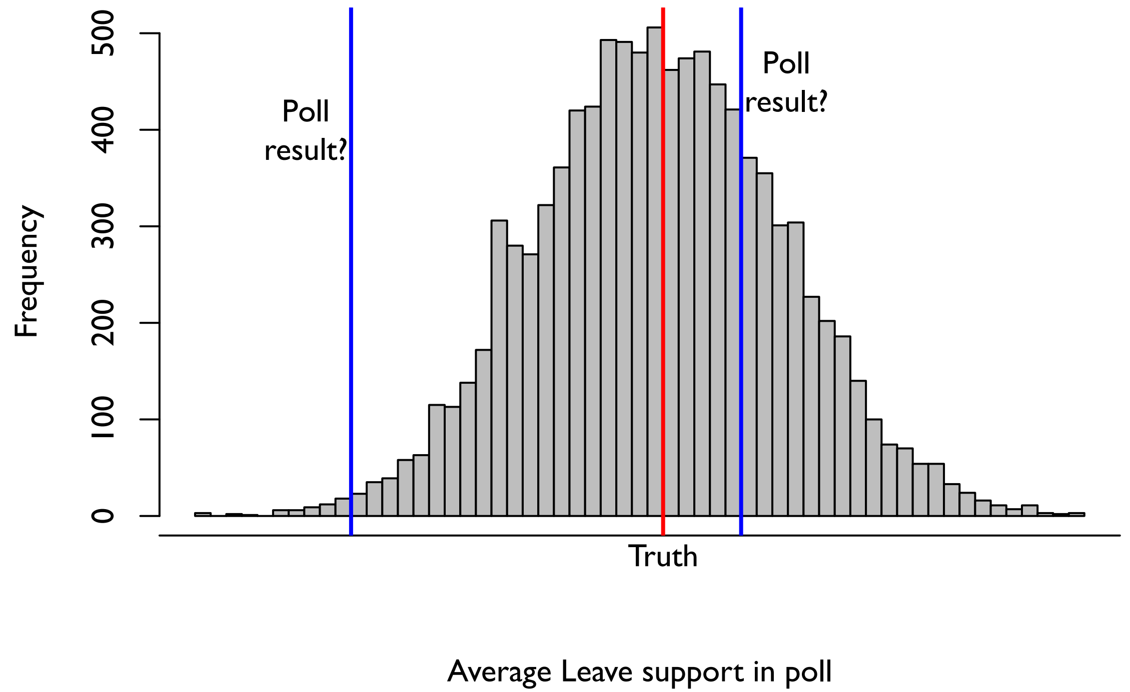

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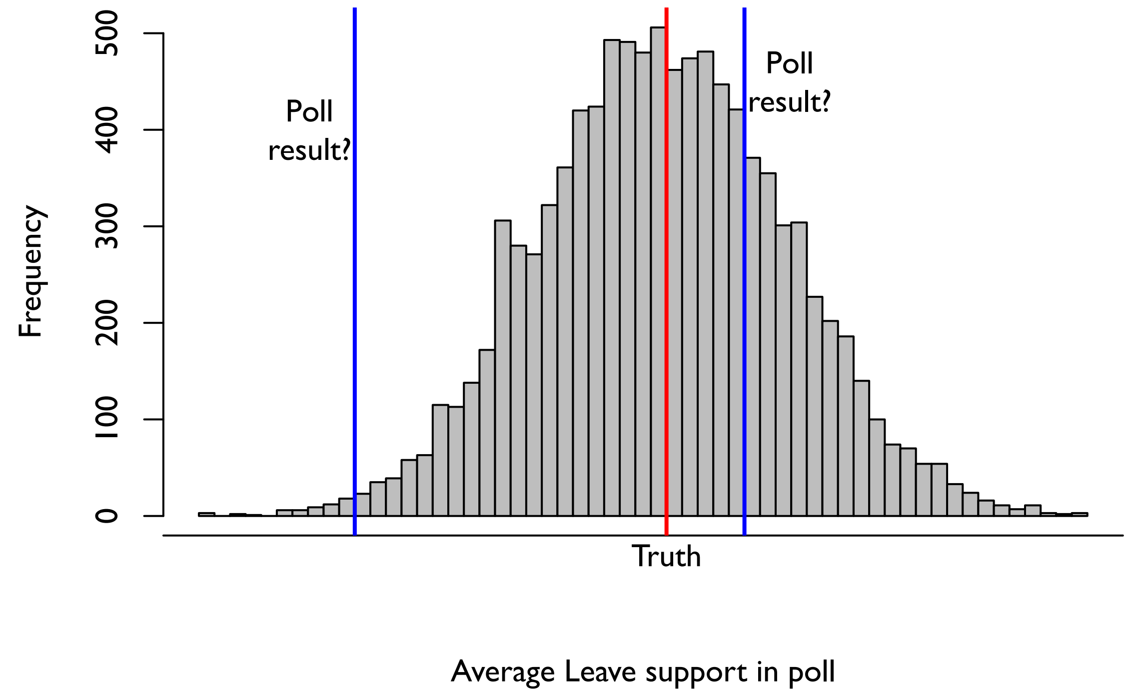

