

Regression and inference

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International Relations

I want you to understand:

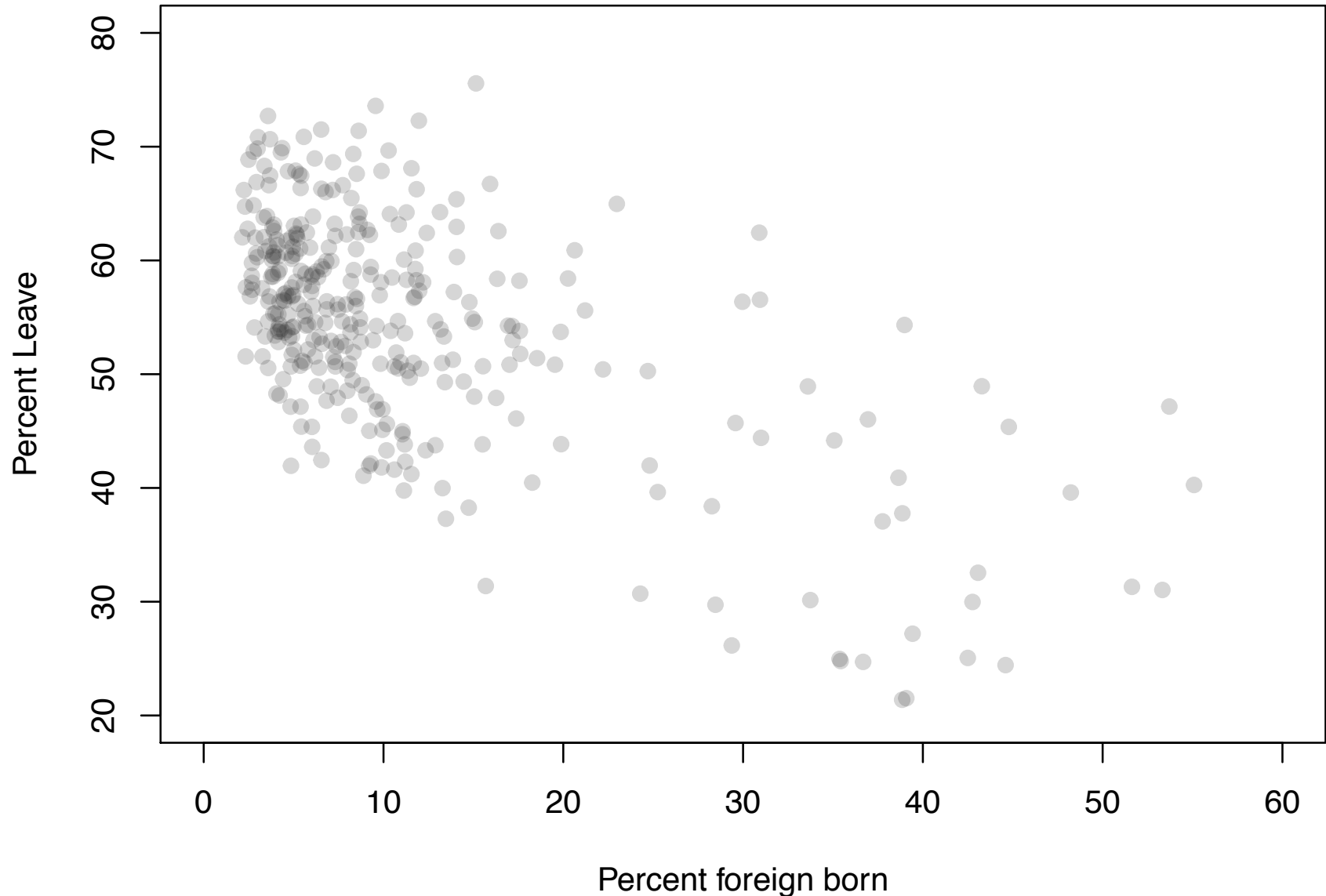
Dependent variable: Nobel Prizes awarded per capita (in log scale)

	(1)	(2)	(3)
Intercept	-1.629* (0.509)	-3.166* (0.511)	-2.982* (0.527)
Chocolate consumption per capita (log scale)	2.092* (0.298)	1.026* (0.326)	0.709 (0.415)
GDP/capita (thousands of USD)		0.105* (0.024)	0.106* (0.024)
NW Europe			0.549 (0.452)
R ²	0.70	0.85	0.86
N	34	34	34

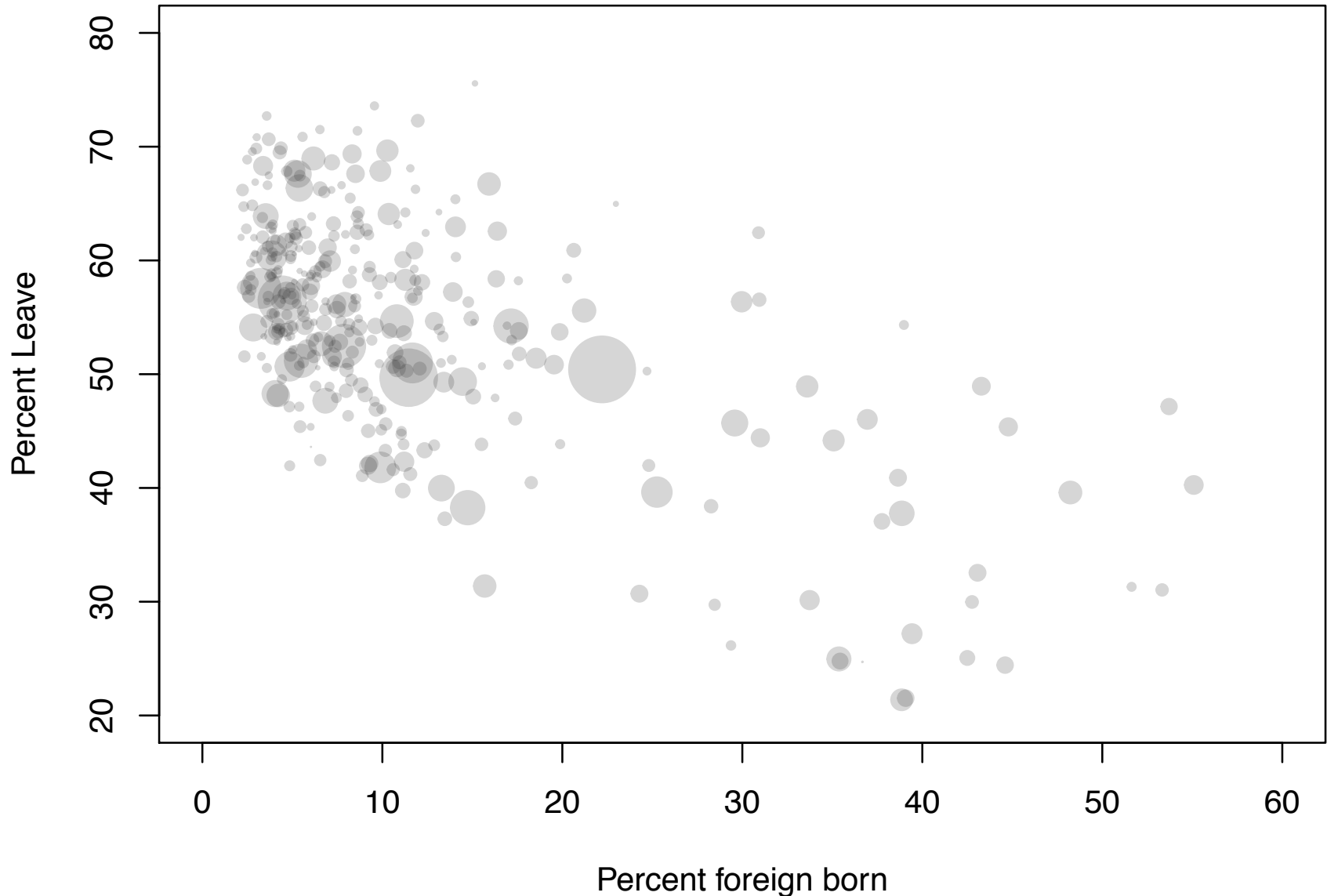
- what a dependent variable is
- what an independent variable is
- what the coefficients mean (intercept, slopes)
- what the stars mean (i.e. what $p < 0.05$ means)
- what the standard errors mean

Standard errors in parentheses. * Indicates $p < 0.05$

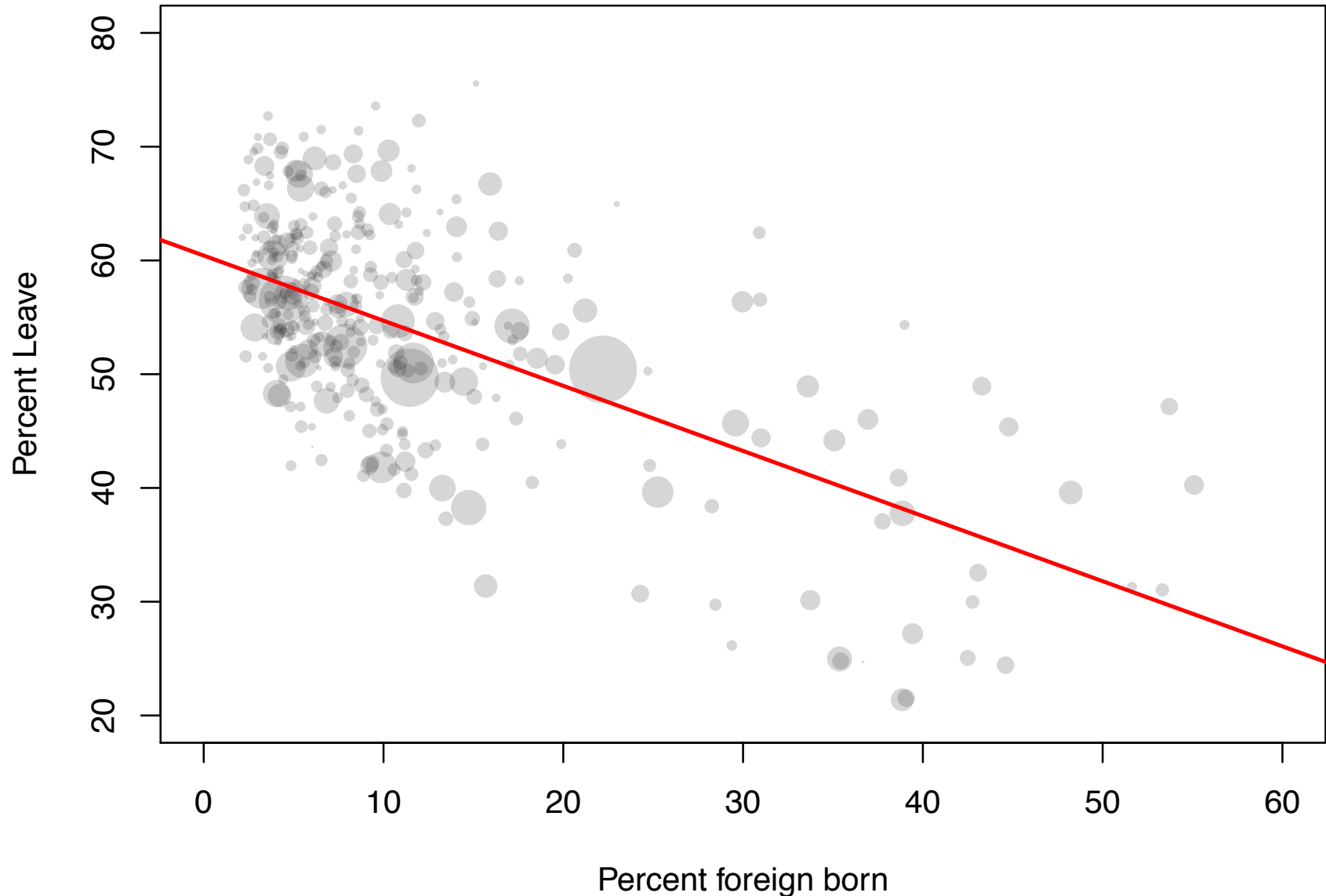
Local authorities with more foreign-born residents were less supportive of Brexit



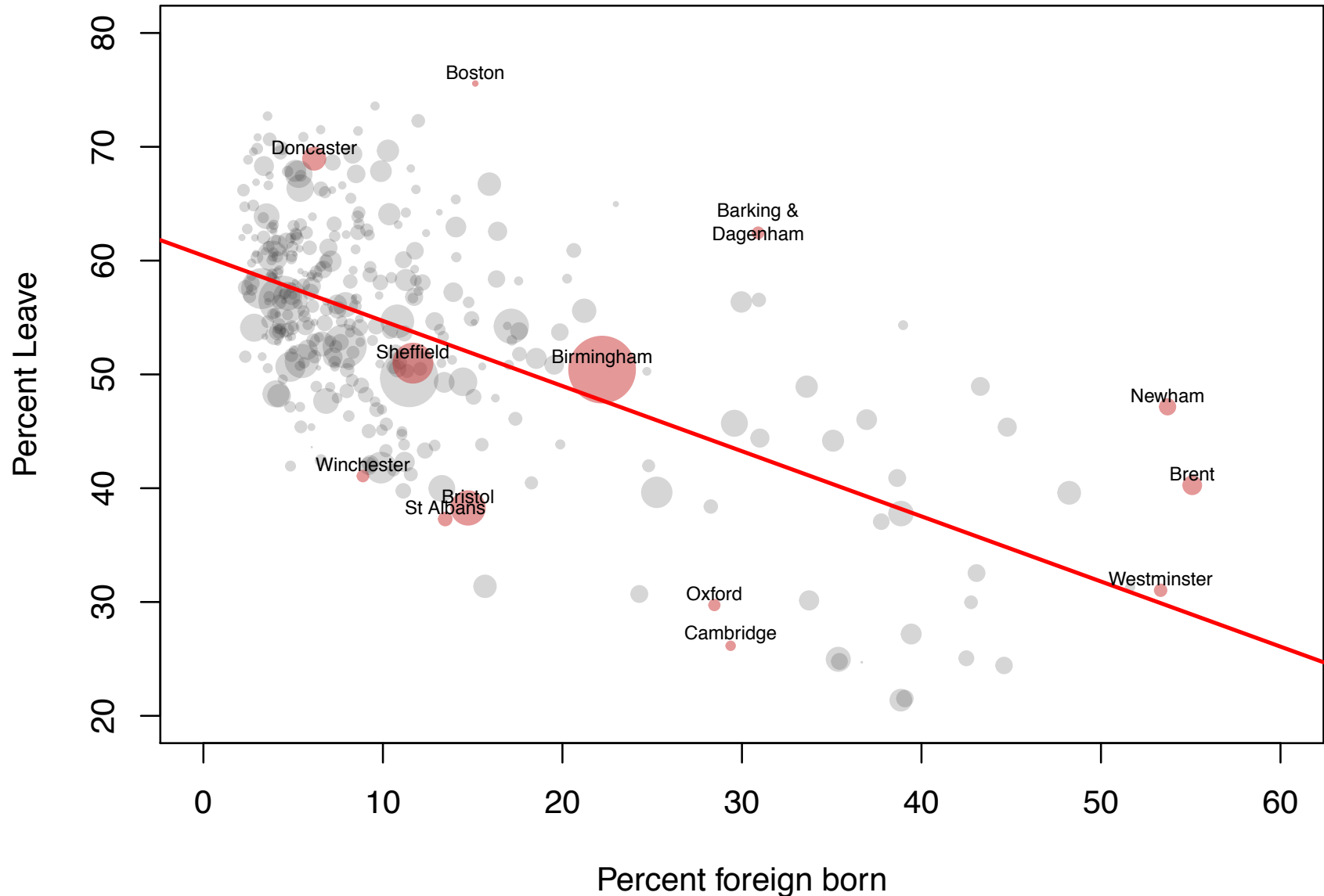
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Contact hypothesis

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“Prejudice (unless deeply rooted in the character structure of the individual) may be reduced by equal status contact between majority and minority groups in the pursuit of common goals. The effect is greatly enhanced if this contact is sanctioned by institutional supports (i.e., by law, custom or local atmosphere), and provided it is of a sort that leads to the perception of common interests and common humanity between members of the two groups.”

— Gordon Allport (1954) *The Nature of Prejudice*

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- How could this pattern be explained by the contact hypothesis? (easy)
- How could this pattern be explained by other factors? (harder)

Today and tomorrow

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Running question: Why is there such a strong relationship between % foreign born and opposition to Brexit?

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 - bivariate OLS regression as main focus

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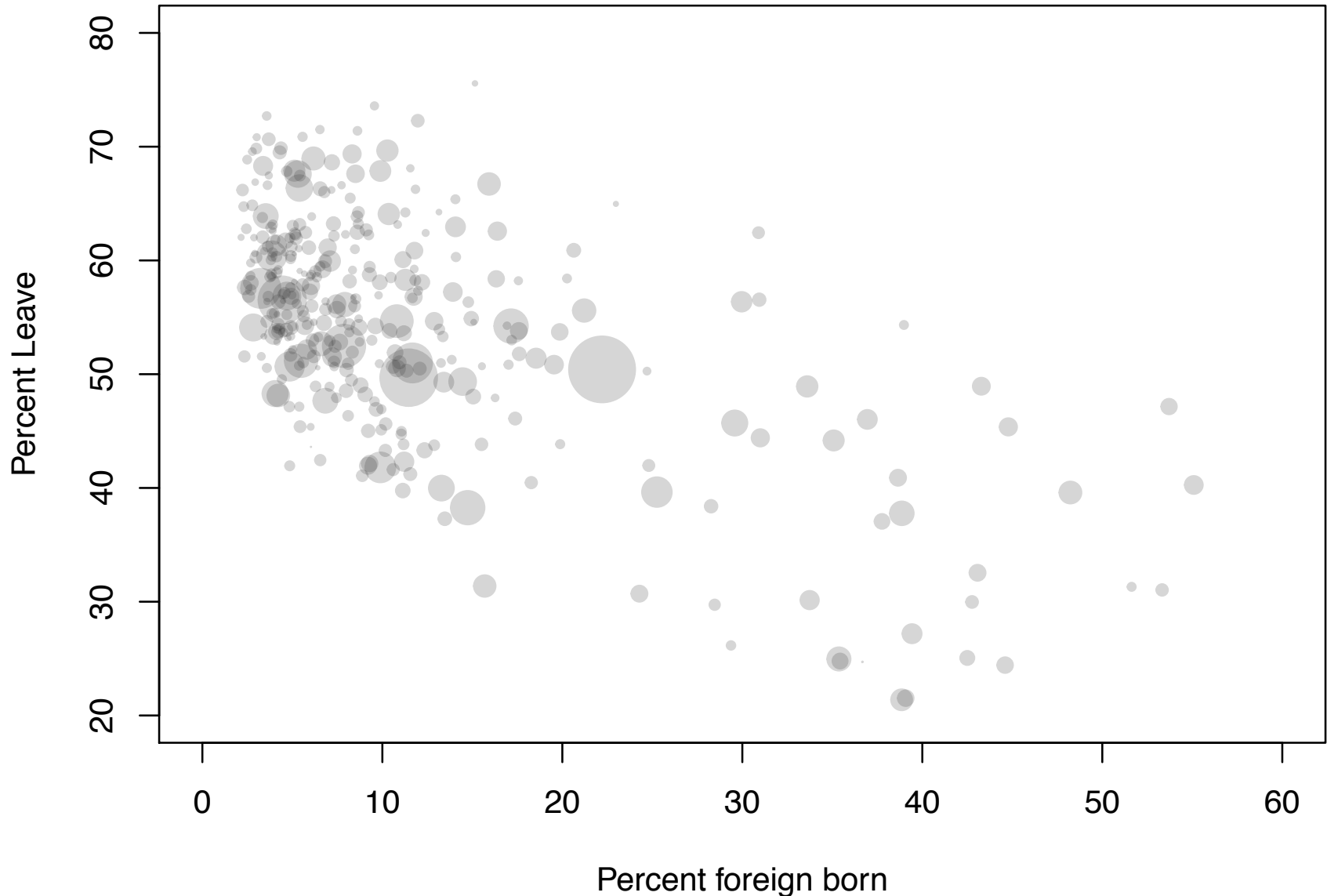
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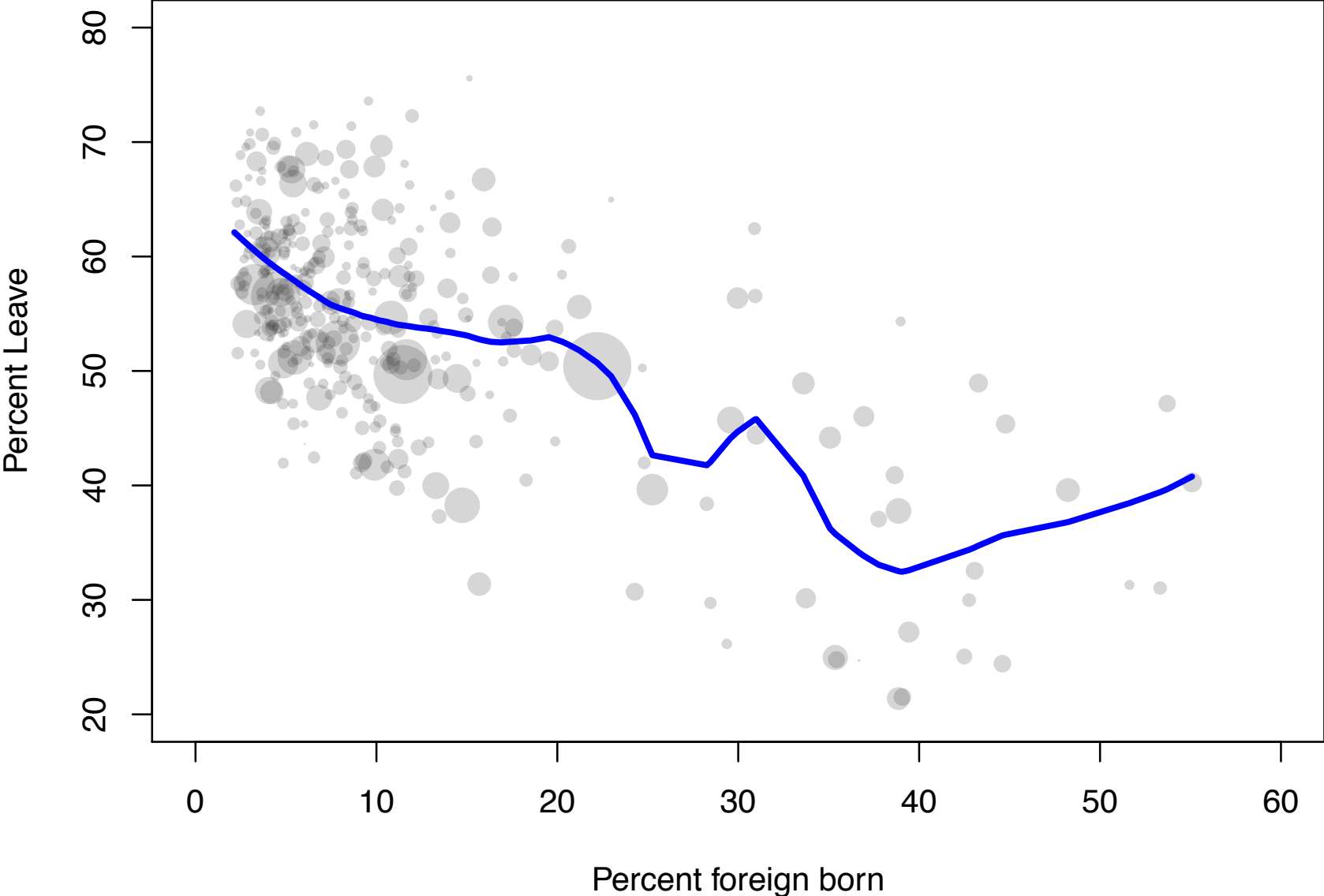
- How do we summarize the relationship between two variables?
 - bivariate OLS regression as main focus
- How do we summarize the relationship between two variables controlling for a third variable?
 - multivariate OLS regression as main focus
- How do we summarize our uncertainty about our conclusions?
 - standard errors, p-values, confidence intervals

Summarizing bivariate relationships: non-OLS options

Local authorities with more foreign-born residents were less supportive of Brexit



Kernel smoother (lokern function in R)



Single-number summaries: covariance

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How do x and y tend to move together, i.e. how do they covary?

