Inference

Week 7

29 February, 2016

Prof. Andrew Eggers

What we're trying to understand today

Dependent variable: Nobel Prizes awarded per capita (in log scale)

	(1)	(2)	(3)
Intercept	-1.629* (0.509)	-3.166* (0.511)	-2.982* (0.527)
Chocolate consumption per capita (log scale)	2.092* (0.298)	1.026* (0.326)	0.709 (0.415)
GDP/capita (thousands of USD)		0.105* (0.024)	0.106* (0.024)
NW Europe			0.549 (0.452)
R^2	0.70	0.85	0.86
N	34	34	34

Standard errors in parentheses. * Indicates p<0.05

- What do the stars mean on regression tables? Numbers in parentheses?
- What is the "margin of error" of a poll?
- What statistical findings are reliable? Which might be just a fluke?

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TABLE 15.2

Multivariate regression analyses of the effect of consensus democracy (executives-parties dimension) on five indicators of violence, with controls for the effects of the level of economic development, logged population size, and degree of societal division, and with extreme outliers removed

Performance variables	Estimated regression coefficient	Absolute t-value	Countries (N)
Political stability and absence of violence (1996–2009)	0.189***	3.360	34
Internal conflict risk (1990–2004)	0.346**	2.097	32
Weighted domestic conflict index (1981–2009)	−105.0*	1.611	30
Weighted domestic conflict index (1990–2009)	−119.7**	2.177	33
Deaths from domestic terrorism (1985–2010)	−2.357**	1.728	33

* Statistically significant at the 10 percent level (one-tailed test)

** Statistically significant at the 5 percent level (one-tailed test)

*** Statistically significant at the 1 percent level (one-tailed test)

Source: Based on data in Kaufmann, Kraay, and Mastruzzi 2010; PRS Group 2004; Banks, 2010; and GTD Team 2010

- What do the stars mean on regression tables? "t-values"?
- What is the "margin of error" of a poll?
- What statistical findings are reliable? Which might be just a fluke?

First task: understanding margin of error

ComRes

The margin of error shows the level of accuracy that a random sample of a given population has.

Our calculator gives the percentage points of error either side of a result for a chosen sample size.

It is calculated at the standard 95% confidence level. Therefore we can be 95% confident that the sample result reflects the actual population result to within the margin of error. This calculator is based on a 50% result in a poll, which is where the margin of error is at its maximum.

This means that, according to the law of statistical probability, for 19 out of every 20 polls the 'true' result will be within the margin of error shown.

http://www.comres.co.uk/our-work/margin-of-error-calculator/

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"Margin of error" tries to summarize the **magnitude of random error due to sampling**.

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- true level of support (what if 100% supported remaining in EU?)