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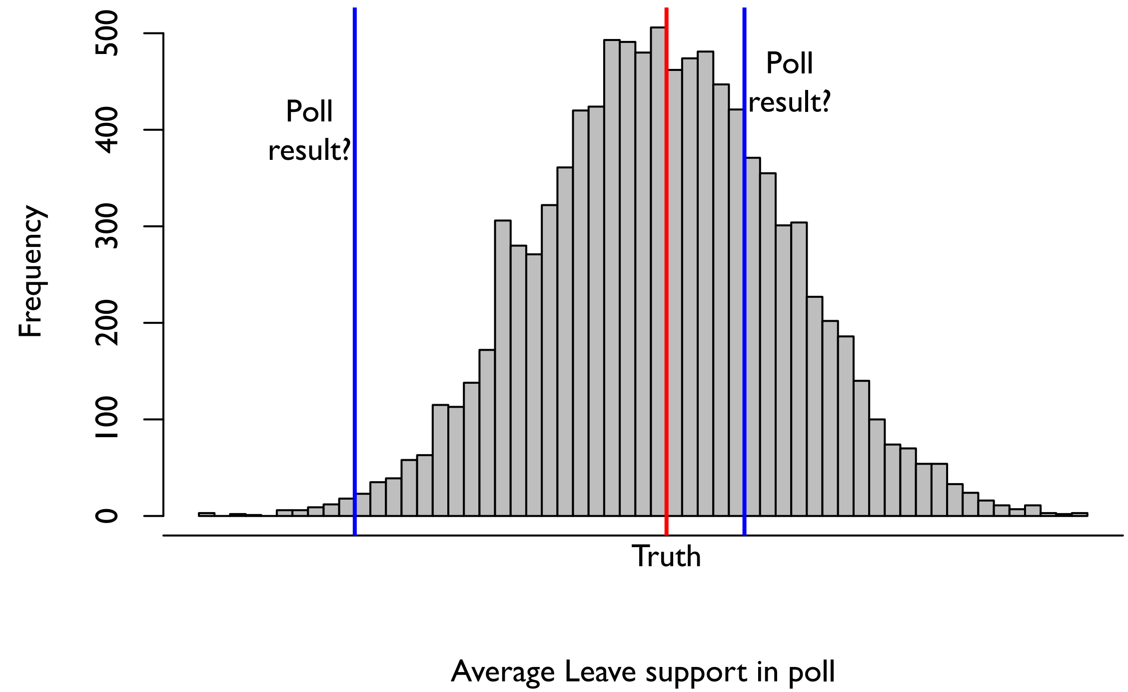

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(Combining the two:) A 95% confidence interval, which we expect to include the truth in 95% of samples: e.g. 49% ± 3.1% (3.1% is the **margin of error** of the poll)

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Compare: the standard deviation of our simulations was 0.0157