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```
> cov(d$Percent_foreign_born, d$Percent_Leave, use = "complete")  
[1] -62.17755
```

Single-number summaries: correlation

If you plot x and y , how closely are the points arranged on a line (and is the slope of that line positive or negative)?

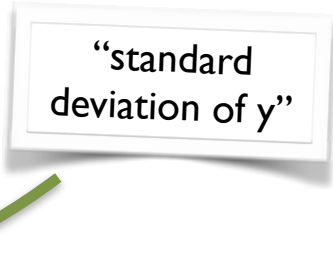
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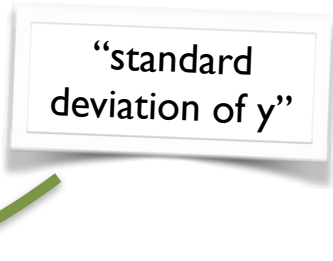
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“standard deviation of y ”

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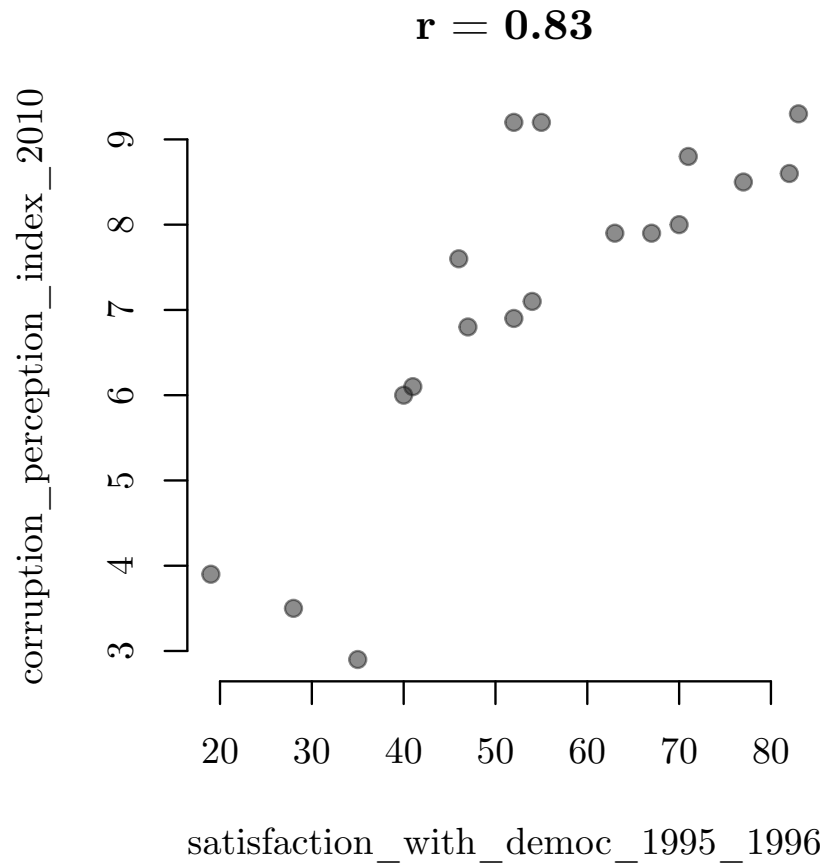
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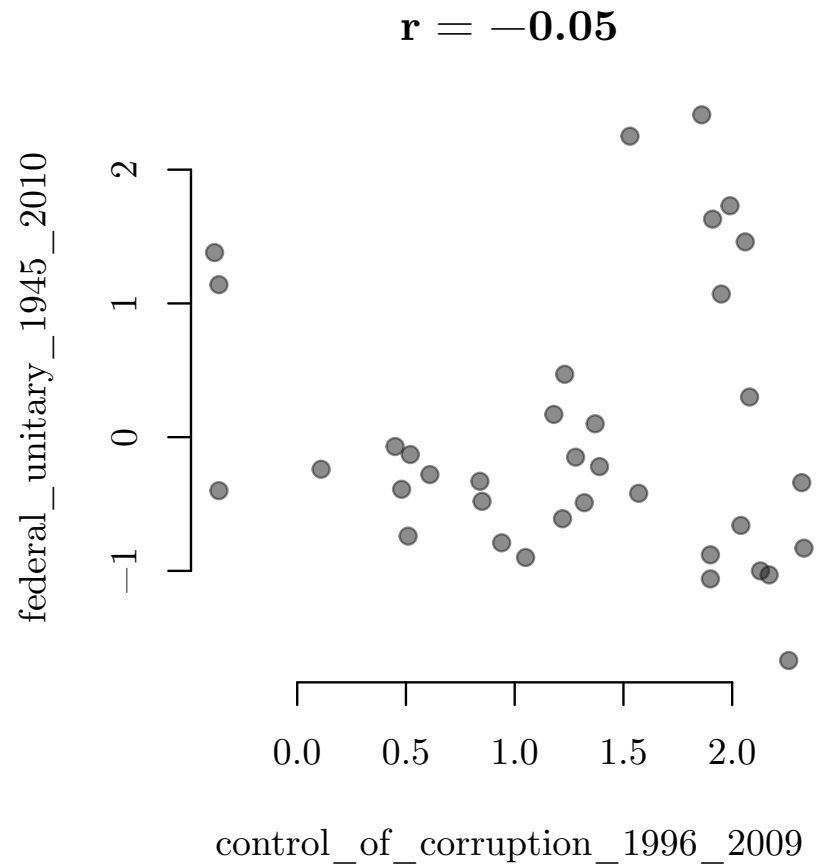
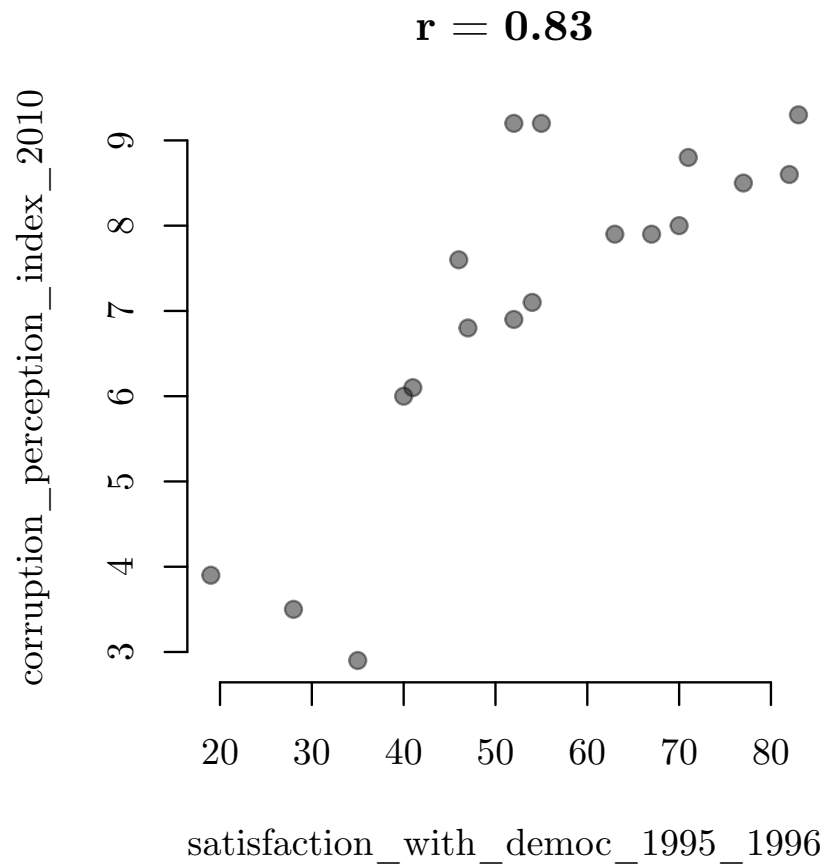
```
> cor(d$Percent_foreign_born, d$Percent_Leave, use = "complete")  
[1] -0.6125353
```

Correlation examples

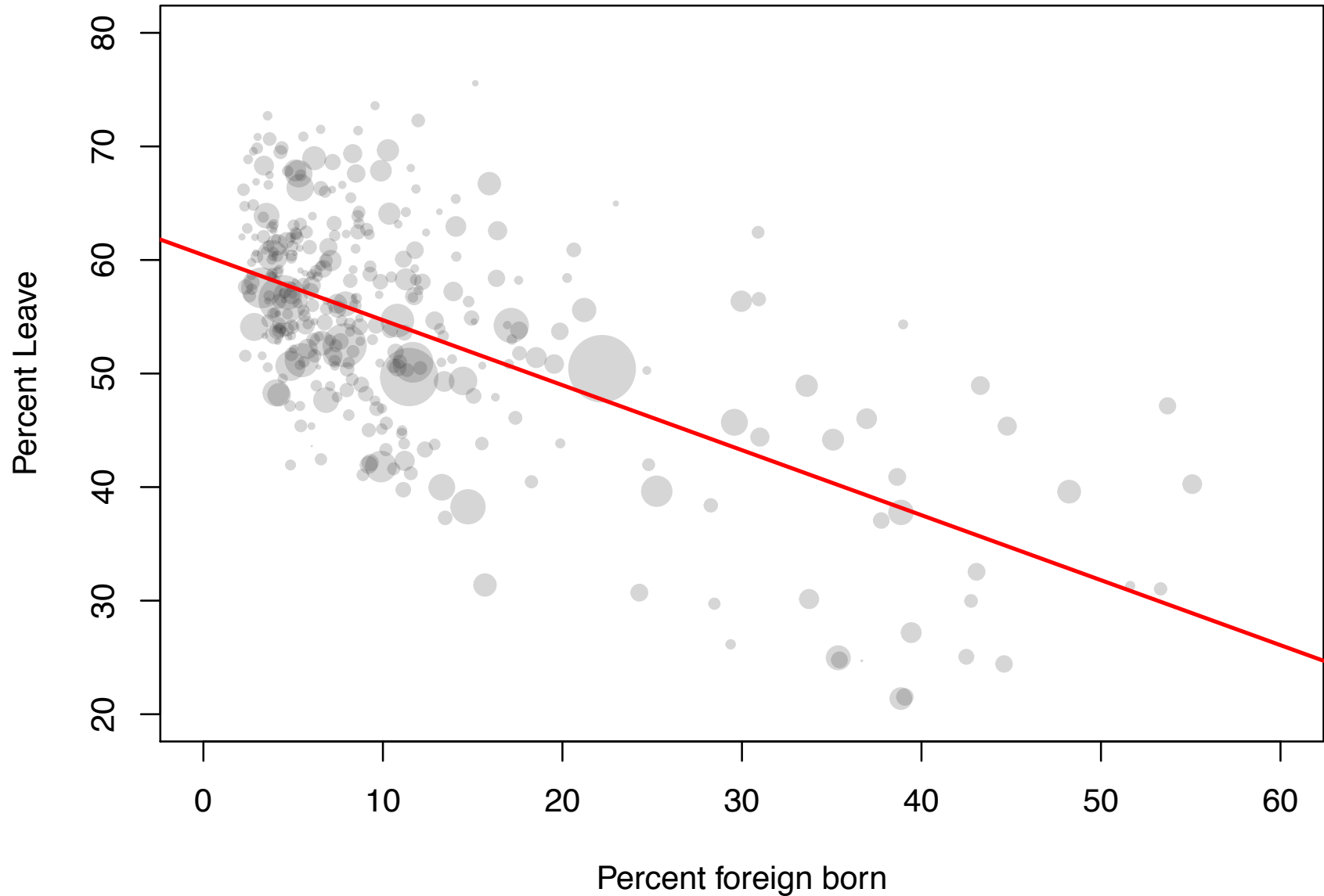
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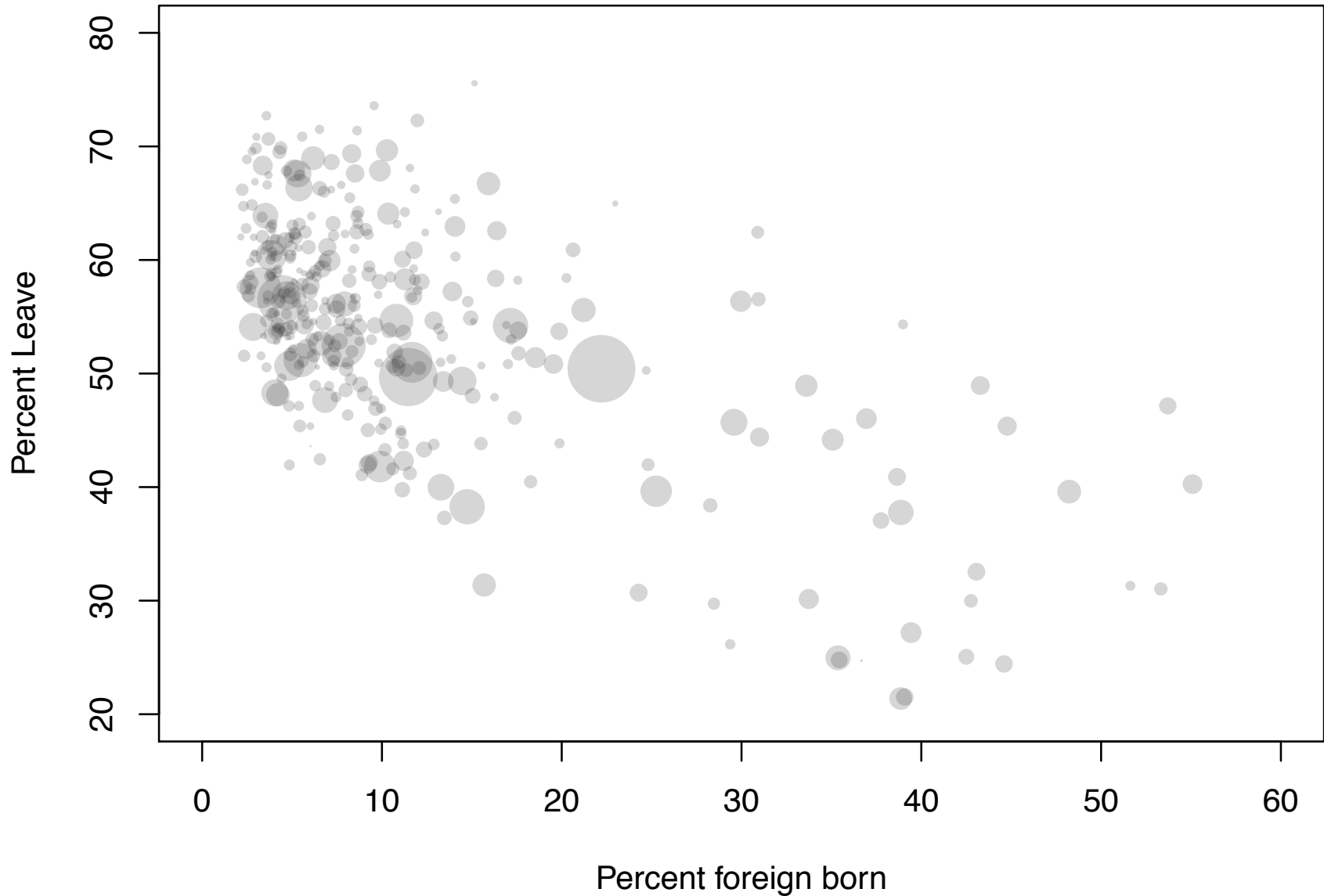
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The most important summary: OLS regression

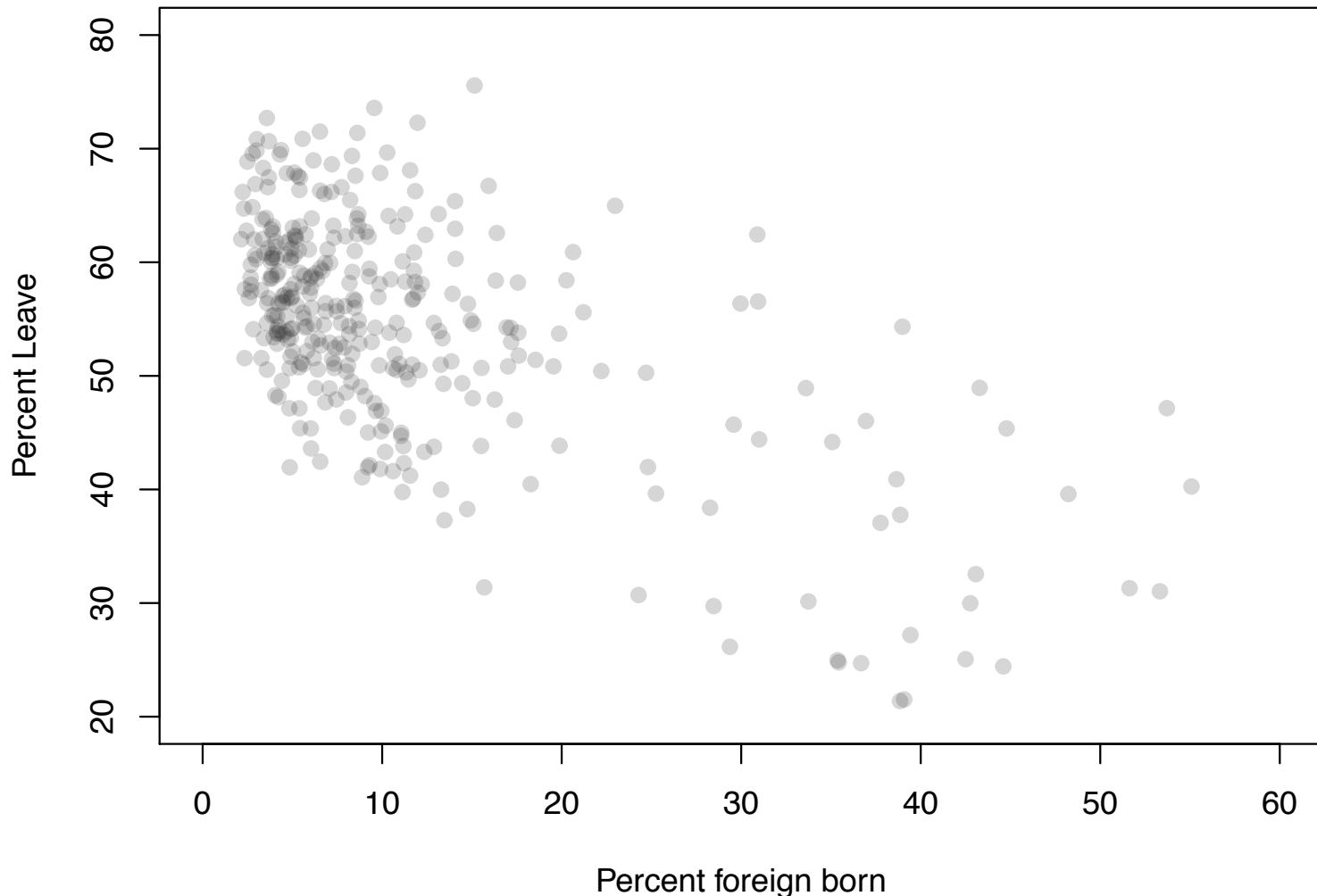


The most important summary: OLS regression



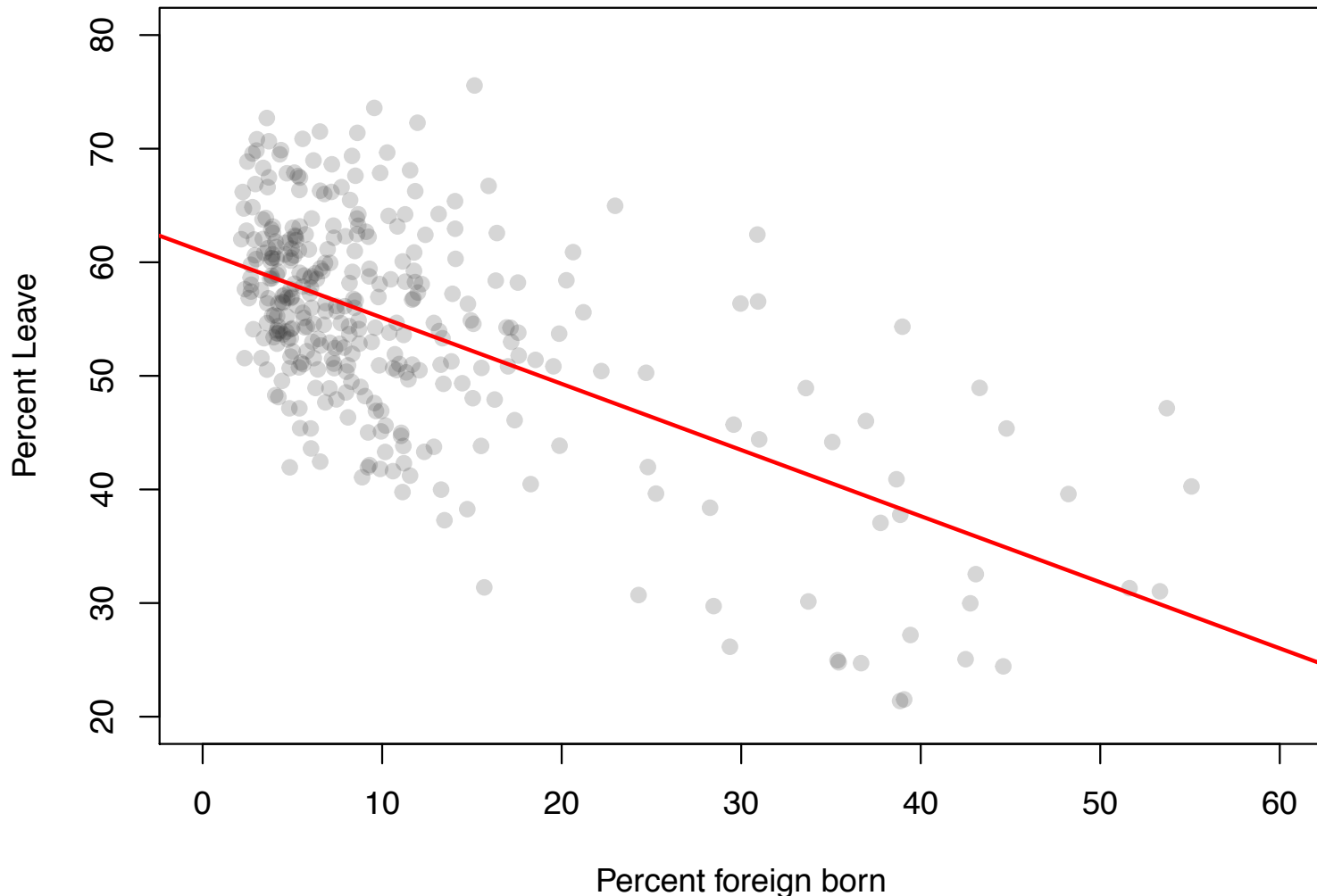