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I can increase the number of "respondents" to 1,006:

```
> sample(x = c(0,1), size = 1006, replace = T, prob = c(.43, .57))
   [1] 0 1 1 1 0 1 1 0 1 1 0 1 1 1 1 0 0 1 1 1 1 0 0 1 1 0 1 1 1 0 1 1 1 1 0
  [50] 0 0 1 0 1 0 1 0 1 1 1 0 0 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 0 1 0 0 1
  [99] 1 1 1 0 0 0 1 1 0 0 1 0 1 1 0 1 1 0 1 1 0 1 0 0 0 1 1 1 0 0 1 0 1 1 0
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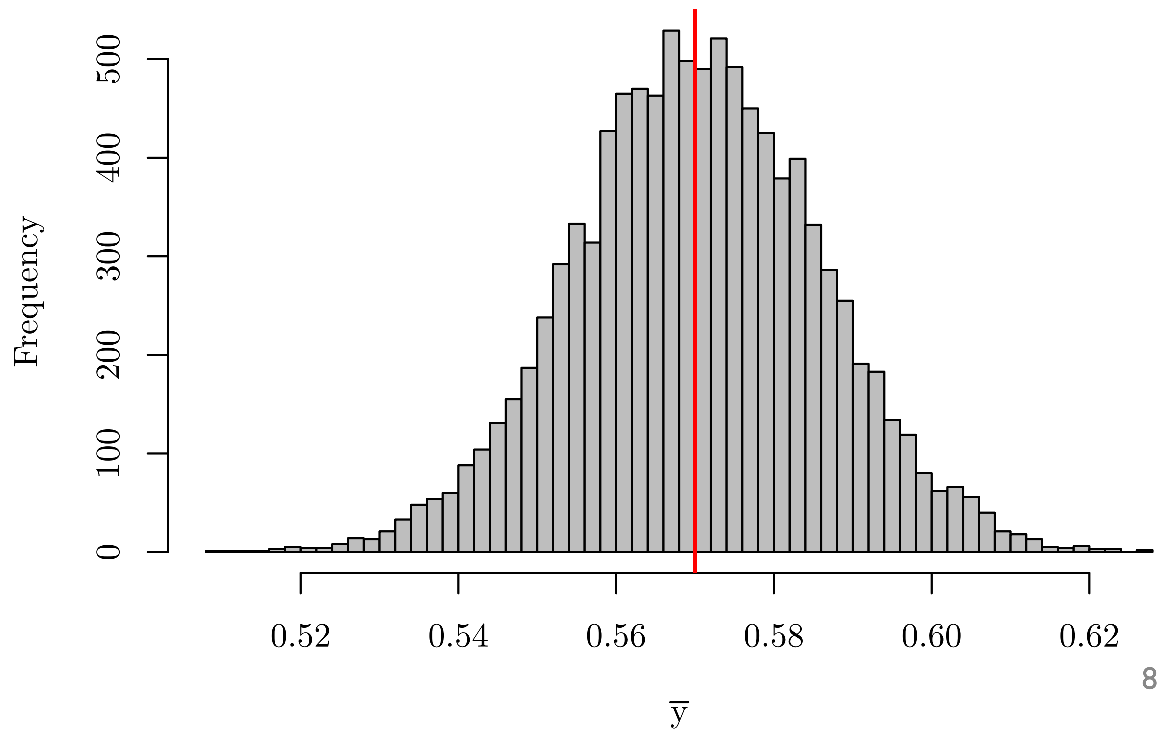