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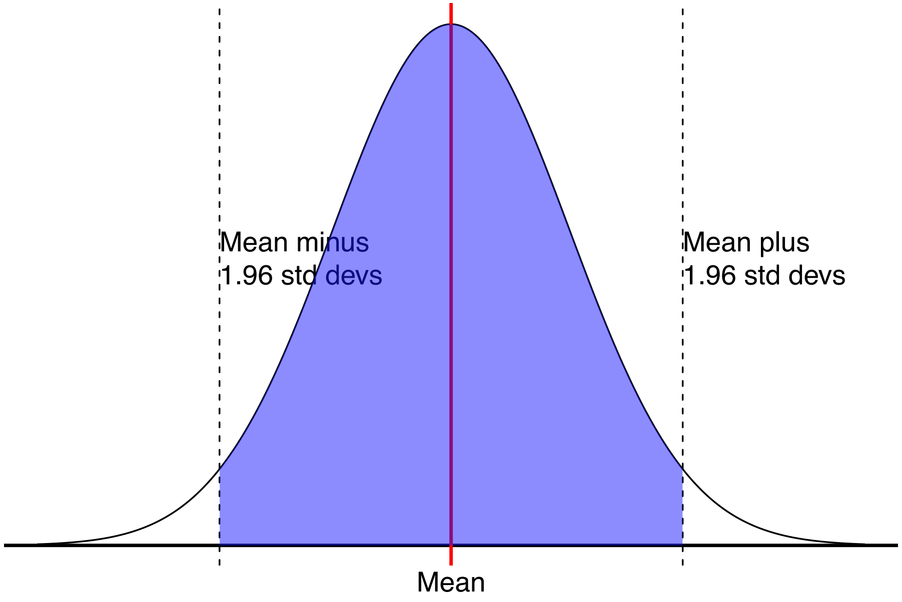

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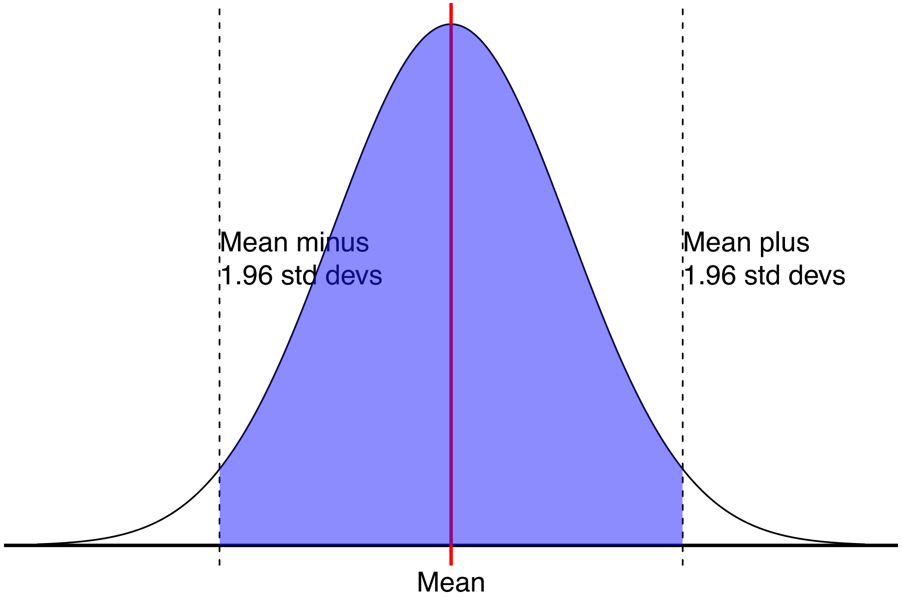

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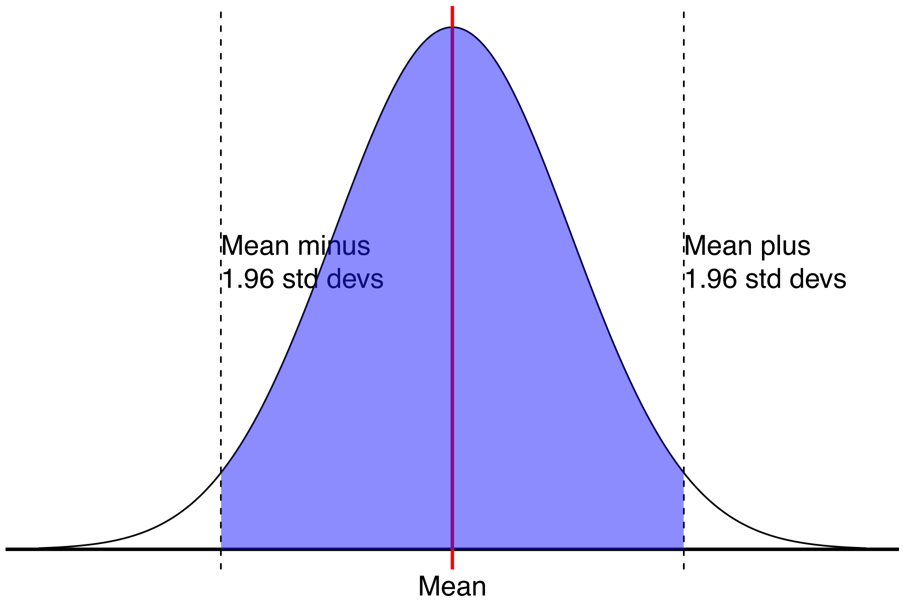