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A **residual** is the difference between the *actual* y -value and the *predicted* y -value (i.e. vertical diff. btw point and line).



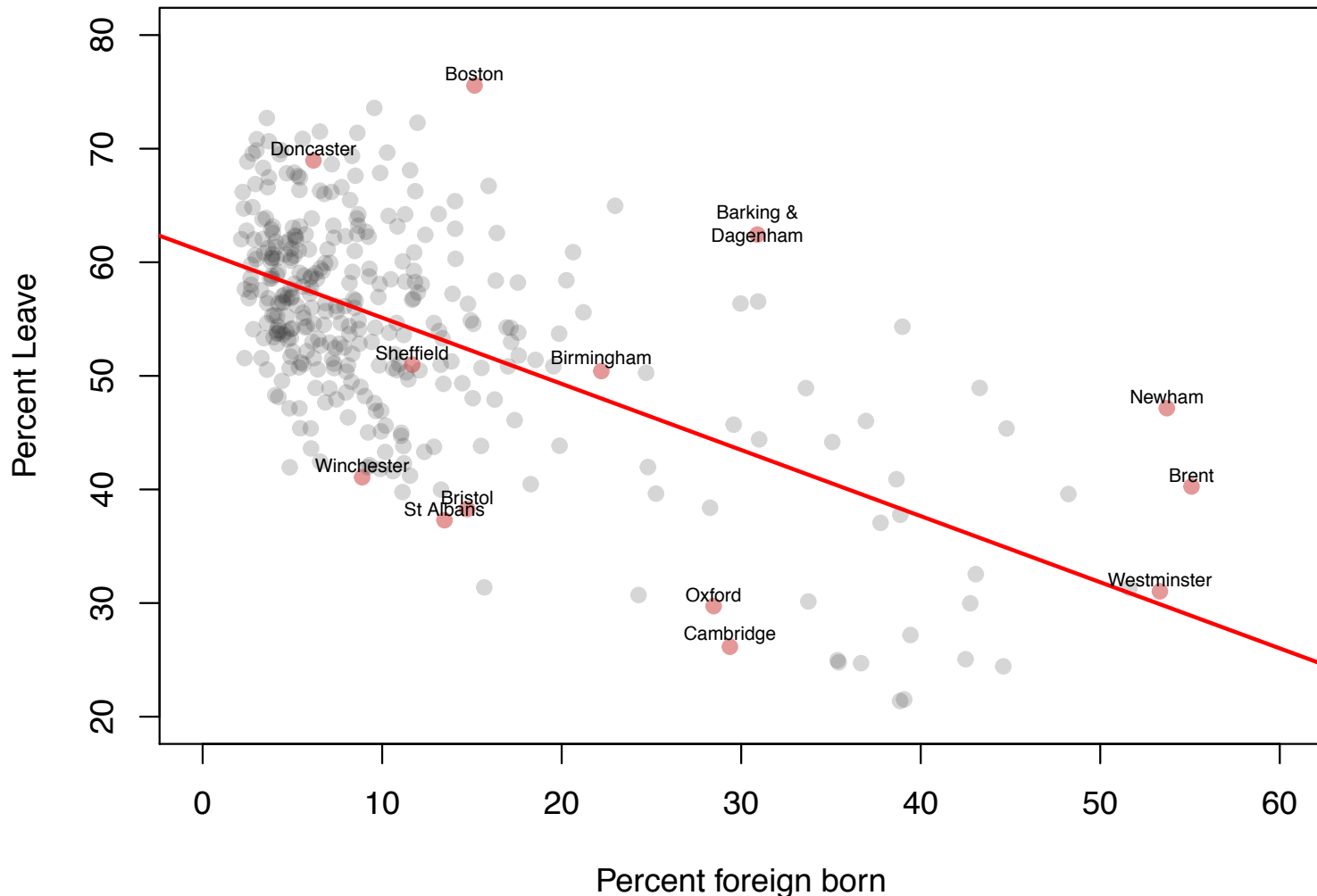
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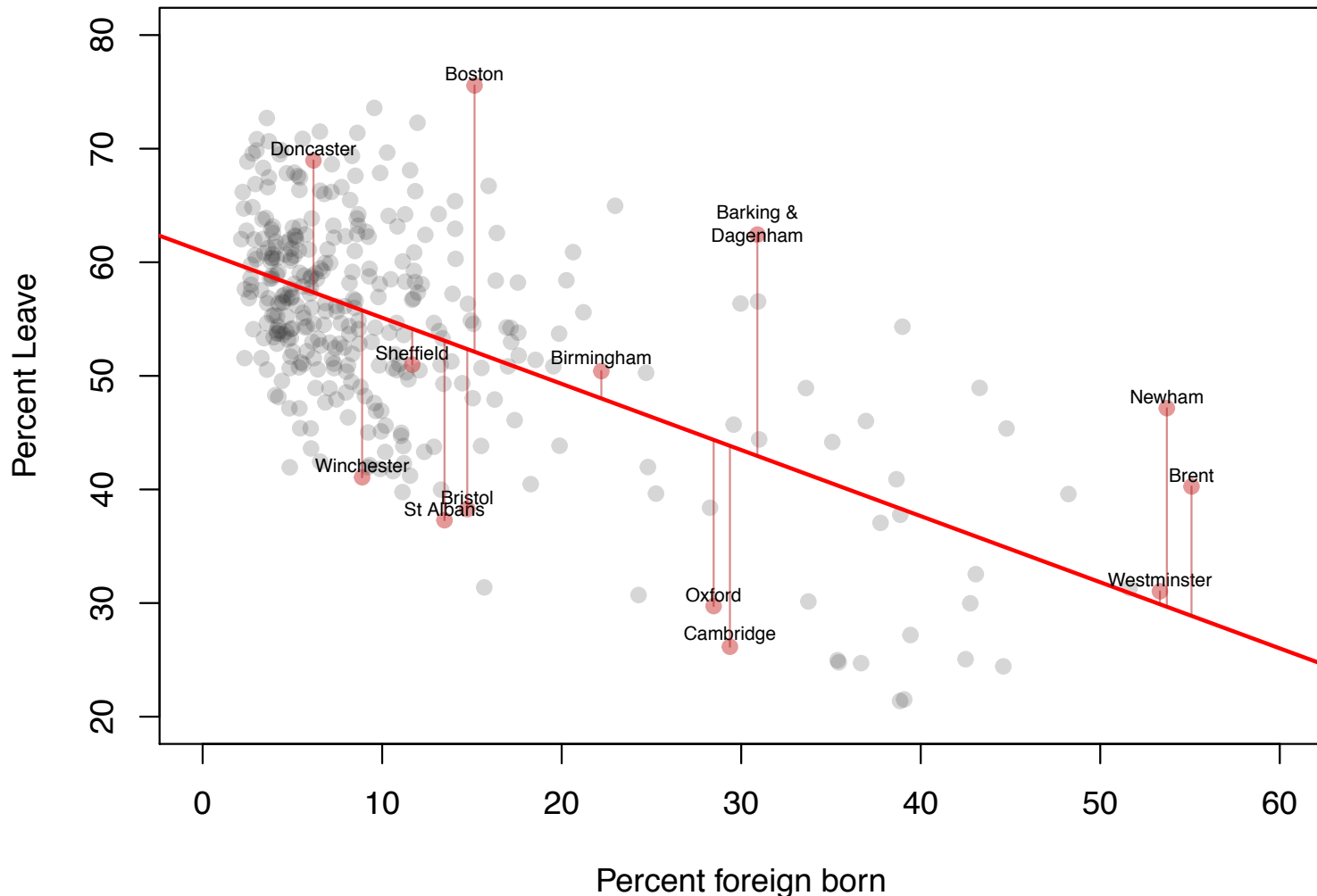
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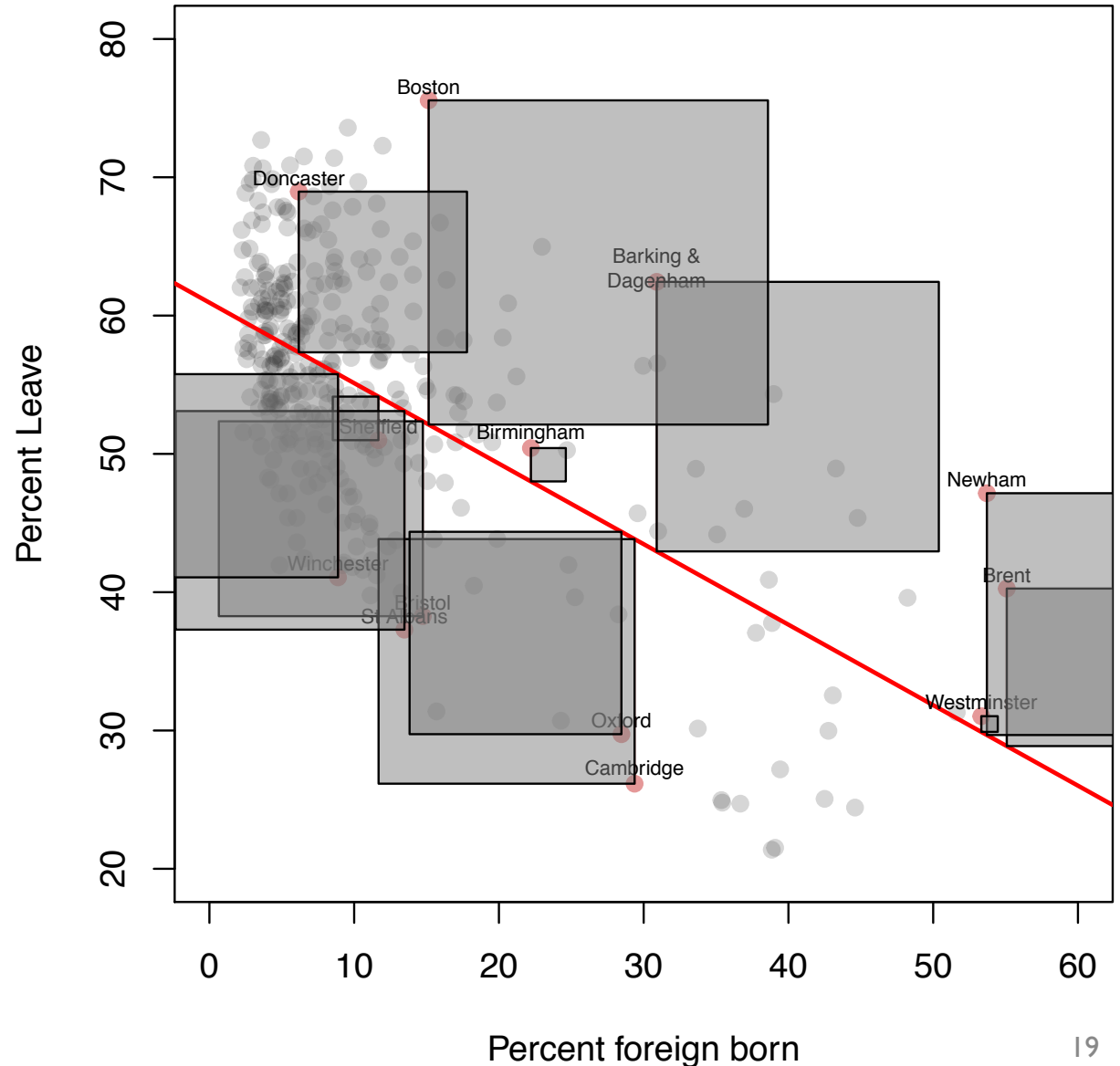
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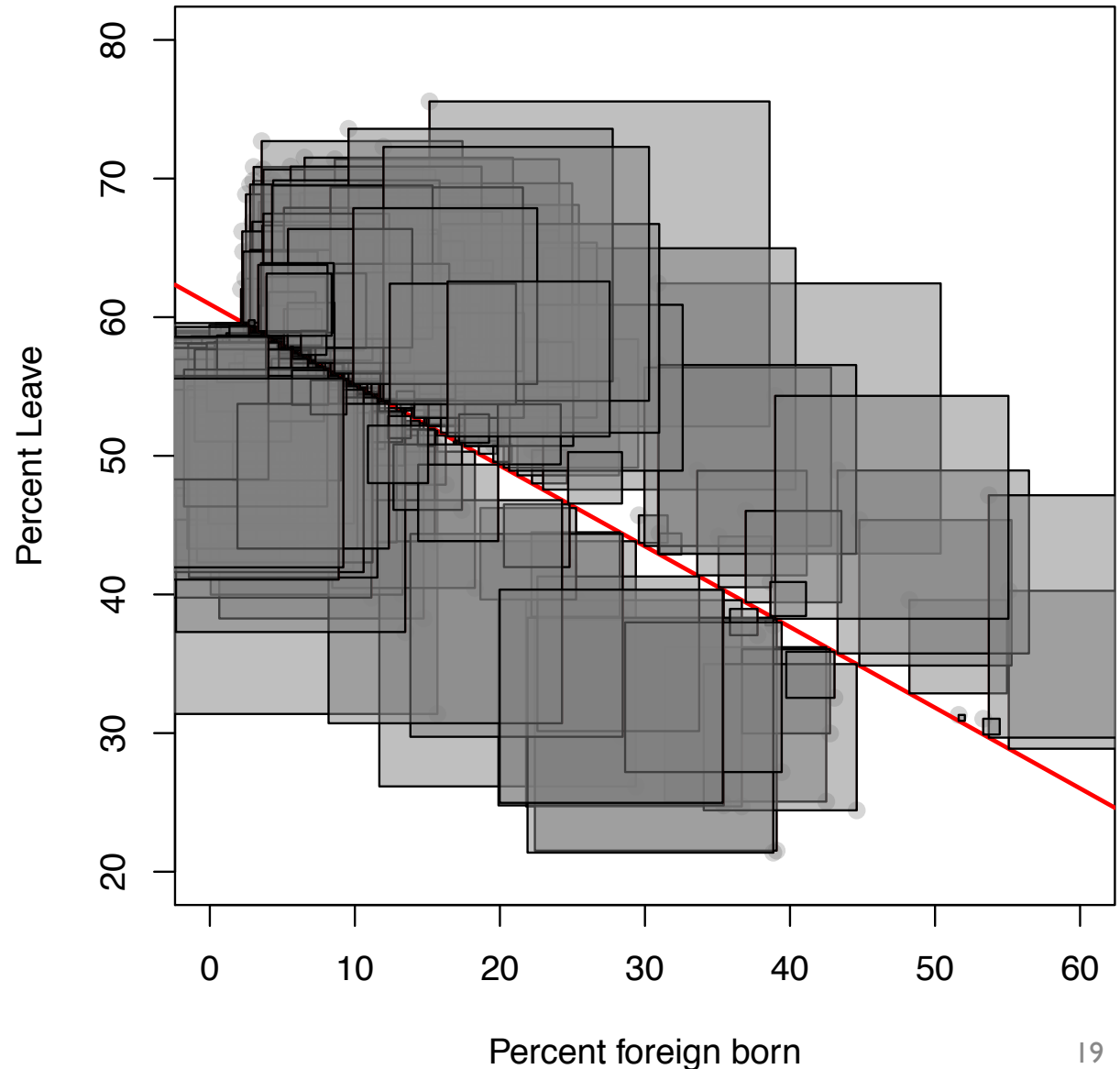
Step 2 for understanding OLS: sum of squared residuals

For any prediction line you draw, you can calculate residuals, square them, and sum them.



Step 2 for understanding OLS: sum of squared residuals

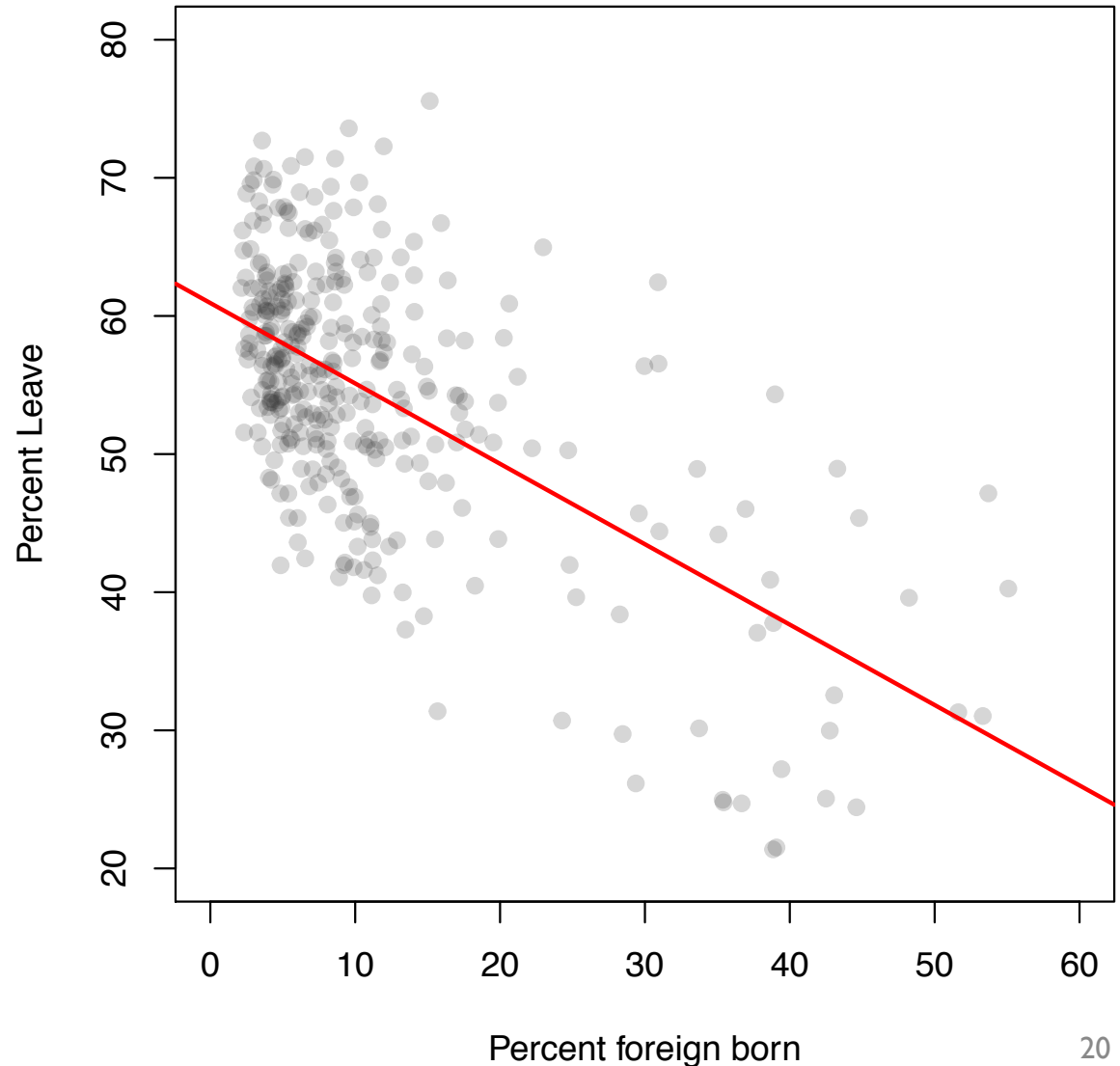
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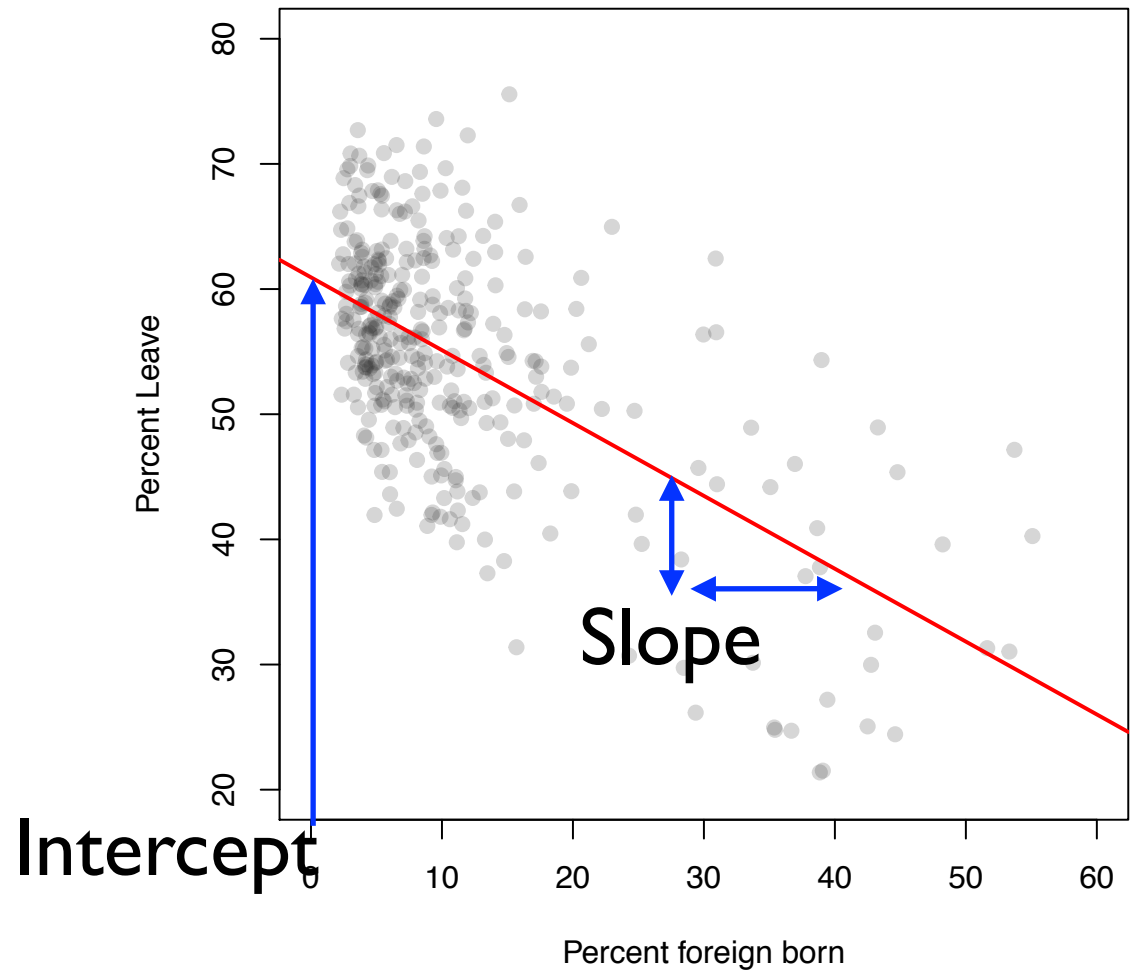
Step 3 for understanding OLS: minimizing the sum of squared residuals

The OLS regression line minimizes the sum of squared residuals (SSR).

Hence ordinary least squares.



Any line can be summarized by an **intercept** and a **slope**.



The intercept and slope of a bivariate regression are called the regression **coefficients**.

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2. Use calculus to find the slope and intercept that minimize the sum of squared residuals.
3. Use `lm()` function in R:

```
> lm(d$Percent_Leave ~ d$Percent_foreign_born)
```

```
Call:
```

```
lm(formula = d$Percent_Leave ~ d$Percent_foreign_born)
```

```
Coefficients:
```

```
      (Intercept)  d$Percent_foreign_born  
          60.9373             -0.5821
```

A (surprising?) fact about the slope coefficient

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Covariance of x
and y :

$$\text{Cov}(x, y) = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{n - 1}$$

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Slope from OLS
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$$\hat{\beta} = \frac{\text{Cov}(x, y)}{\text{Var}(x)}$$

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Slope from OLS regression of y on x :

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```
> cov(d$Percent_Leave, d$Percent_foreign_born, use = "complete")/var(d$Percent_foreign_born, na.rm = T)
[1] -0.582101
> lm(d$Percent_Leave ~ d$Percent_foreign_born)
```

Call:

```
lm(formula = d$Percent_Leave ~ d$Percent_foreign_born)
```

Coefficients:

```
(Intercept)  d$Percent_foreign_born
 60.9373      -0.5821
```

How well does our regression line predict the outcome? R^2

```
> summary(lm(d$Percent_Leave ~ d$Percent_foreign_born))
```

Call:

```
lm(formula = d$Percent_Leave ~ d$Percent_foreign_born)
```

Residuals:

Min	1Q	Median	3Q	Max
-20.4253	-4.7247	-0.0025	4.4336	23.4417

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	60.93732	0.61845	98.53	<2e-16	***
d\$Percent_foreign_born	-0.58210	0.04062	-14.33	<2e-16	***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

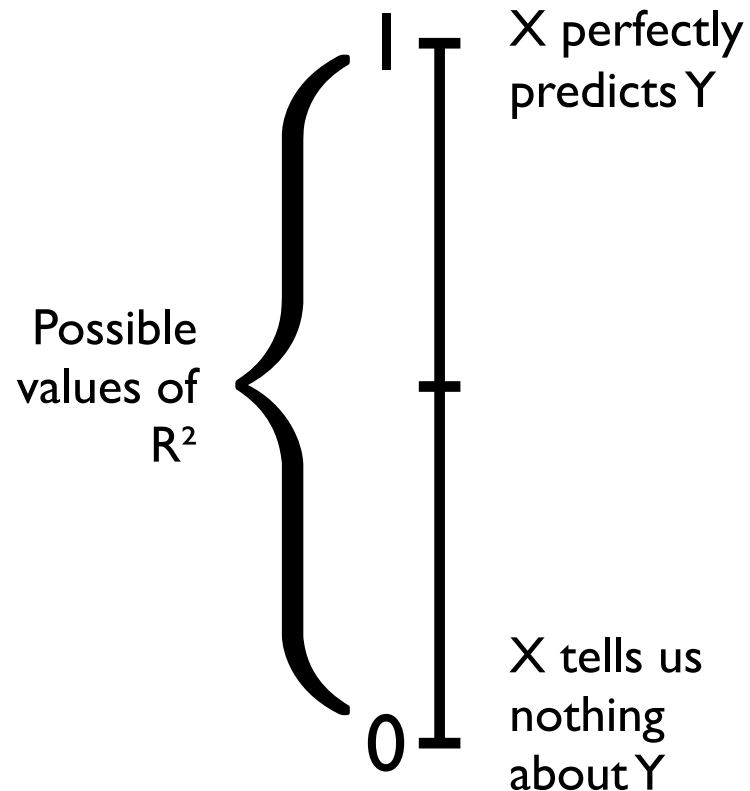
Residual standard error: 7.775 on 342 degrees of freedom
(38 observations deleted due to missingness)

Multiple R-squared: 0.3752, Adjusted R-squared: 0.3734

F-statistic: 205.4 on 1 and 342 DF, p-value: < 2.2e-16

R^2 : intuition

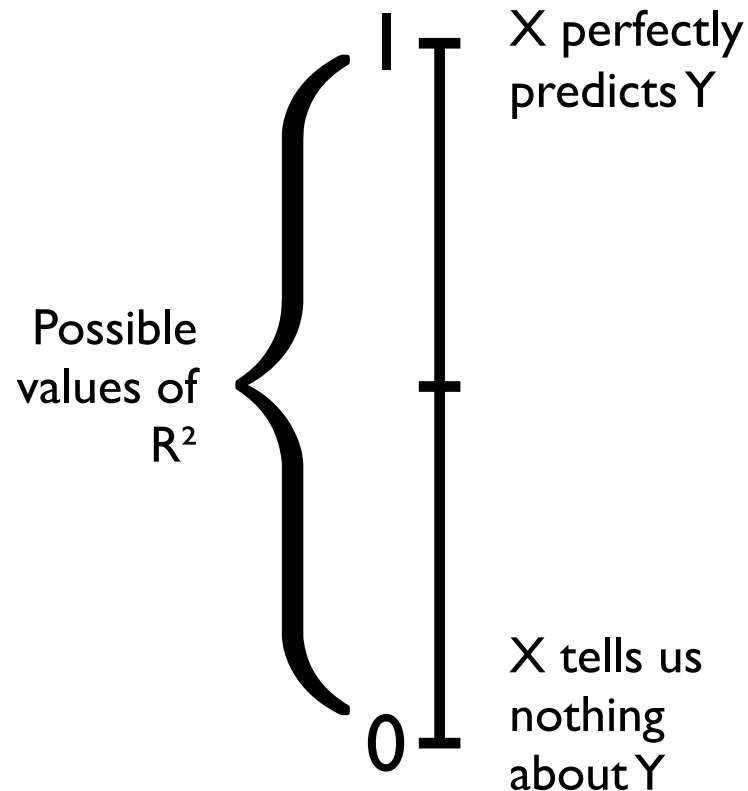
How much better are the predictions from our OLS regression line than the predictions from a flat line (i.e. not using X at all)?



R²: intuition

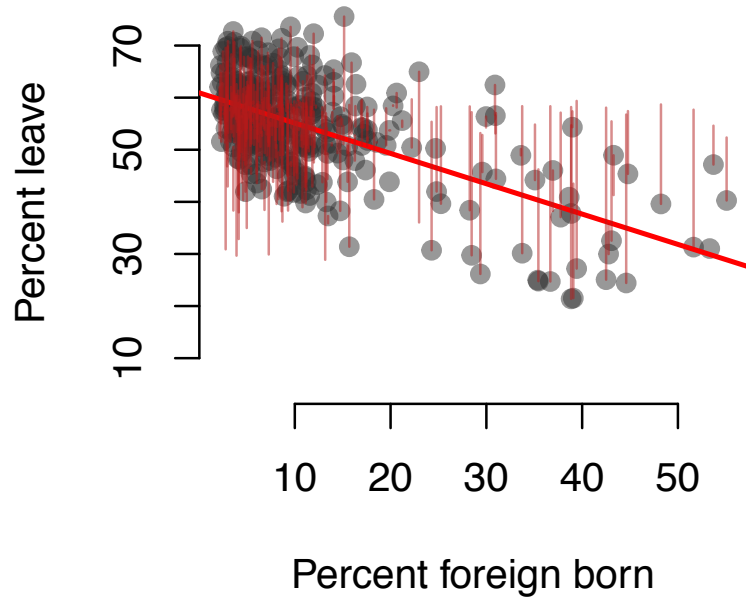
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How much of the variation in Y is “explained” by the variation in X ?

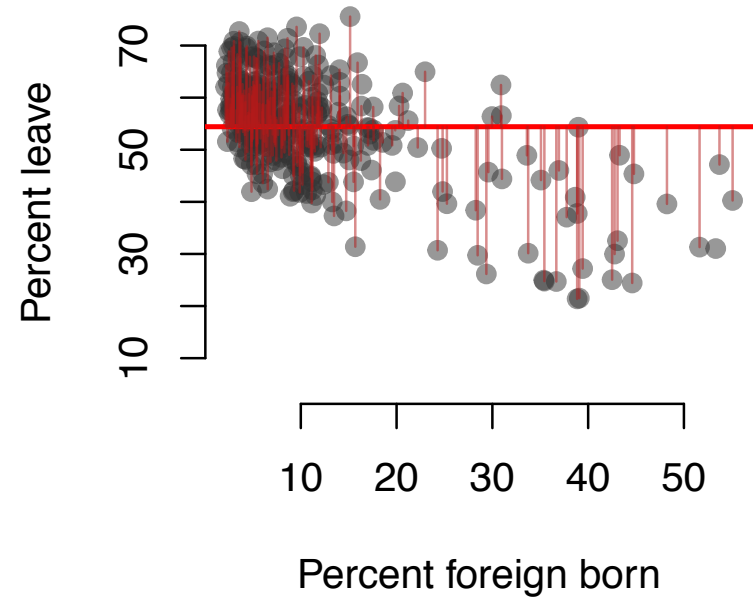


R²: calculation

Sum of squared residuals:
20673.074



Sum of squared residuals:
33087.482



```
> 1 - (20673.074/33087.482)
[1] 0.3751995
```

Connections between measures of bivariate relationships

Key measures:

- covariance
- correlation
- OLS regression

output:

- intercept
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Covariance and regression slope (but not correlation) depend on the units

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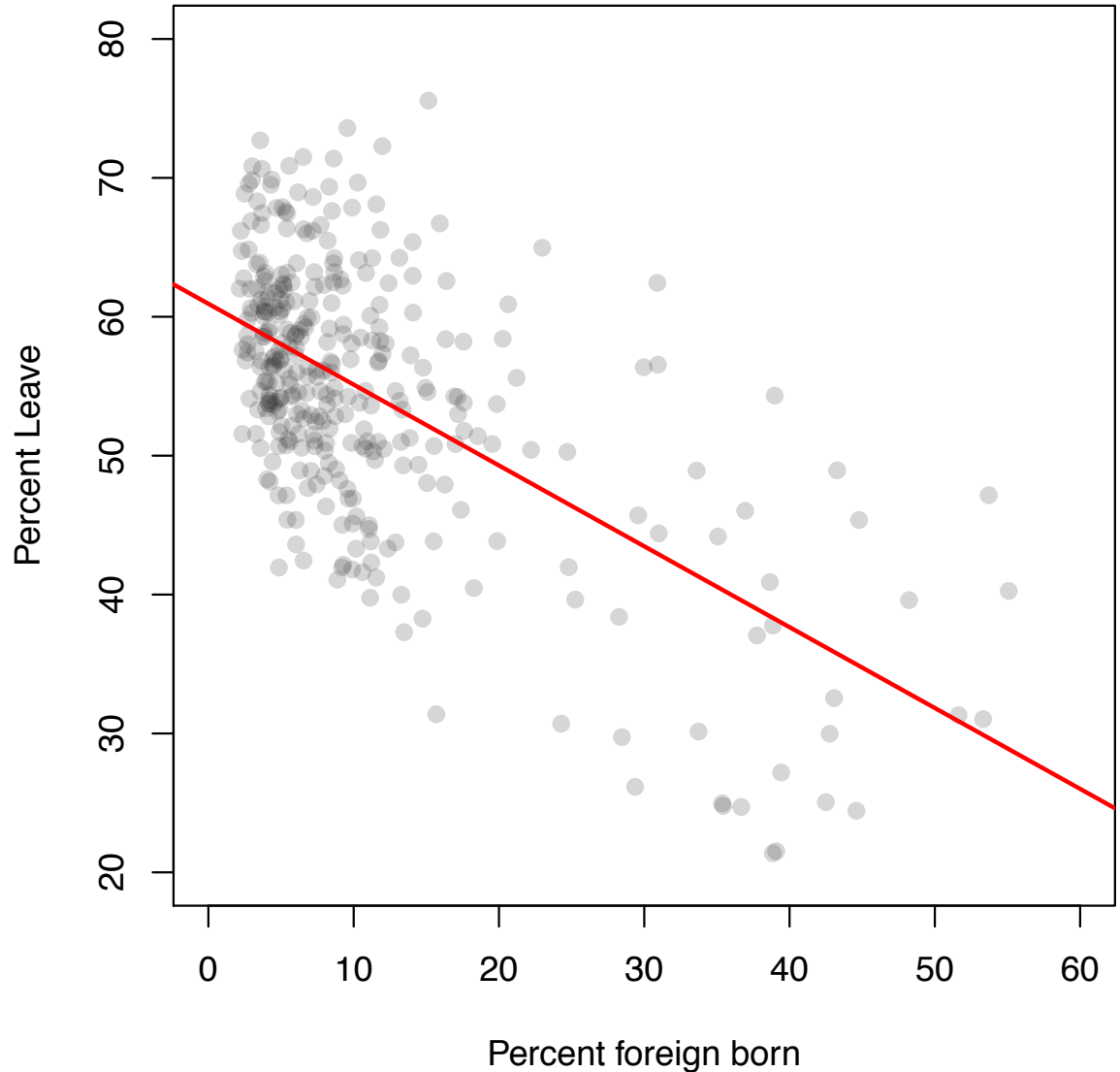
Regression slope (but not covariance or correlation) depends on which is Y and which is X

Covariance and regression slope (but not correlation) depend on the units

To discuss

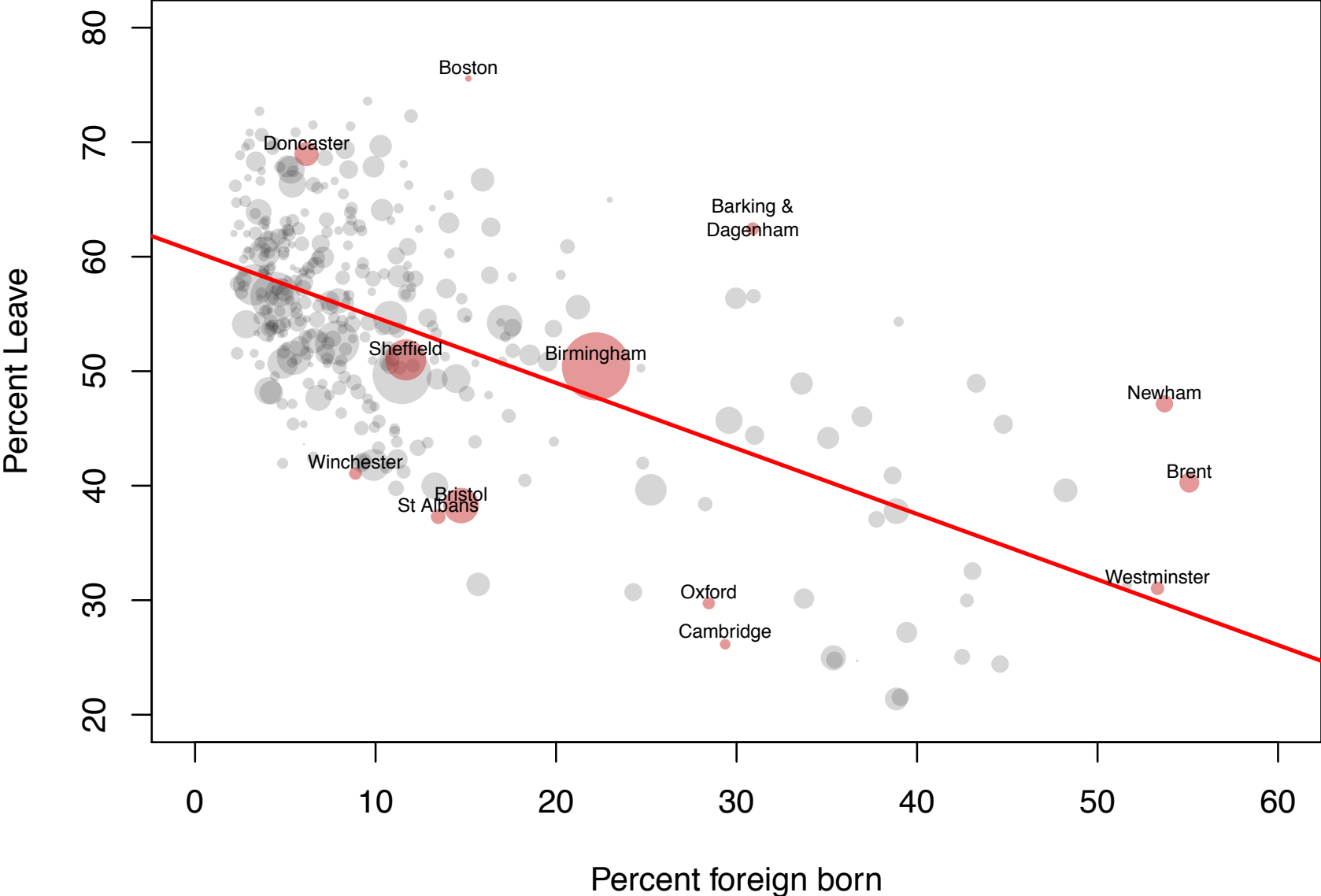
What other options could you imagine for deciding on a predictive line?

What are the advantages of OLS?



**Summarizing multivariate
relationships:
motivation and one non-OLS solution**

Did this pattern arise because contact with immigrants makes people less opposed to immigration?



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Ice cream
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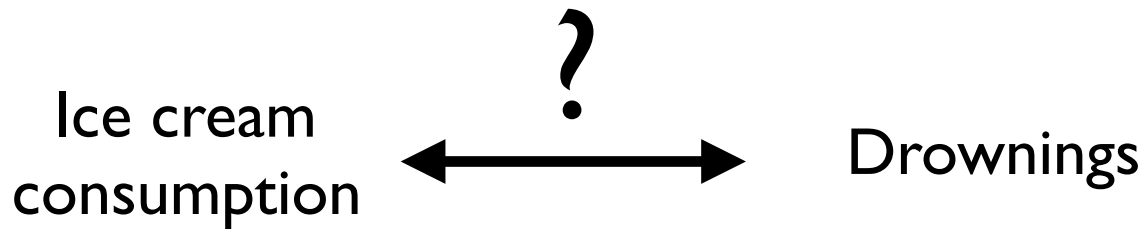
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Drownings

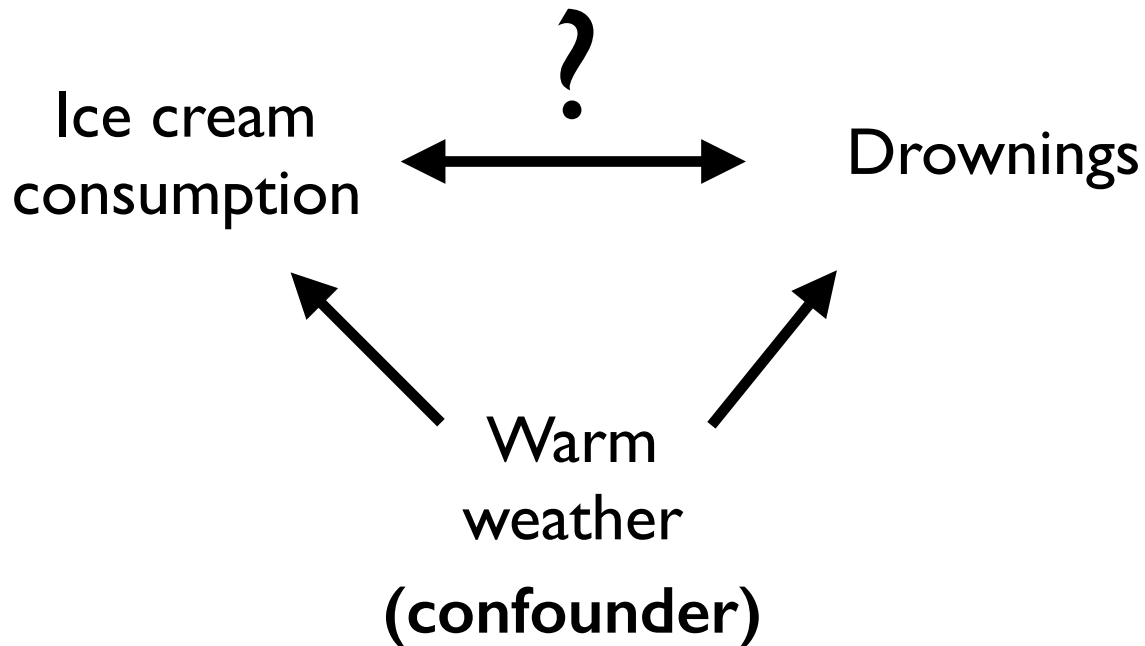
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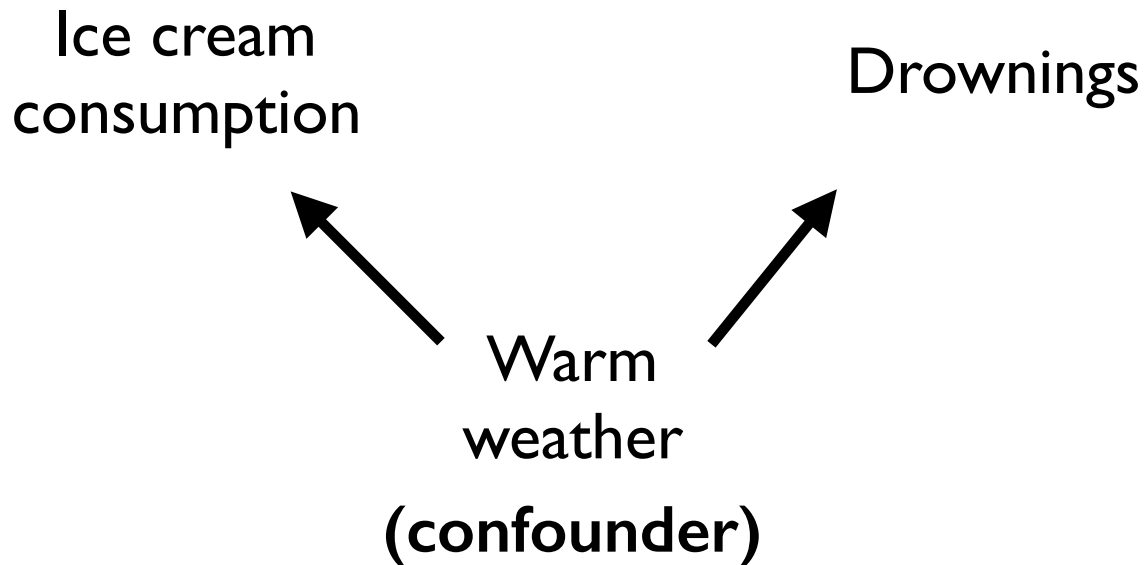
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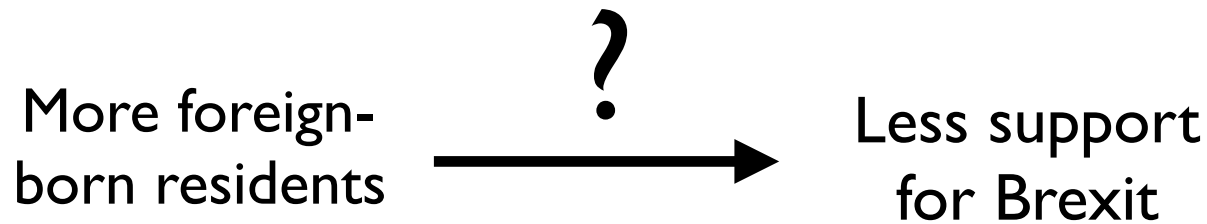
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More foreign-
born residents

Less support
for Brexit

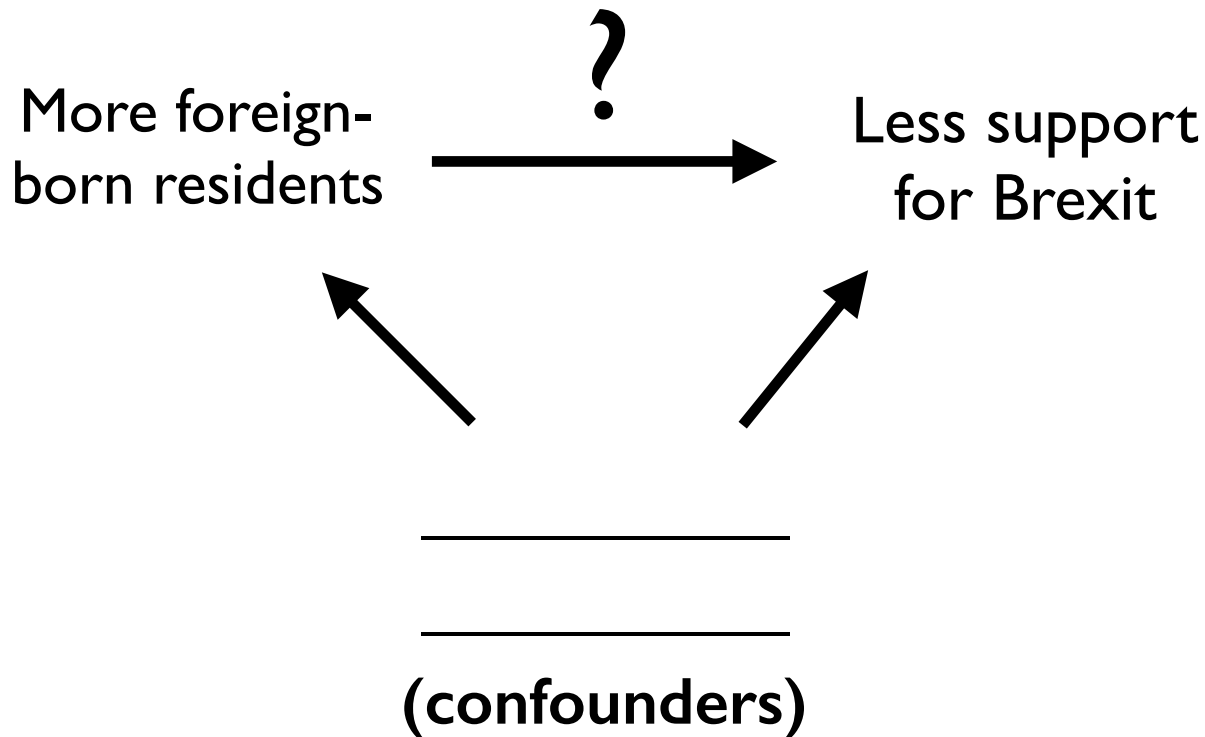
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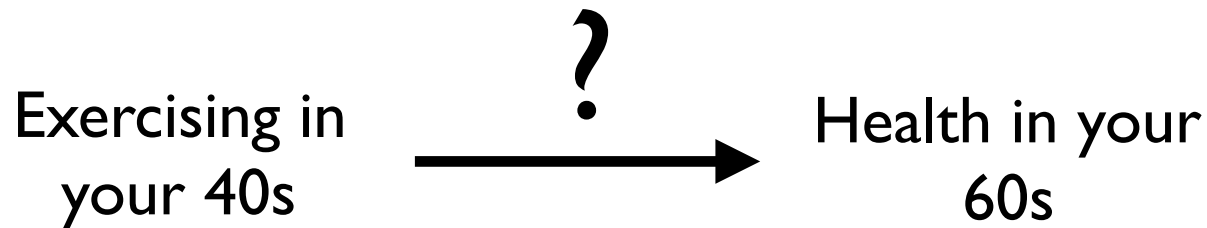
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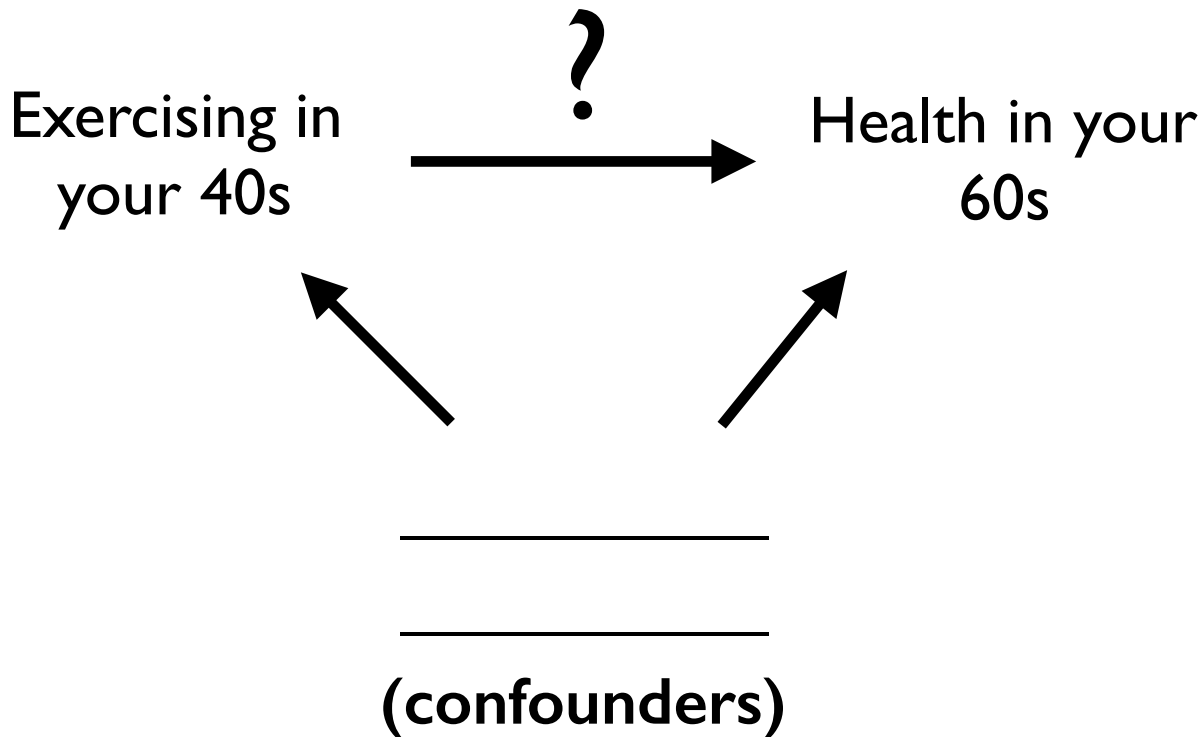
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- Are countries with more inclusive political systems less likely to experience violence, controlling for economic development and the number of ethnic groups?

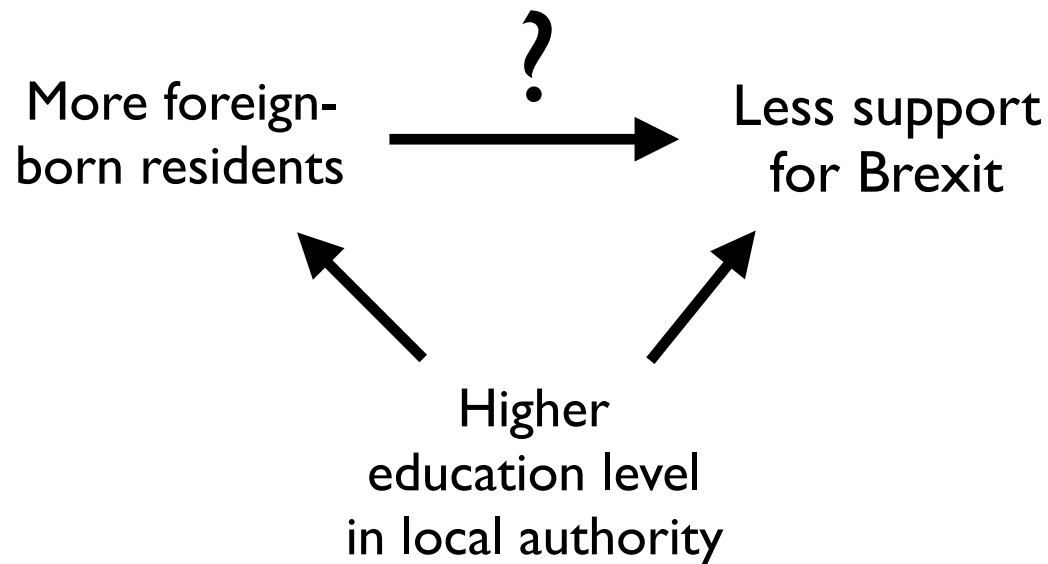
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- Are people who exercise less likely to develop dementia, controlling for diet and age?
- Are countries with more inclusive political systems less likely to experience violence, controlling for economic development and the number of ethnic groups?
- Are local authorities with more foreign-born residents less likely to support Brexit, controlling for _____?

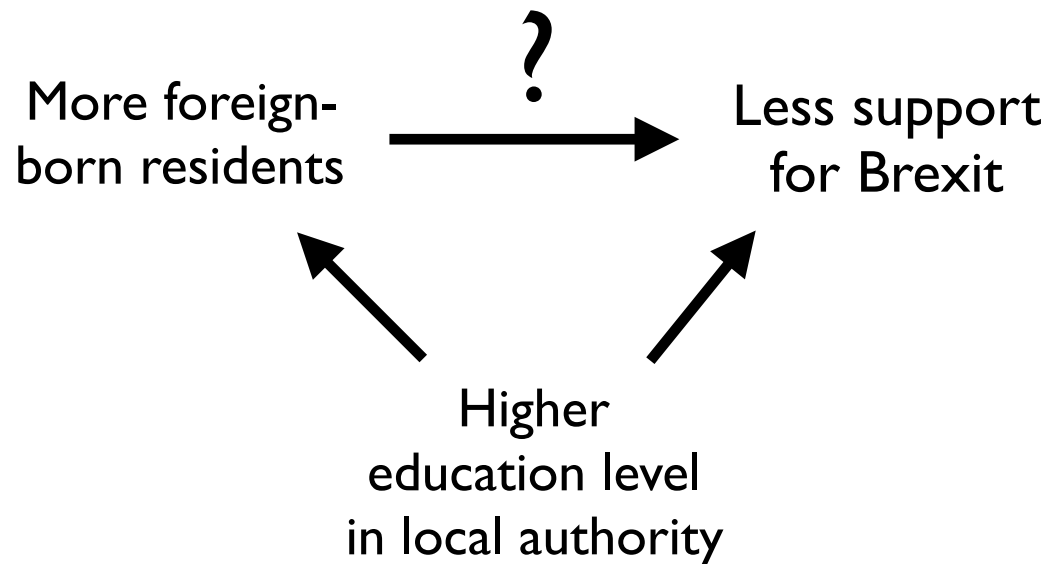
How do we control for confounders?

Let's focus on education as a confounder in our Brexit example:



How do we control for confounders?

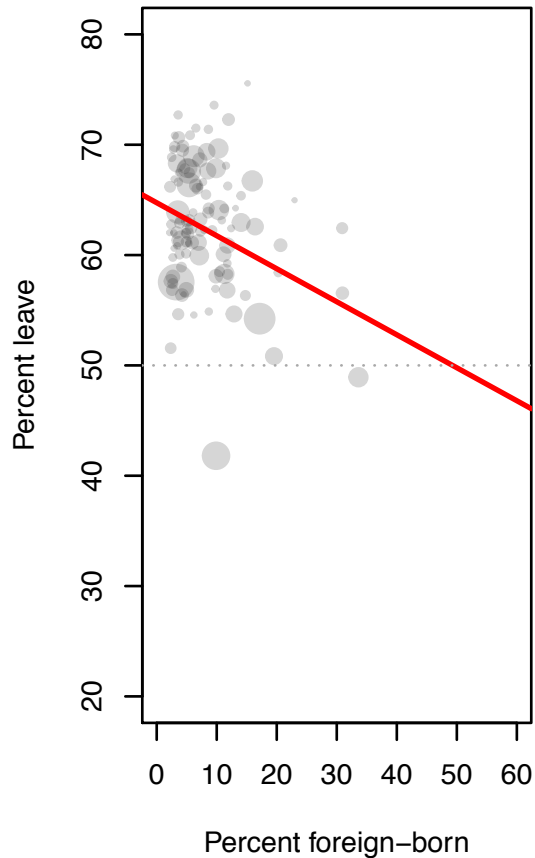
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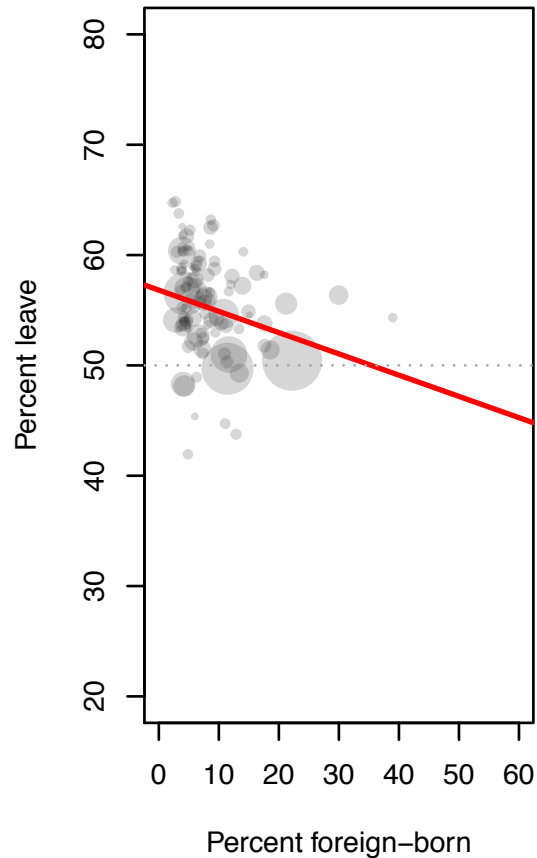
How can we measure the relationship between a local authority's proportion of foreign-born residents and its support for Brexit, controlling for its education level?

One idea: stratify by education level

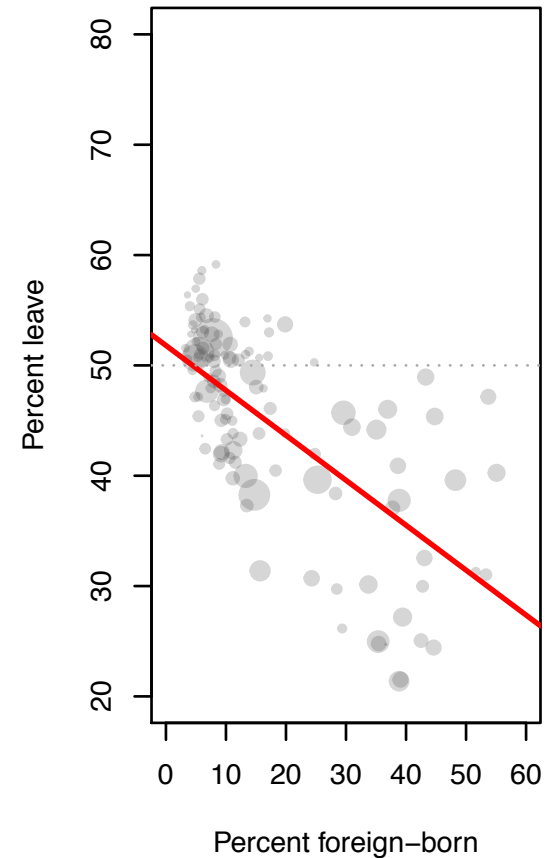
**Percent with bachelors:
Lowest third**



**Percent with bachelors:
Middle third**



**Percent with bachelors:
Highest third**



**Summarizing multivariate
relationships:
multivariate regression**

A more general approach: multivariate regression

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Goal: measure relationship between

- “support for Leave” and
- “% foreign-born”

controlling for “% bachelors degree”.

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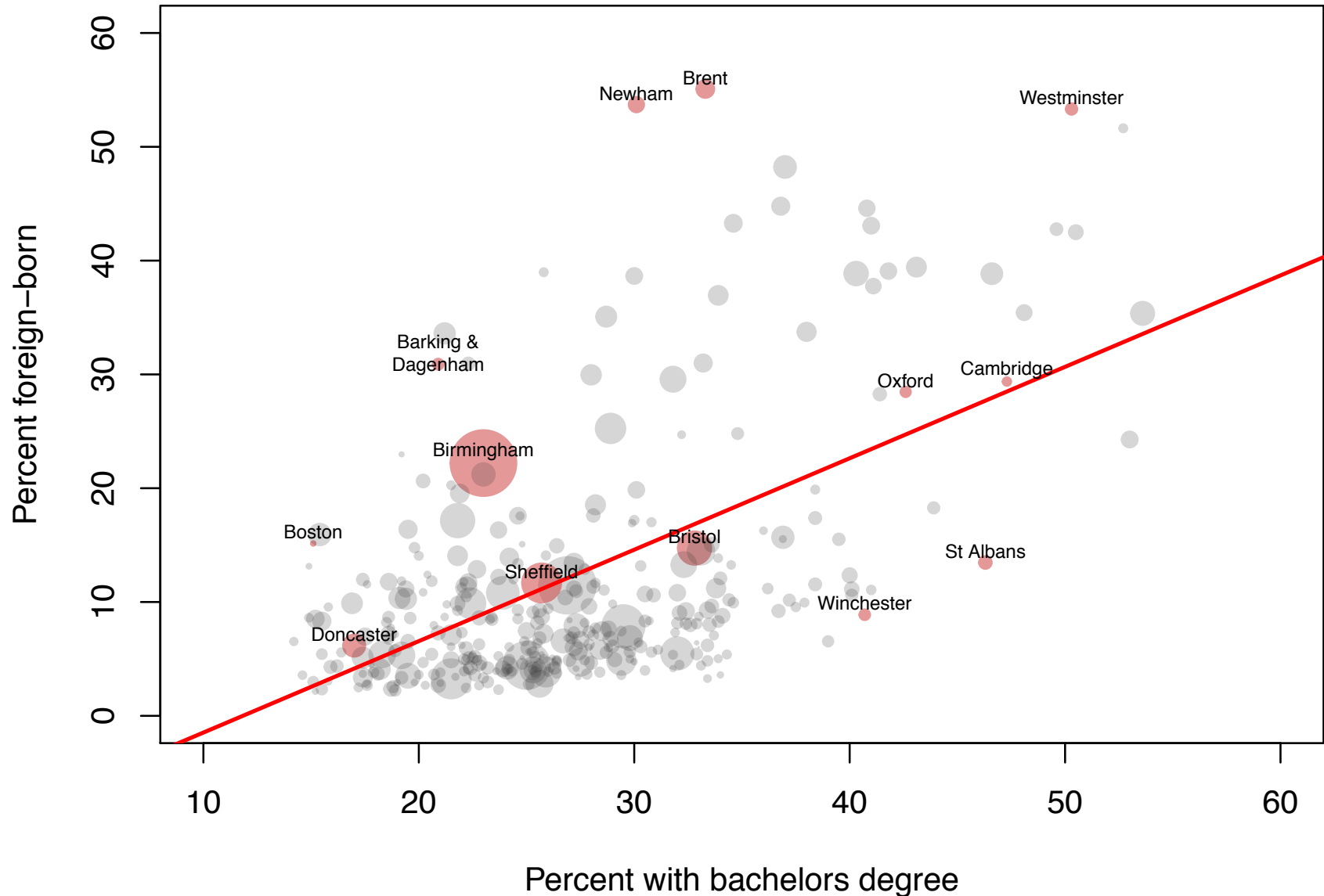
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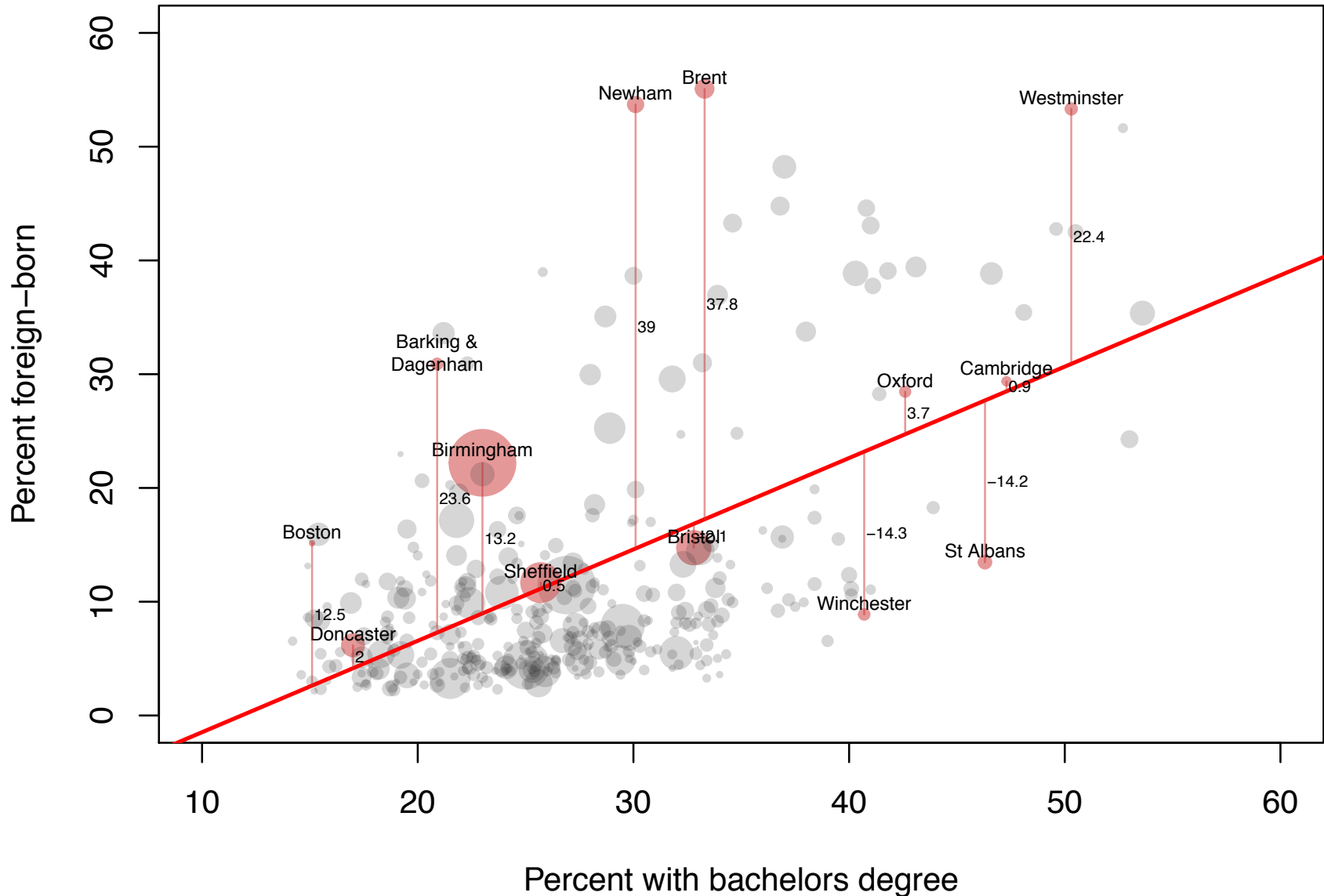
Basic idea: measure relationship between