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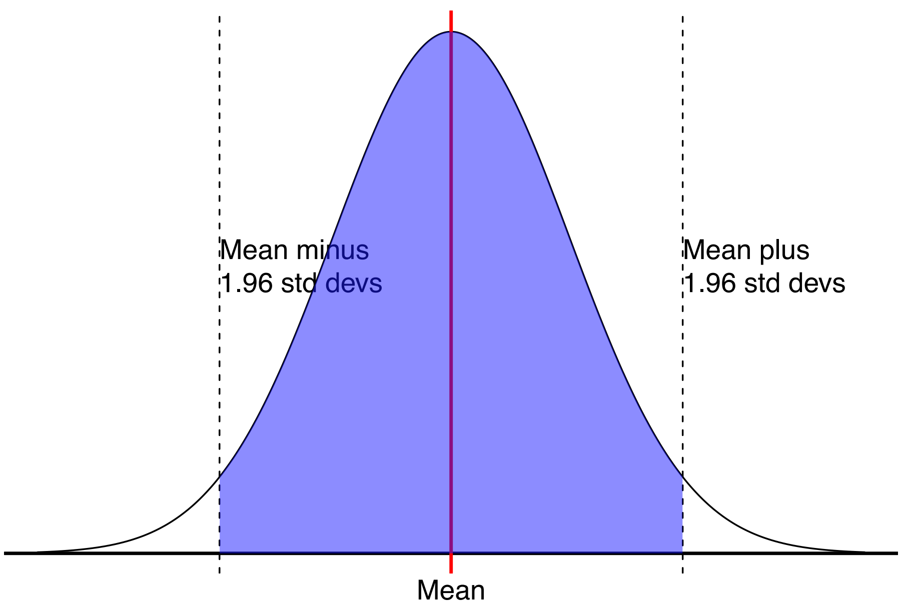

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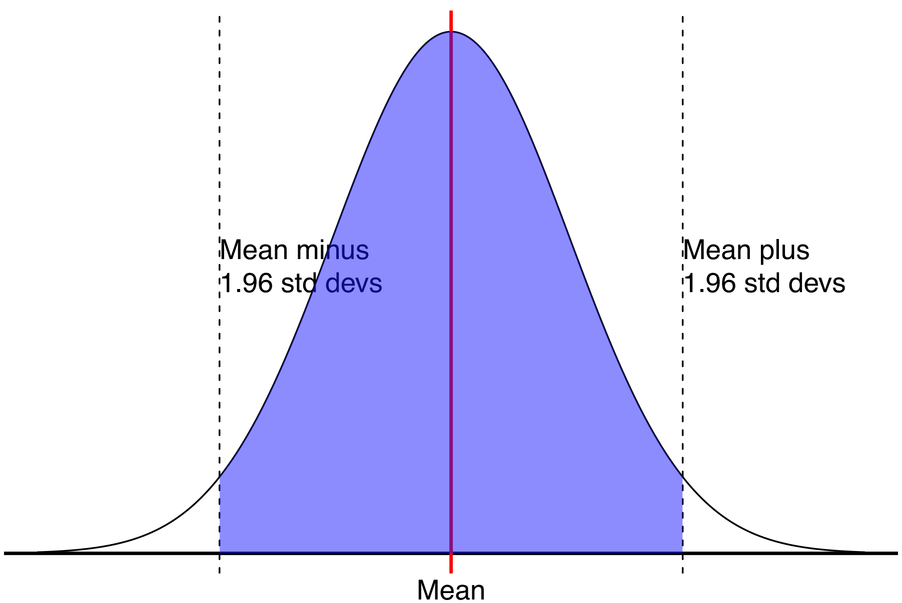

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But what if we draw a random sample and run this regression in our sample? How far off might the coefficients be?