- “support for Leave” and
- the part of “% foreign-born” that is not explained by “% bachelors degree”

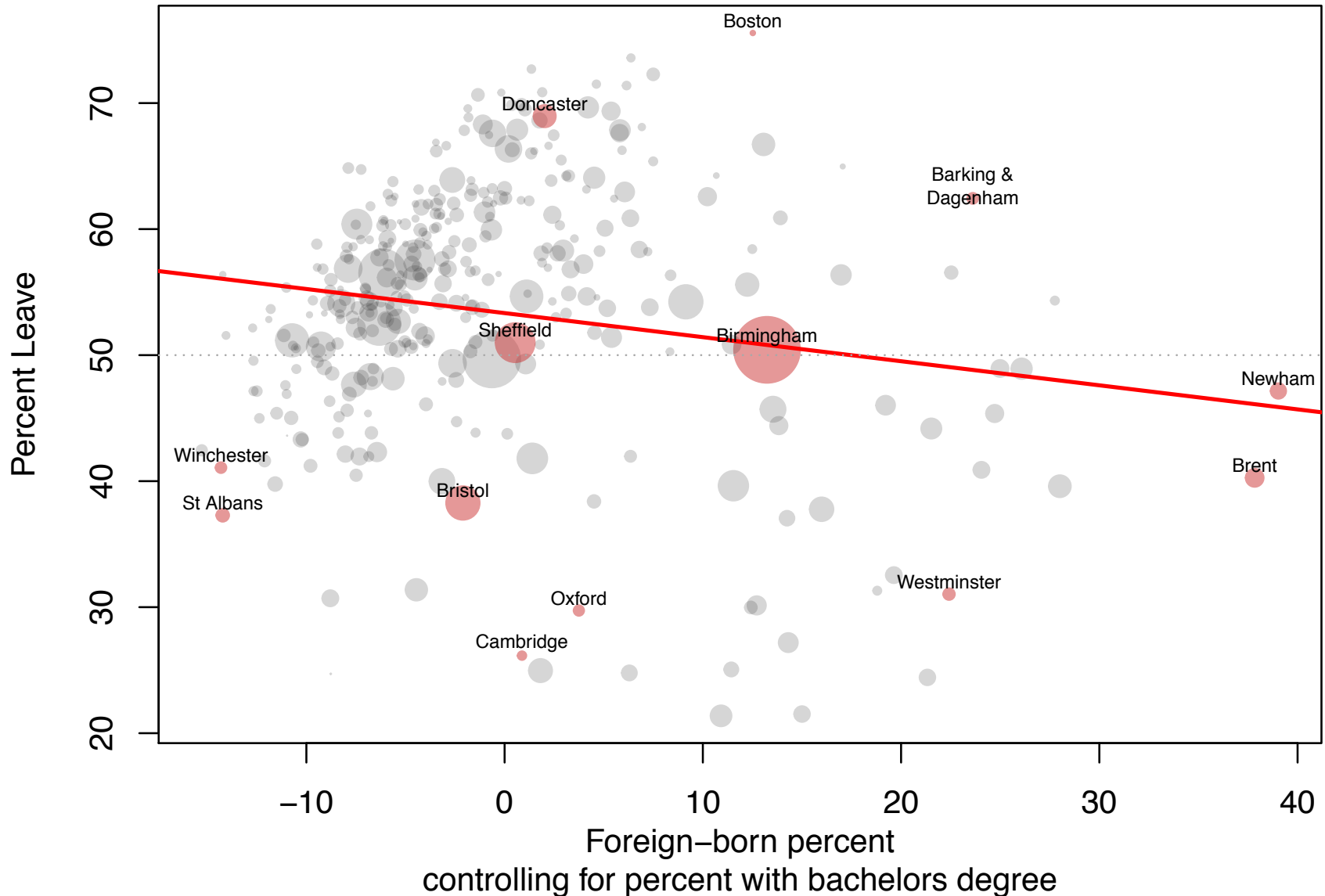
Step I: regress explanatory variable (%foreign-born) on confounder (education)



Step 2: calculate residuals, i.e. the part of %foreign-born not “explained” by education



Step 3: regress outcome (%leave) on those residuals



Group activity

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Using the data sheet on the handout, find each group member's local authority (or another if we've already highlighted yours!) on Figures 1, 2, and 3.

1. Was your local authority more supportive of Brexit than would be expected given its % foreign born? or less?
2. Does your local authority have a higher % foreign born than would be expected given its % with bachelors? or lower?
3. Was your local authority more supportive of Brexit than would be expected given its % foreign born, controlling for its % with bachelors? or less?

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Bivariate:

$$\text{PercentLeave} = \beta_0 + \beta_1 \text{PercentForeignBorn}$$

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Can also just think about minimizing sum of squared residuals for a different prediction equation.

Bivariate:

$$\text{PercentLeave} = \beta_0 + \beta_1 \text{PercentForeignBorn}$$

Multivariate:

$$\text{PercentLeave} = \beta_0 + \beta_1 \text{PercentForeignBorn} + \beta_2 \text{Education}$$

Implementing multivariate regression

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1. Use R to try every combination of 2 slopes and 1 intercept; choose the combination that has the lowest sum of squared residuals.
2. Use calculus to find the slope and intercept that minimize the sum of squared residuals.
3. Use `lm()` function in R:

```
> lm(d$Percent_Leave ~ d$Percent_foreign_born + d$Bachelors_deg_percent)
```

```
Call:
```

```
lm(formula = d$Percent_Leave ~ d$Percent_foreign_born + d$Bachelors_deg_percent)
```

```
Coefficients:
```

(Intercept)	d\$Percent_foreign_born	d\$Bachelors_deg_percent
83.1386	-0.1742	-0.9875

Inference

i.e. making claims beyond your sample

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How would you answer these questions?

Is there any **uncertainty** in your answers?

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No real uncertainty.

(Maybe about measurement.)

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(About the population)

Uncertainty due to **sampling variation.**

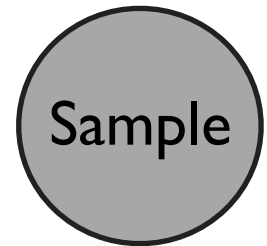
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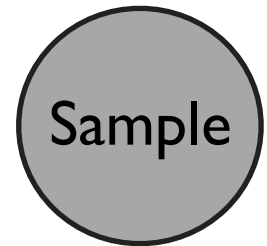
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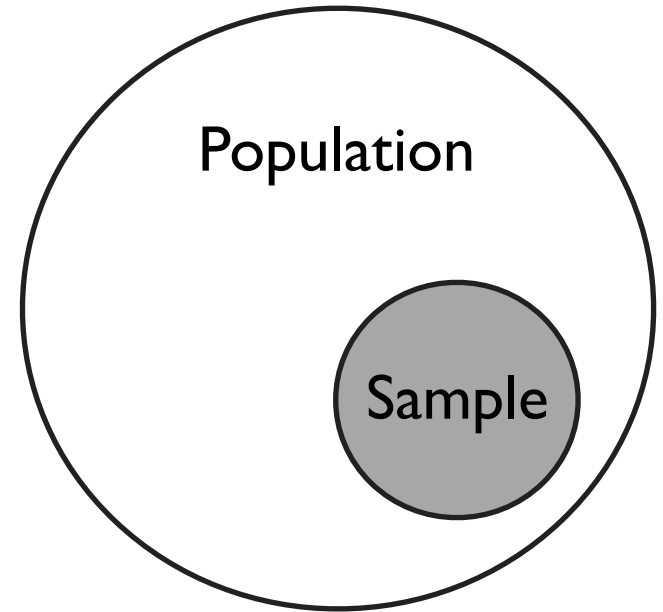


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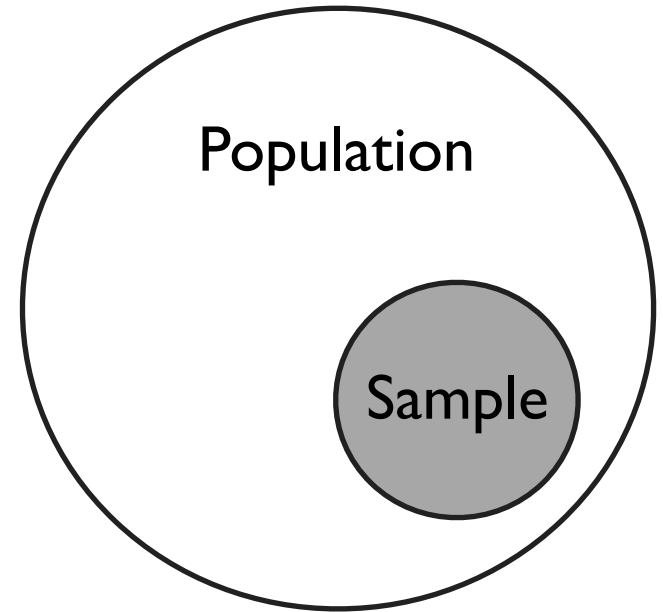
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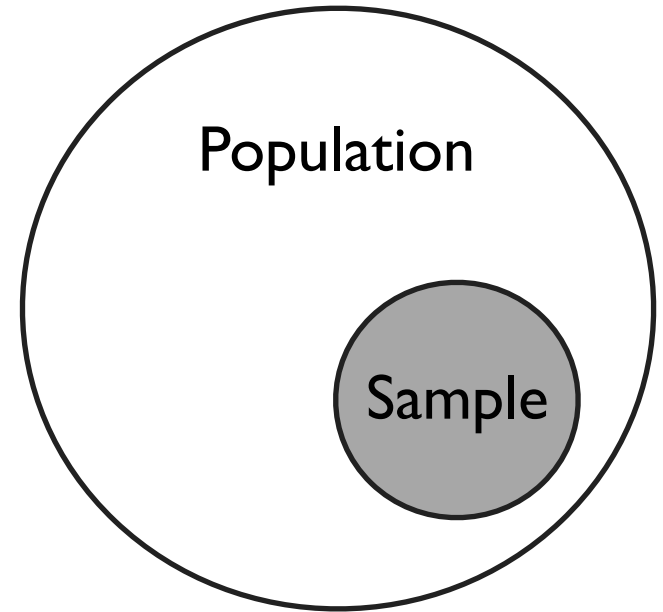
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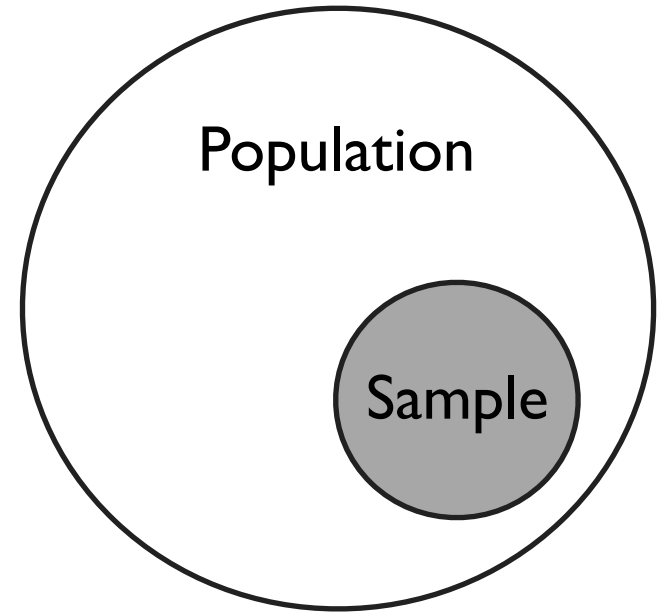


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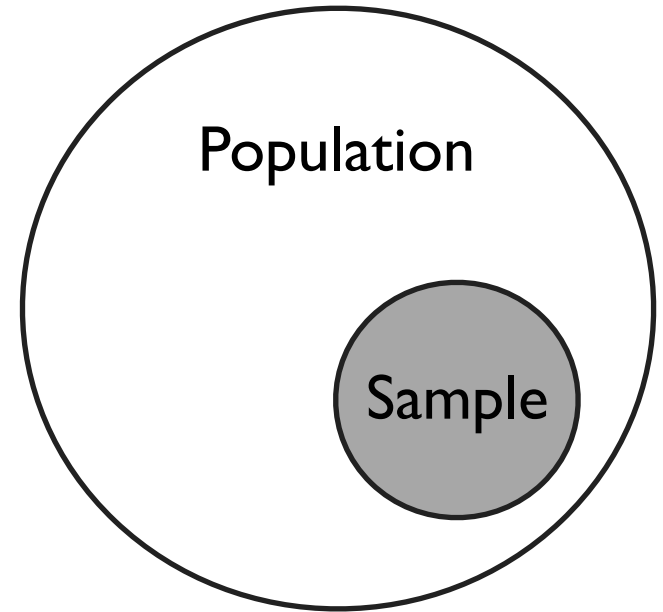
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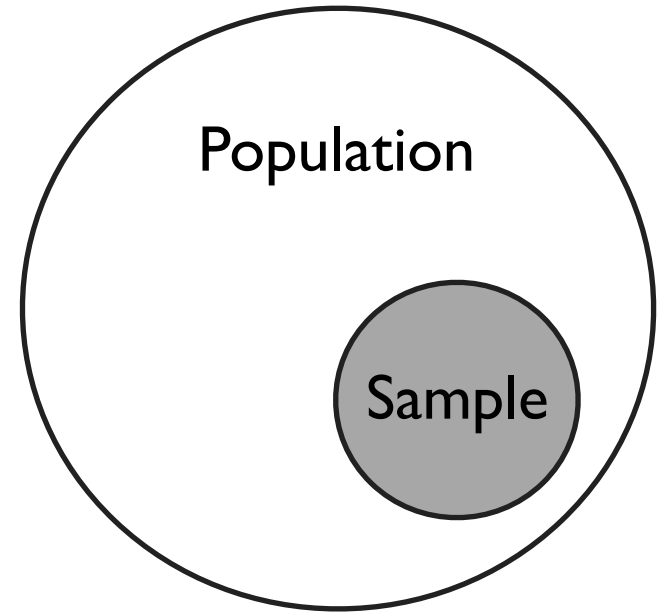
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- polls have a **margin of error**
- regression coefficients have **standard errors**
- our conclusions in hypothesis testing are guesses, with confidence summarized by **p-values**

Thought experiment

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What would the magnitude of this **random error** depend on?

- size of sample (1,006 GB adults vs. 10,000,000)
- true level of support (what if 100% supported remaining in EU?)