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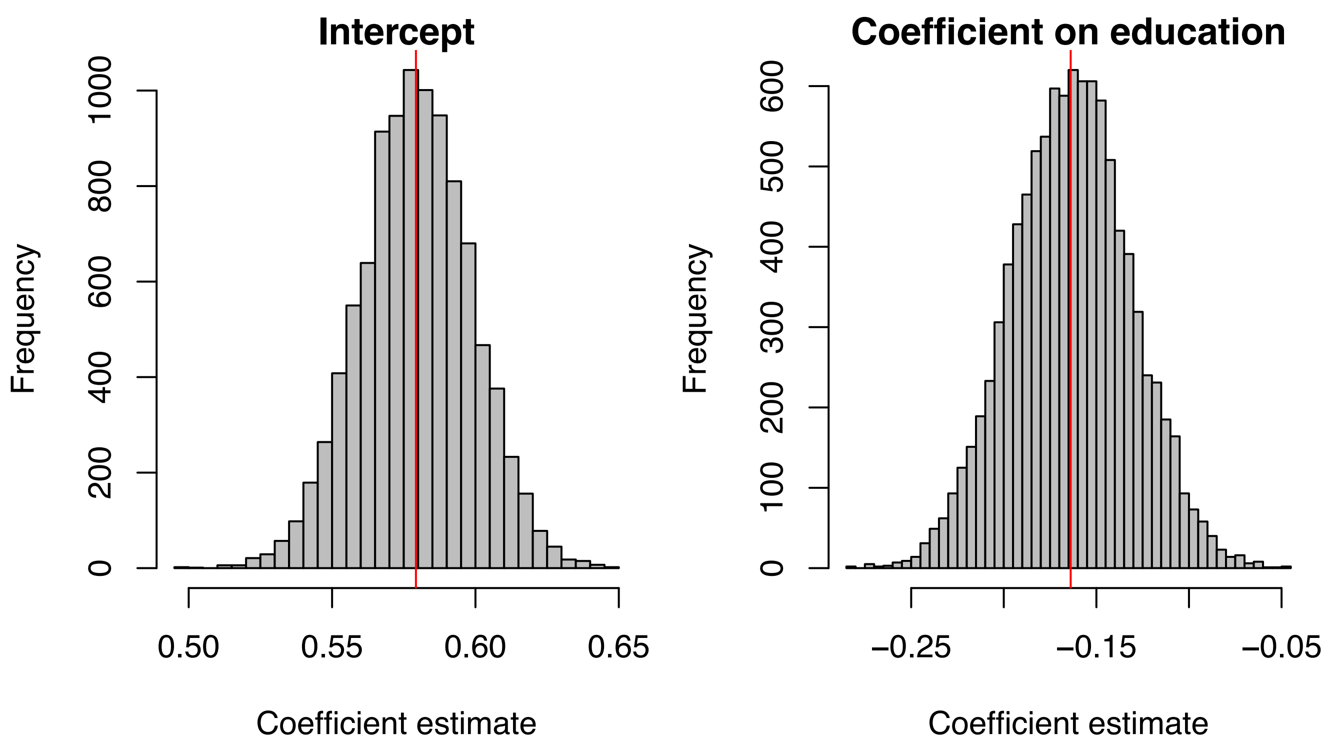

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As with the simple polling case, we can also use some statistical theory to estimate the standard errors given a sample (i.e. without doing a simulation).

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Call:
lm(formula = support.leave[indices.to.sample] ~ attended.uni[indices.to.sample])

Residuals:
    Min      1Q  Median      3Q     Max 
-0.5987 -0.5987  0.4013  0.4013  0.5550 

Coefficients:
                                Estimate Std. Error t value Pr(>|t|)    
(Intercept)                      0.59874    0.01960  30.544  < 2e-16 ***
attended.uni[indices.to.sample] -0.15370    0.03219  -4.774 2.07e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.4932 on 1004 degrees of freedom
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Very close to our estimates of the standard errors from simulation.

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We want to know: have you proven conclusively that university attendance is related to Brexit support *in the population*, or might it just be a fluke *in your sample*?

Put differently: How likely is it that you would get a coefficient that far from 0 in your sample if the true coefficient were in fact 0?