Simulating the thought experiment in R

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I can increase the number of “respondents” to 1,006:

```
> sample(x = c(0,1), size = 1006, replace = T, prob = c(.48, .52))  
[1] 0 0 0 0 0 0 0 1 1 1 1 1 0 1 1 1 1 1 1 1 1 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 1  
[39] 0 0 0 1 1 1 1 0 0 0 1 0 0 1 0 1 1 1 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 1  
[77] 1 0 0 1 1 1 0 0 0 0 0 1 1 1 1 1 1 1 0 0 0 0 1 0 0 0 0 0 1 1 0 0 1 1 0 0 1  
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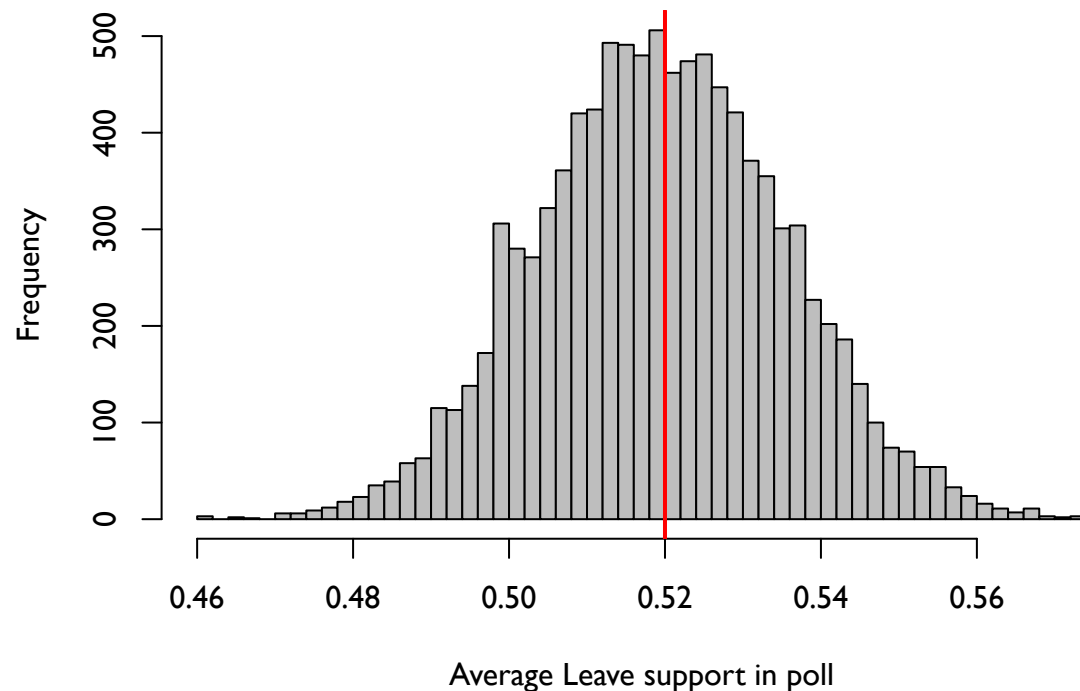
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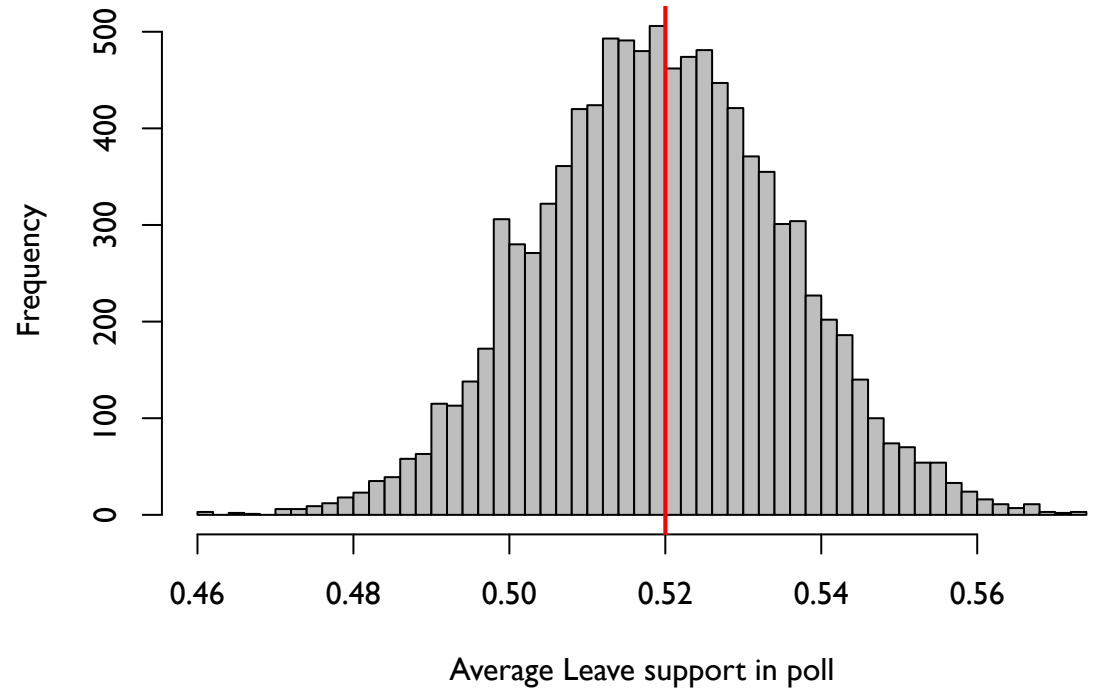
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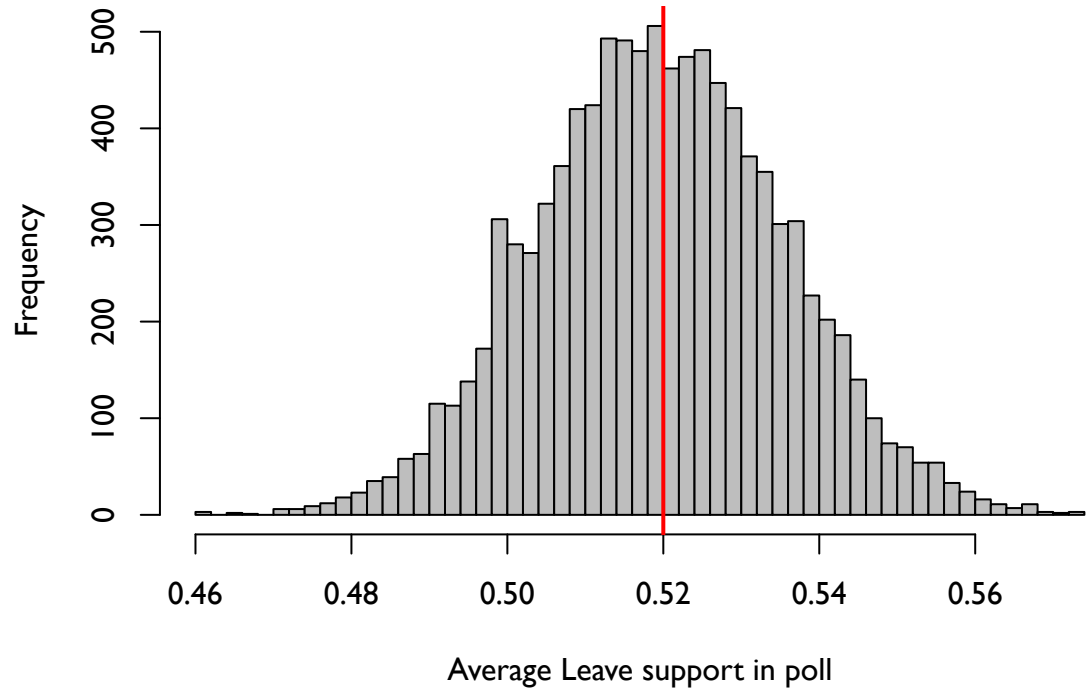


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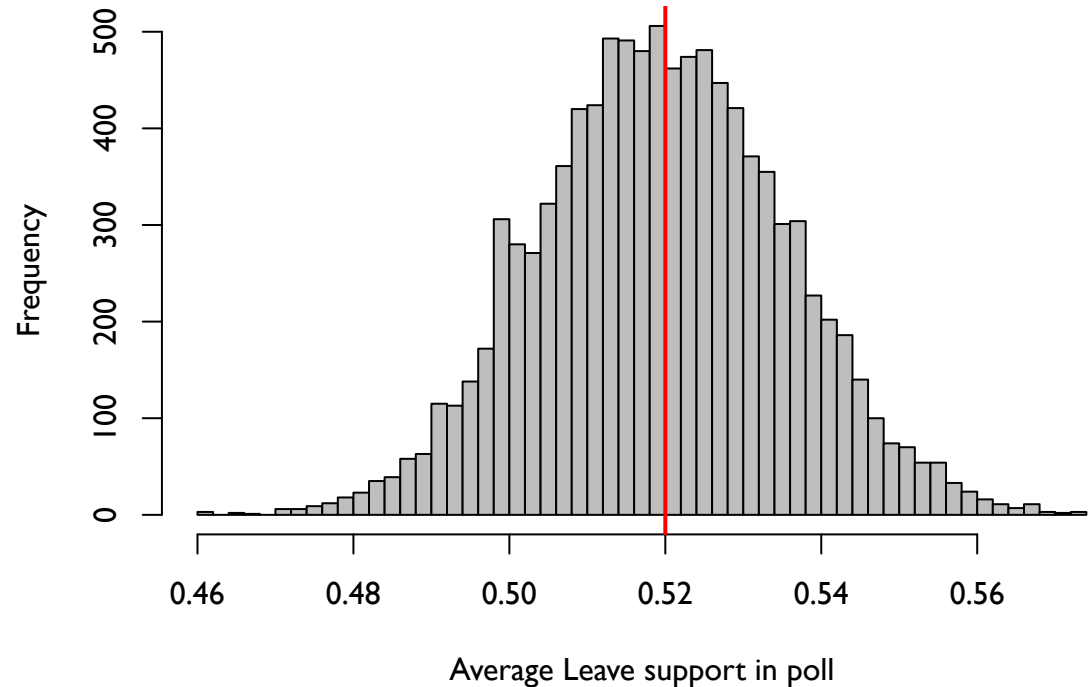
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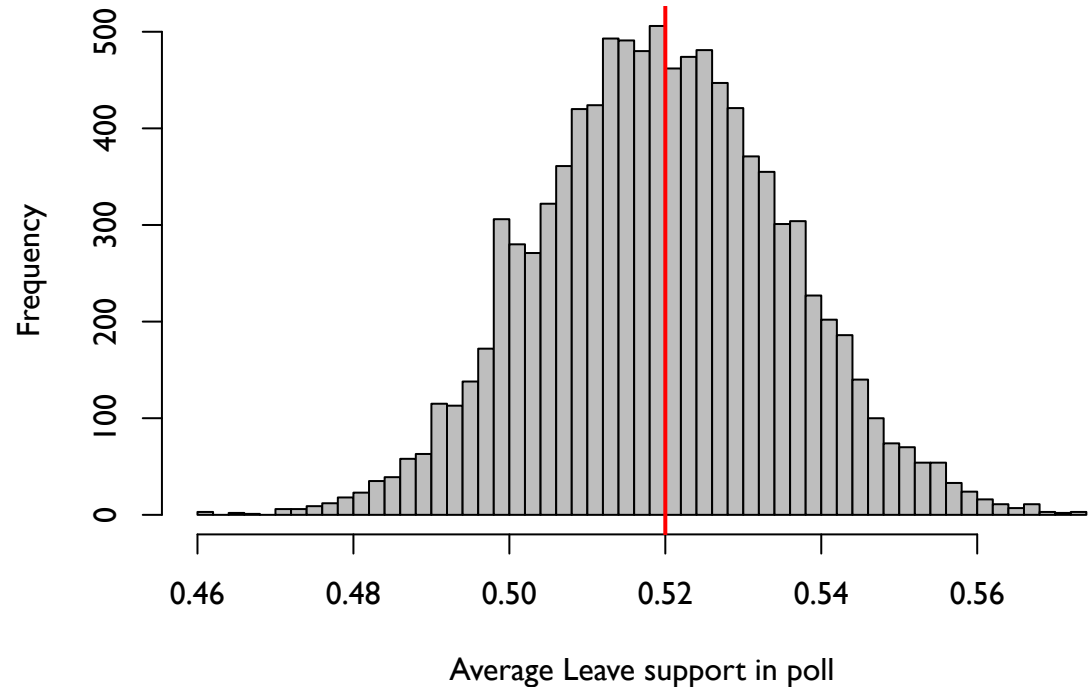
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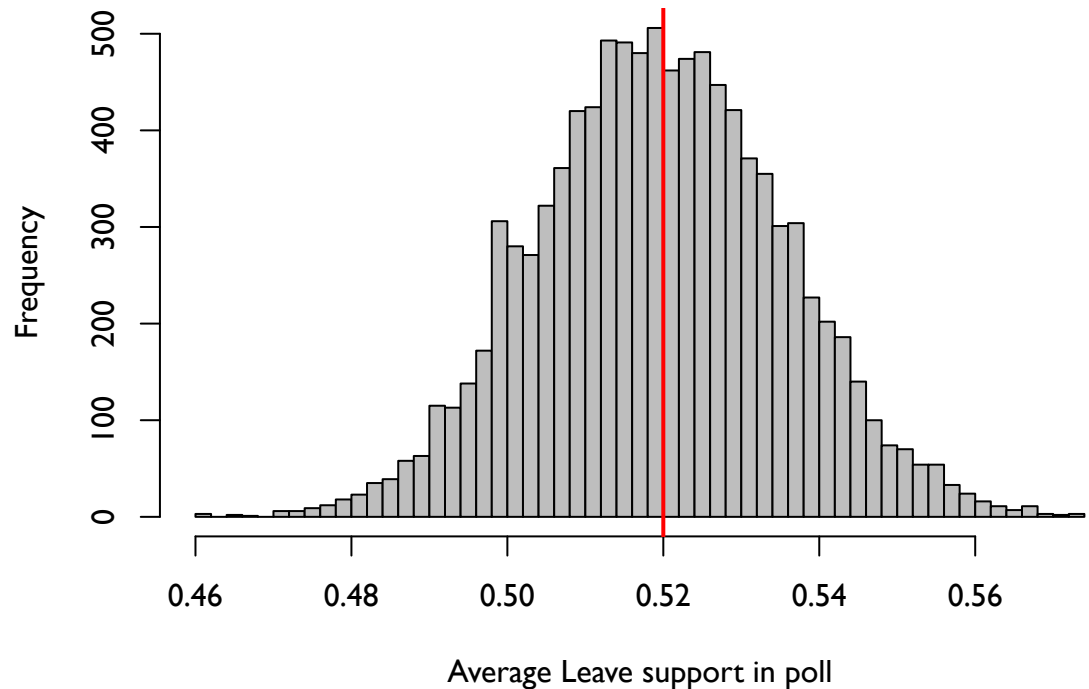
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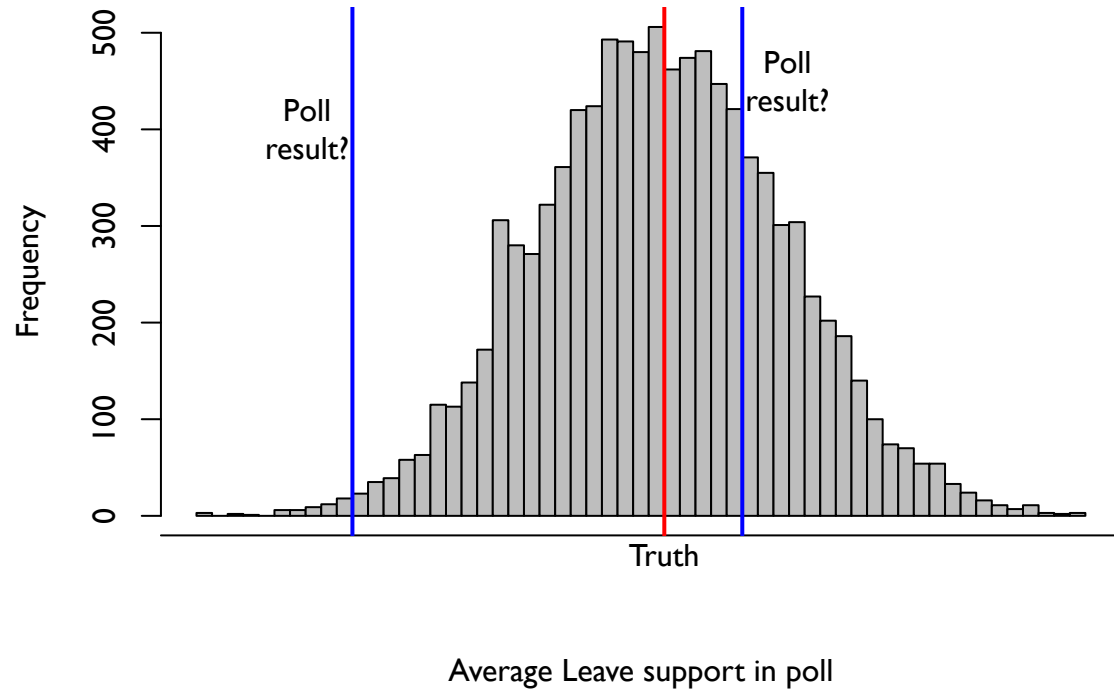
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95% of the samples had a mean between 0.49 and 0.55:

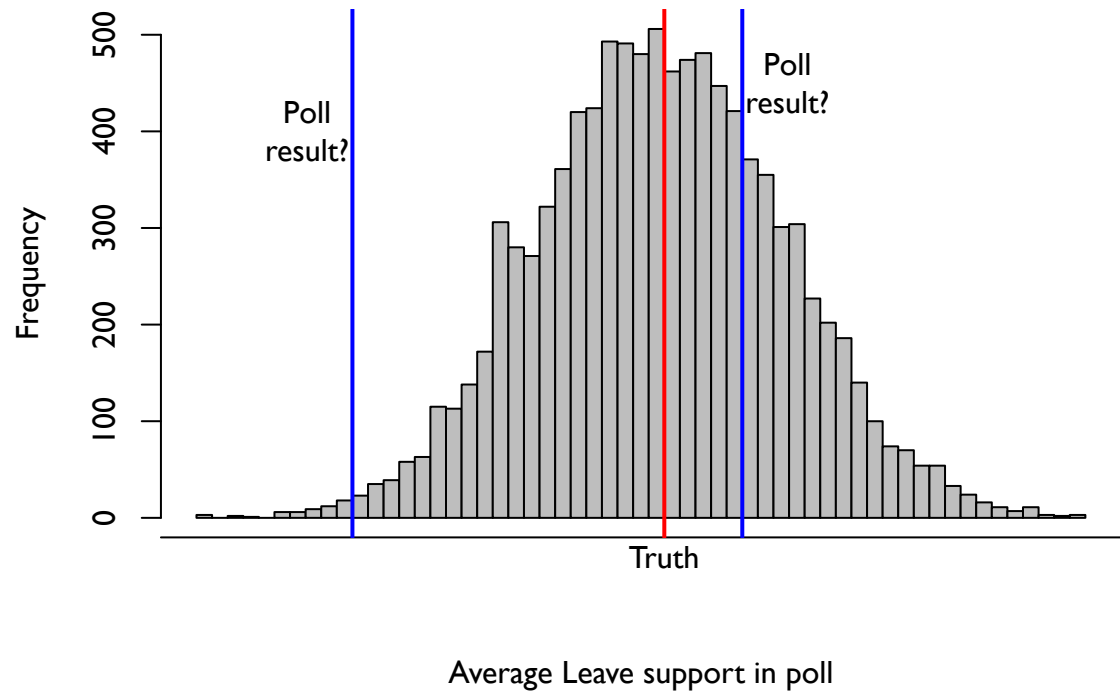
```
> quantile(poll.results, c(.025, .975))
      2.5%      97.5%
0.4890656 0.5497018
```

From thought experiment to margin of error



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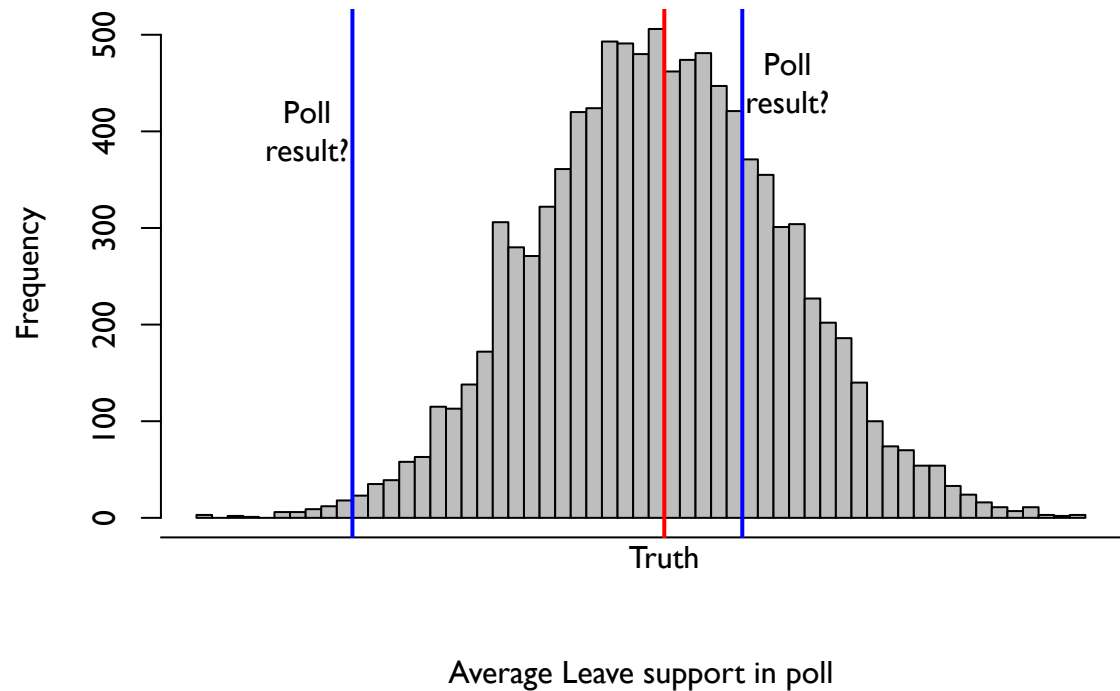
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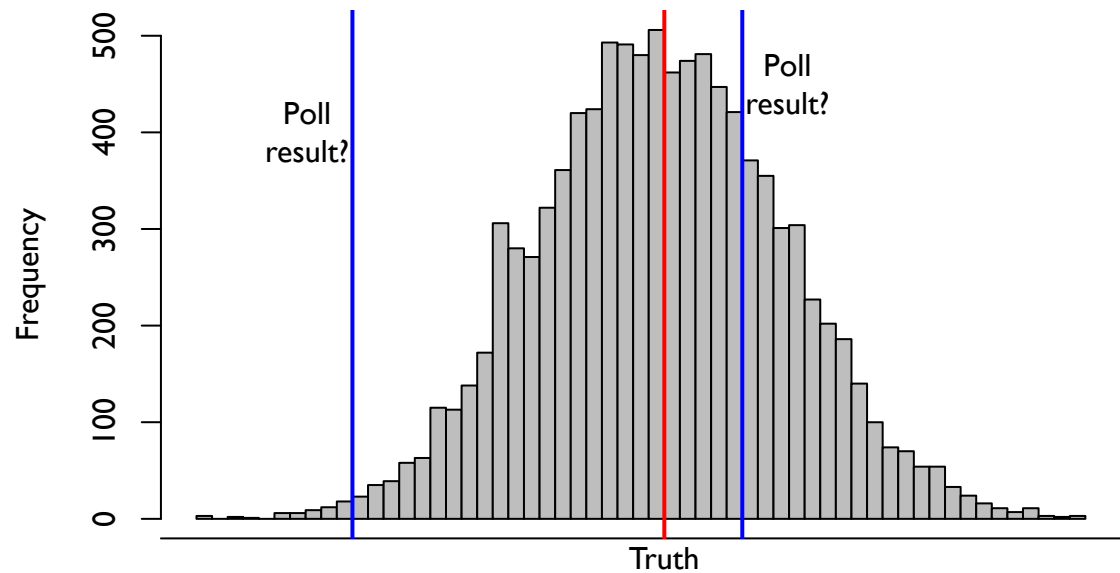
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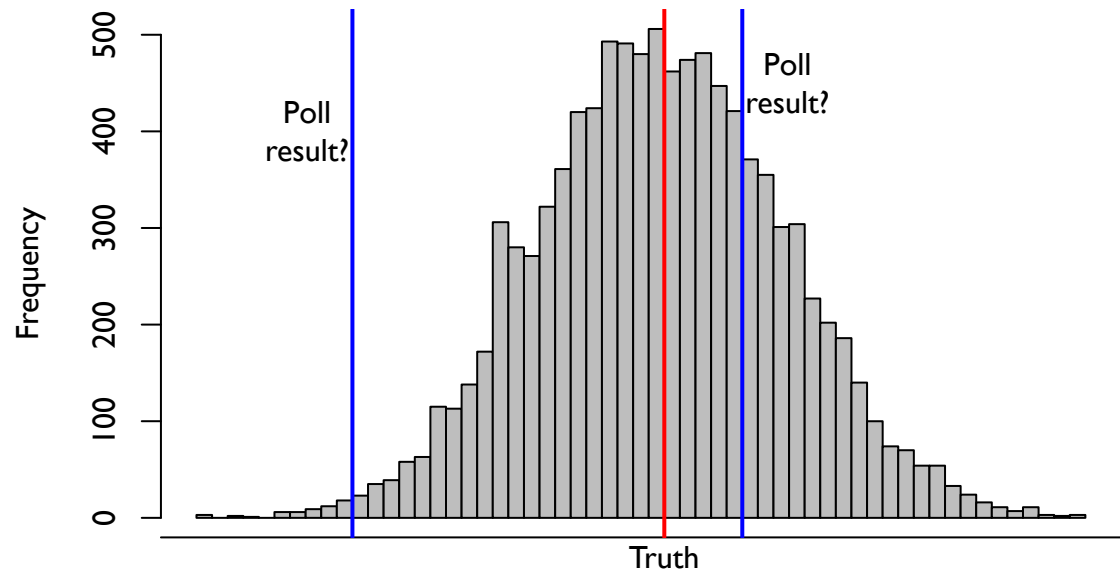
Average Leave support in poll

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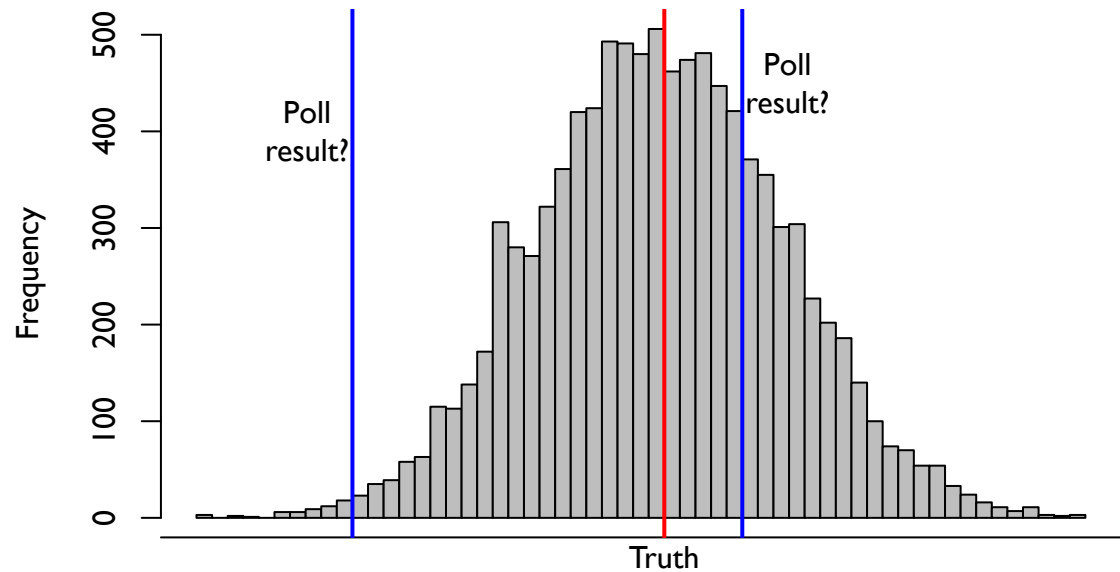
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From thought experiment to margin of error (2)

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An estimate for Leave support (e.g. 49%)

(From the thought experiment:) An estimate of the standard deviation of poll results across samples: 0.0157 (called the **standard error** of the poll)

(Combining the two:) A 95% confidence interval, which we expect to include the truth in 95% of samples: e.g. $49\% \pm 3.1\%$ (3.1% is the **margin of error** of the poll)

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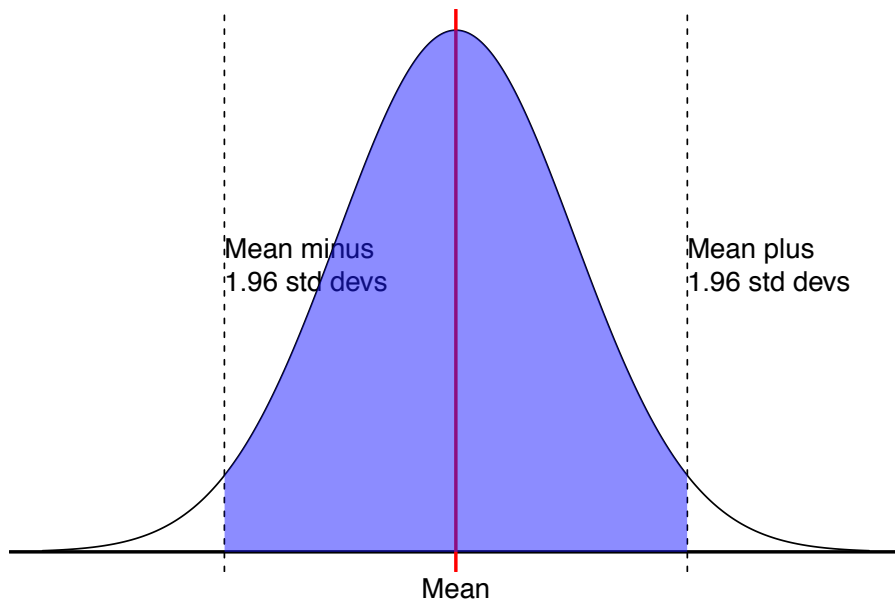
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$$\sqrt{\frac{.248}{1006}} = 0.0157$$

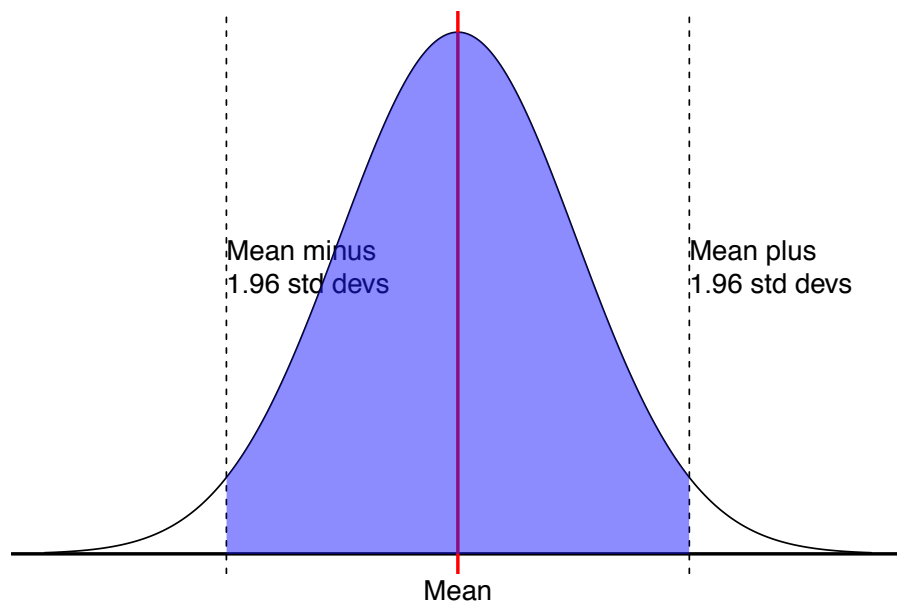
Compare: the standard deviation of our simulations was 0.0157

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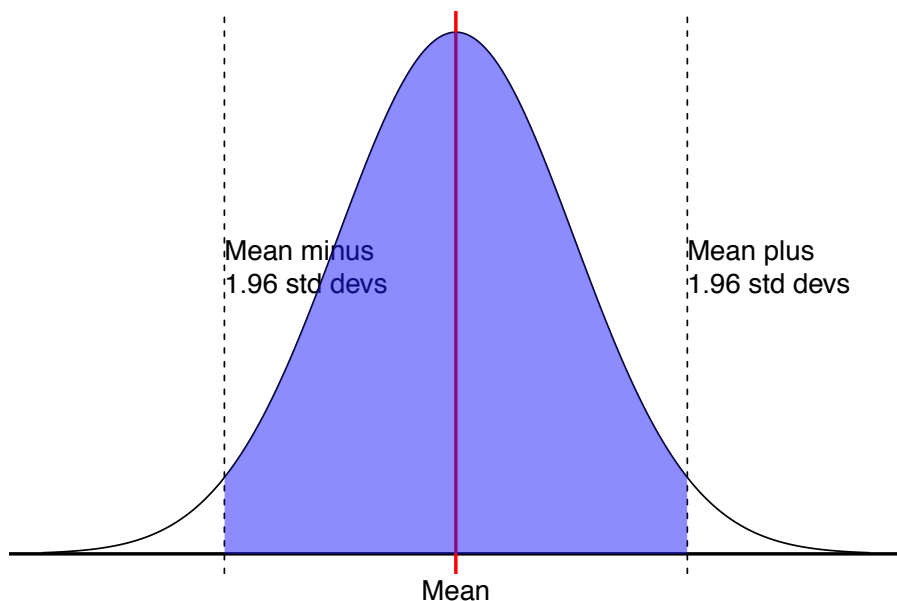
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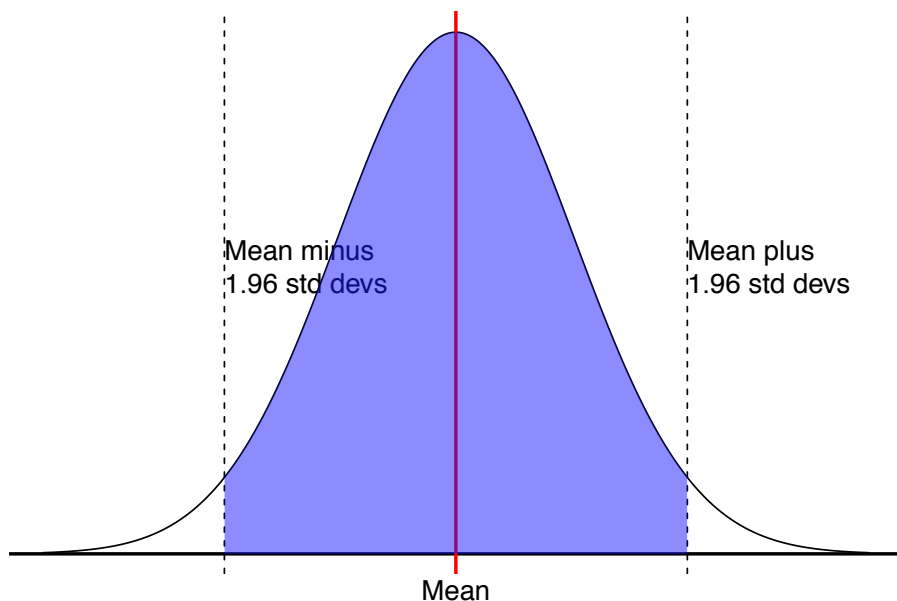


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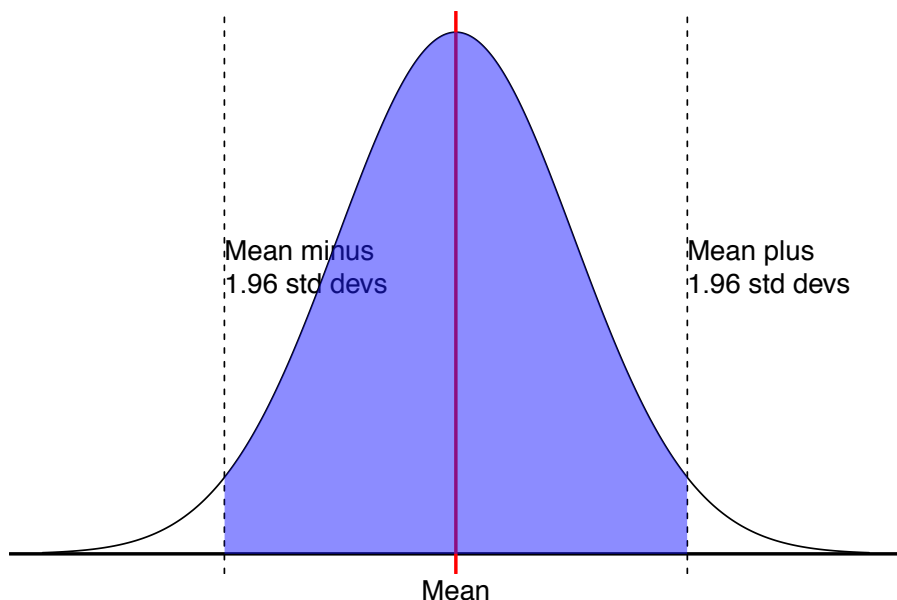
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But what if we draw a random sample and run this regression in our sample? How far off might the coefficients be?

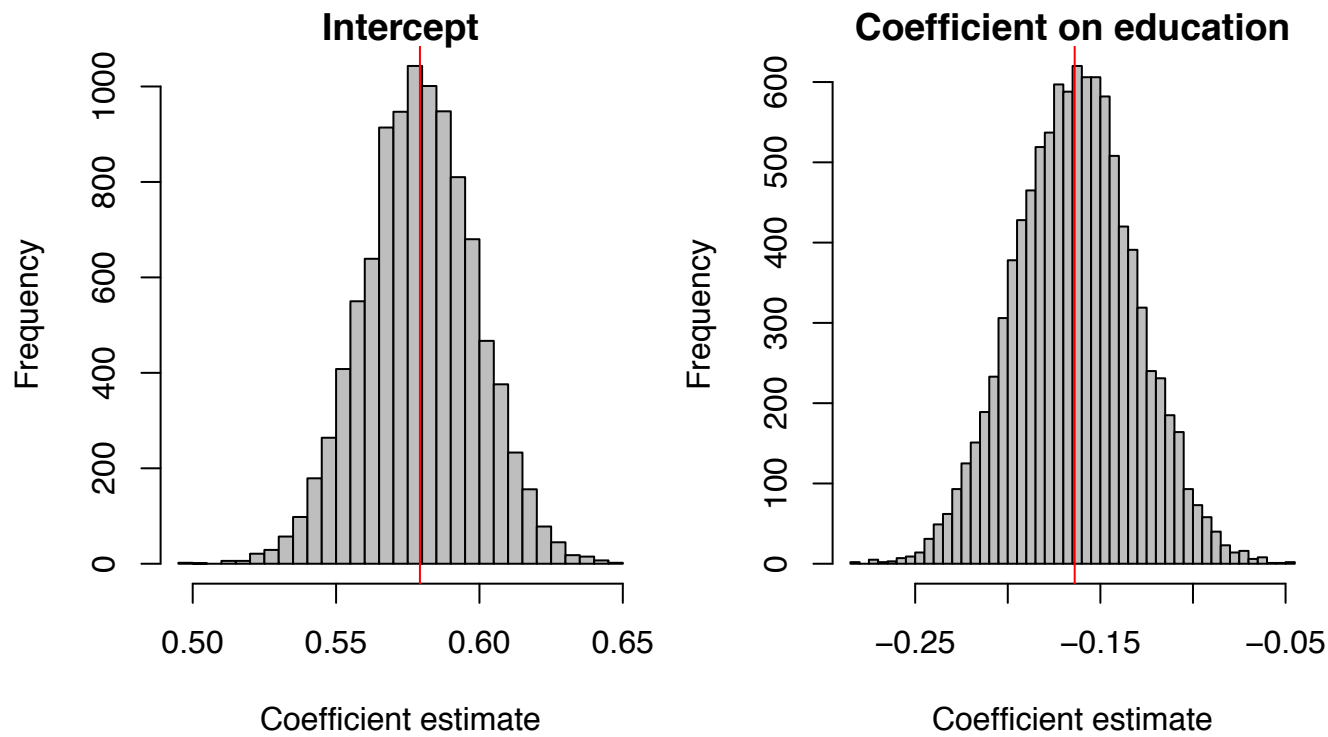
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As with the simple polling case, we can also use some statistical theory to estimate the standard errors given a sample (i.e. without doing a simulation).

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lm(formula = support.leave[indices.to.sample] ~ attended.uni[indices.to.sample])
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Residuals:

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      Min       1Q   Median       3Q      Max
-0.5987 -0.5987  0.4013  0.4013  0.5550
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Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
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Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

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Very close to our estimates of the standard errors from simulation.

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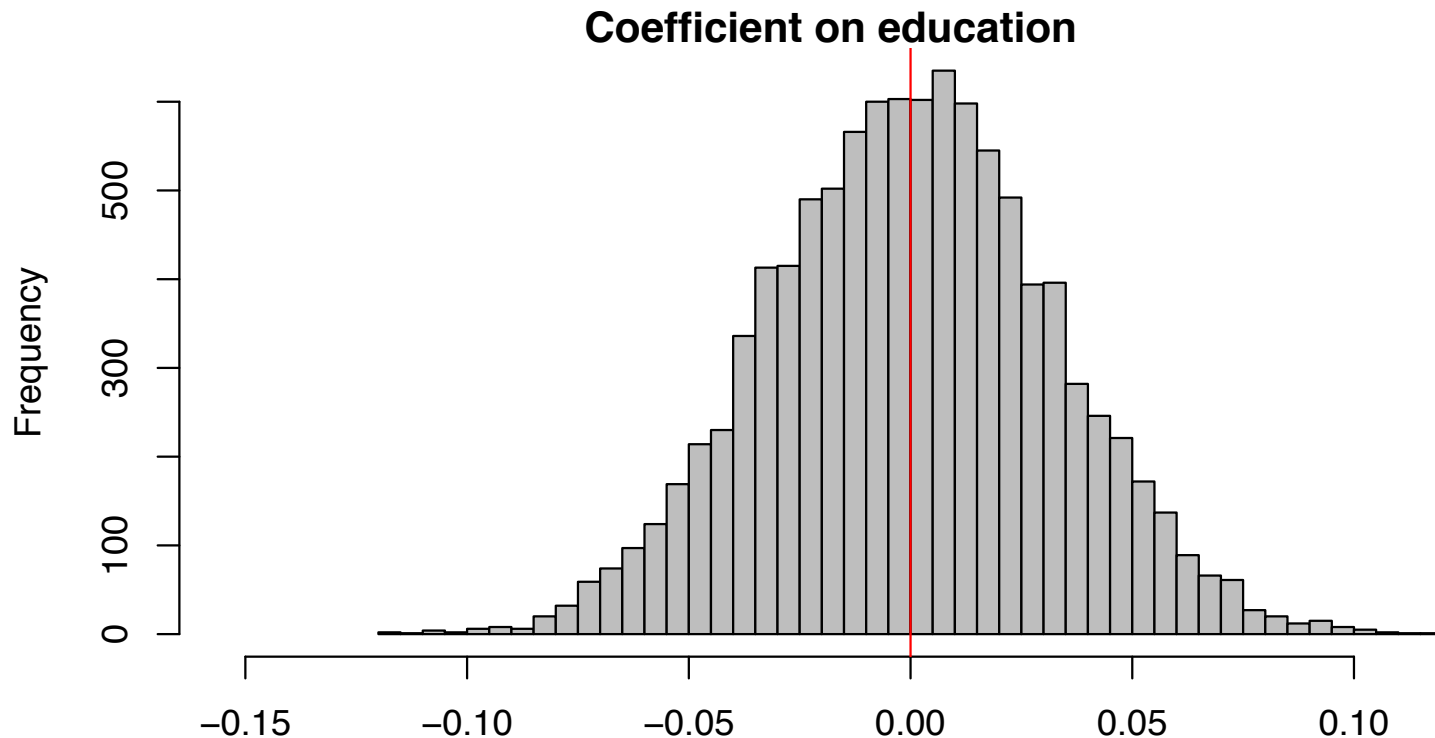
We want to know: have you proven conclusively that university attendance is related to Brexit support *in the population*, or might it just be a fluke *in your sample*?

Put differently: How likely is it that you would get a coefficient that far from 0 in your sample if the true coefficient were in fact 0?

Hypothesis testing for regression coefficients (2)

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The sampling distribution of the coefficient on *AttendedUni*, if the true coefficient were 0, would be something like



Sampling distribution under the null

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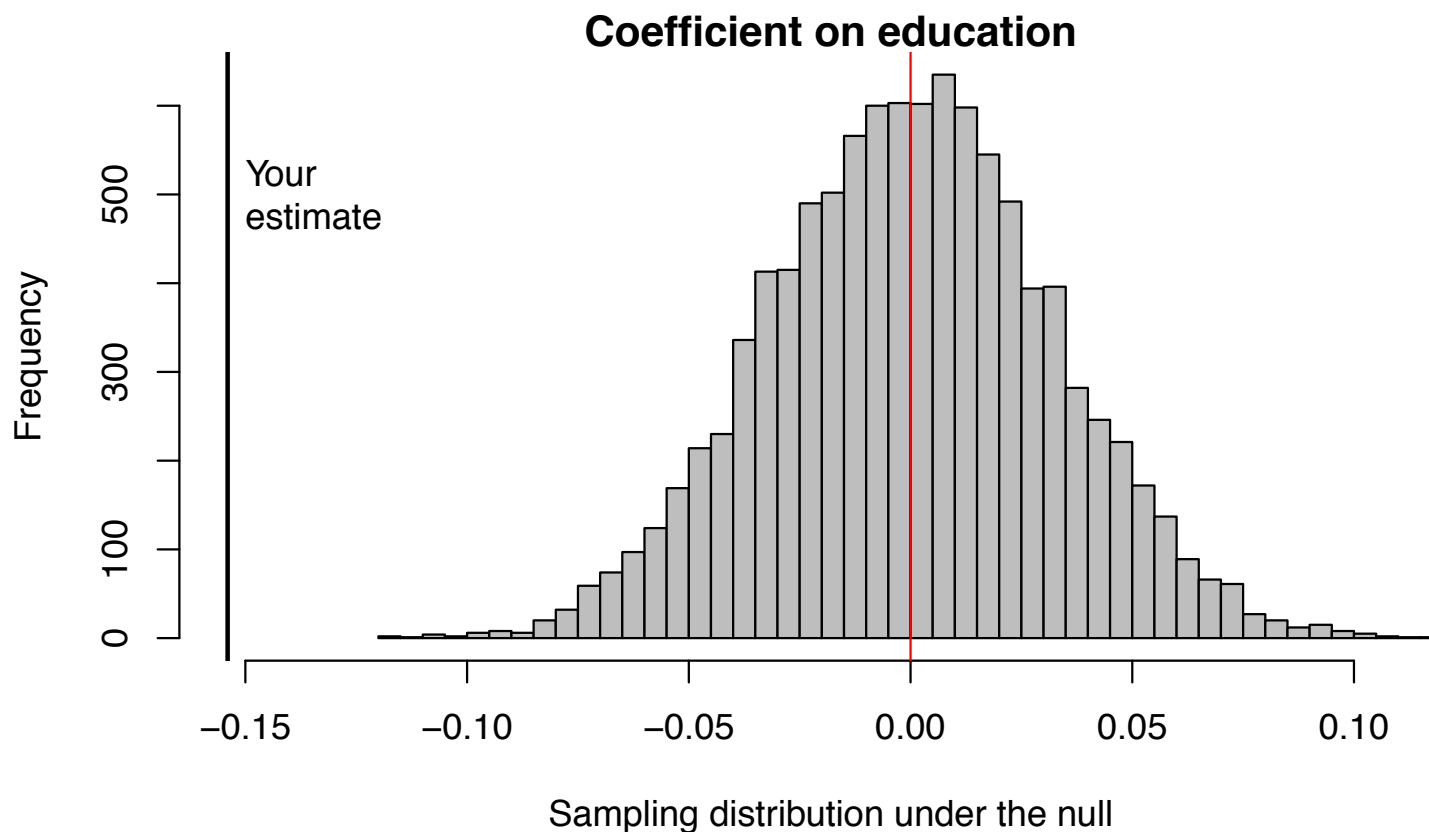
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Basically zero.

Now you should understand:

Dependent variable: Nobel Prizes awarded per capita (in log scale)

	(1)	(2)	(3)
Intercept	-1.629* (0.509)	-3.166* (0.511)	-2.982* (0.527)
Chocolate consumption per capita (log scale)	2.092* (0.298)	1.026* (0.326)	0.709 (0.415)
GDP/capita (thousands of USD)		0.105* (0.024)	0.106* (0.024)
NW Europe			0.549 (0.452)
R ²	0.70	0.85	0.86
N	34	34	34

- what a dependent variable is
- what an independent variable is
- what the coefficients mean (intercept, slopes)
- what the stars mean (i.e. what $p < 0.05$ means)
- what the standard errors mean

Standard errors in parentheses. * Indicates $p < 0.05$

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The broader view is that history offers one sample, but if “re-run” it might have produced another. Less philosophically satisfying!