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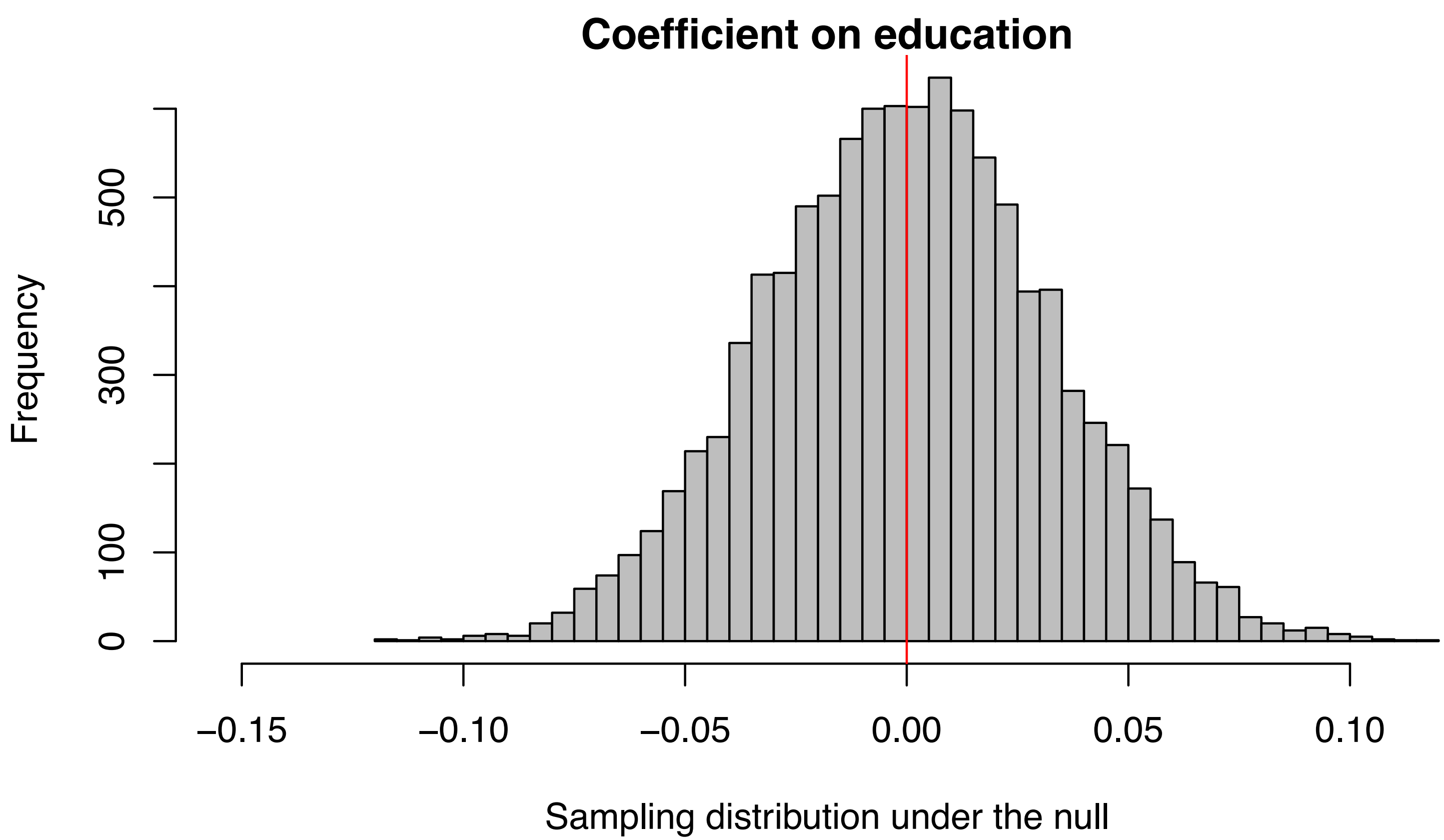
The sampling distribution of the coefficient on AttendedUni, **if the true coefficient were 0**, would be something like



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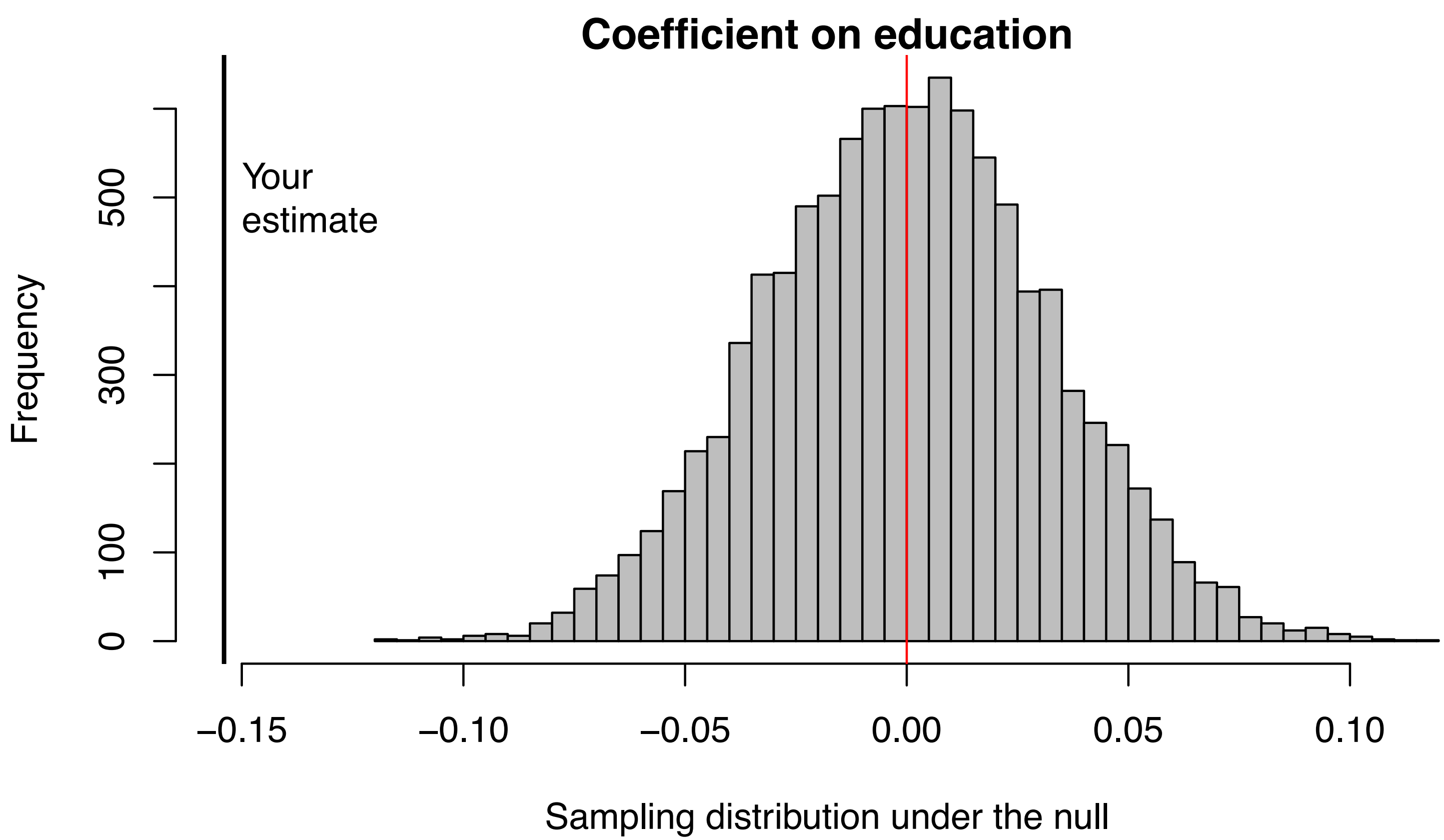

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Now you should understand:

Dependent variable: Nobel Prizes awarded per capita (in log scale)

	(1)	(2)	(3)
Intercept	-1.629* (0.509)	-3.166* (0.511)	-2.982* (0.527)
Chocolate consumption per capita (log scale)	2.092* (0.298)	1.026* (0.326)	0.709 (0.415)
GDP/capita (thousands of USD)		0.105* (0.024)	0.106* (0.024)
NW Europe			0.549 (0.452)
R^2	0.70	0.85	0.86
N	34	34	34

Standard errors in parentheses. * Indicates p<0.05

- what a dependent variable is
- what an independent variable is
- what the coefficients mean (intercept, slopes)
- what the stars mean (i.e. what $p<0.05$ means)
- what the standard errors mean

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The broader view is that history offers one sample, but if "re-run" it might have produced another. Less philosophically satisfying!