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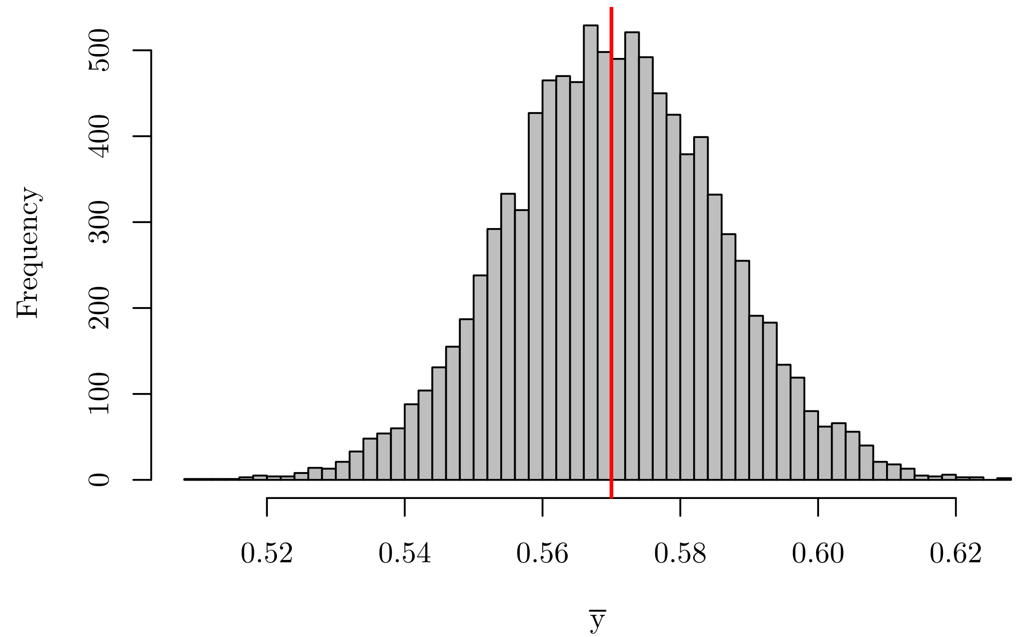

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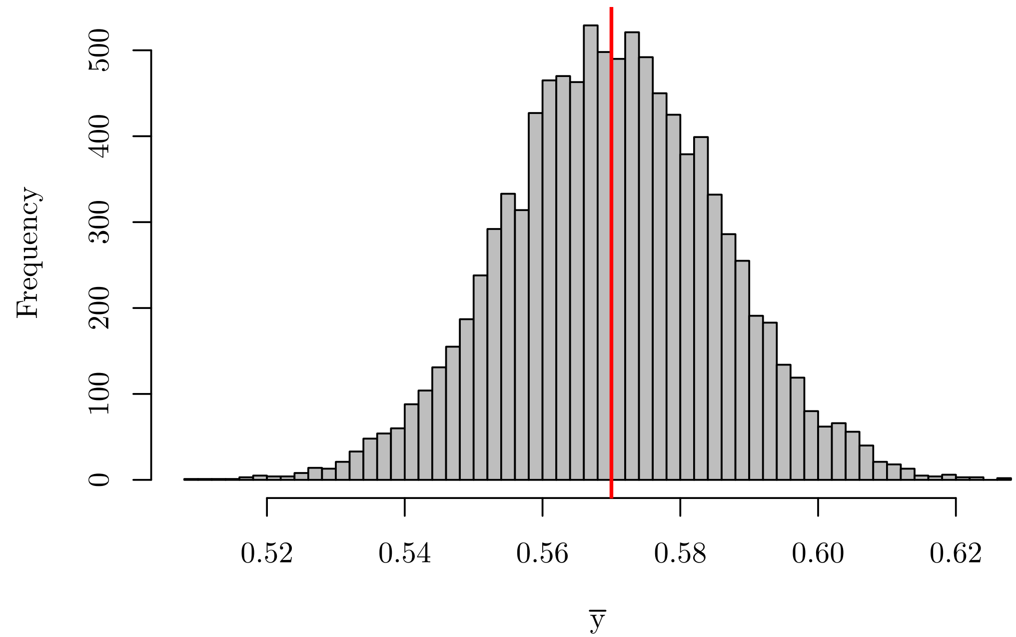

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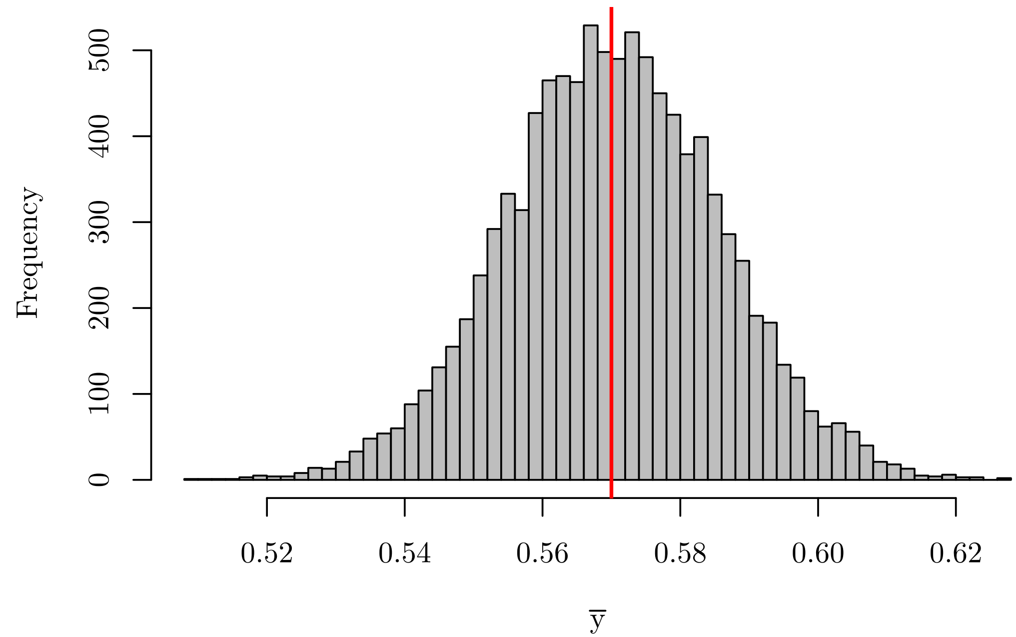

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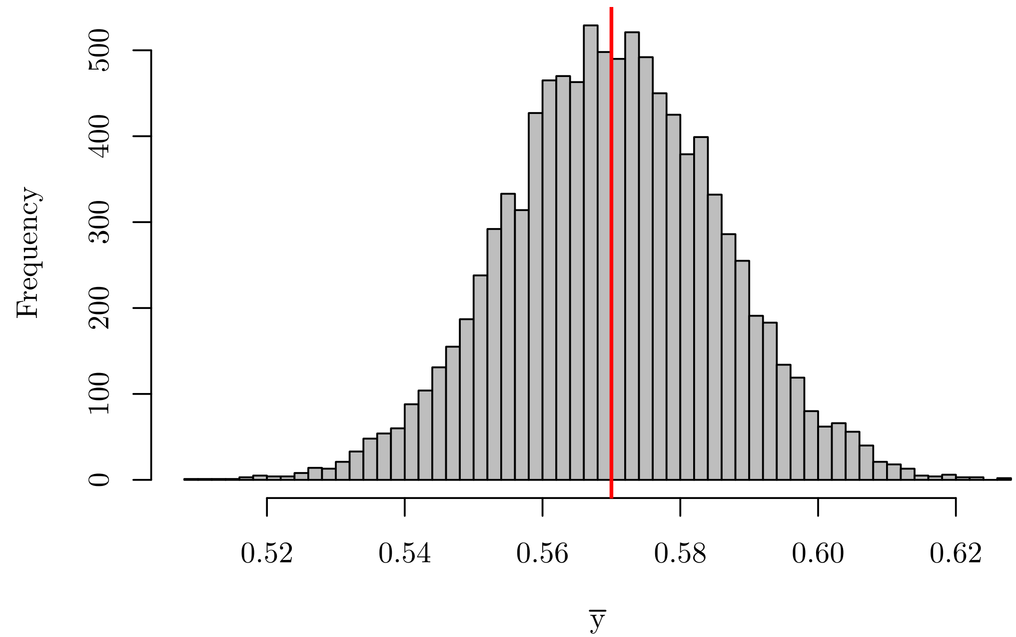

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95% of the samples had a mean between 0.54 and 0.60:

```
> quantile(sms, c(.025, .975))
     2.5%     97.5%
0.5397614 0.6013917
```

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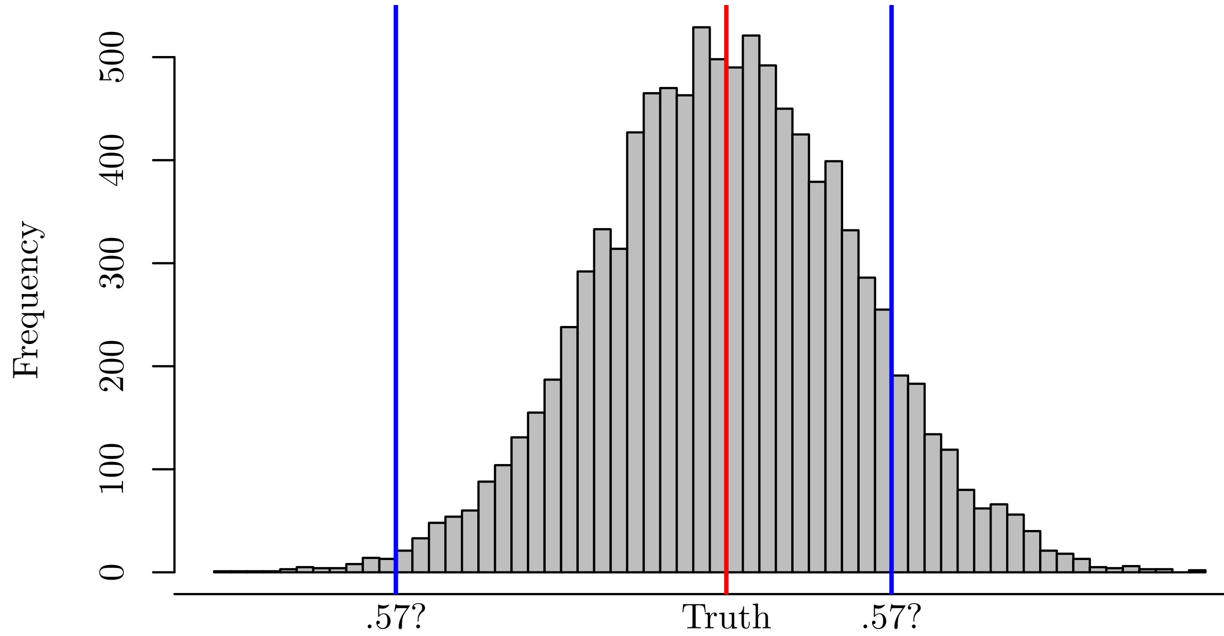

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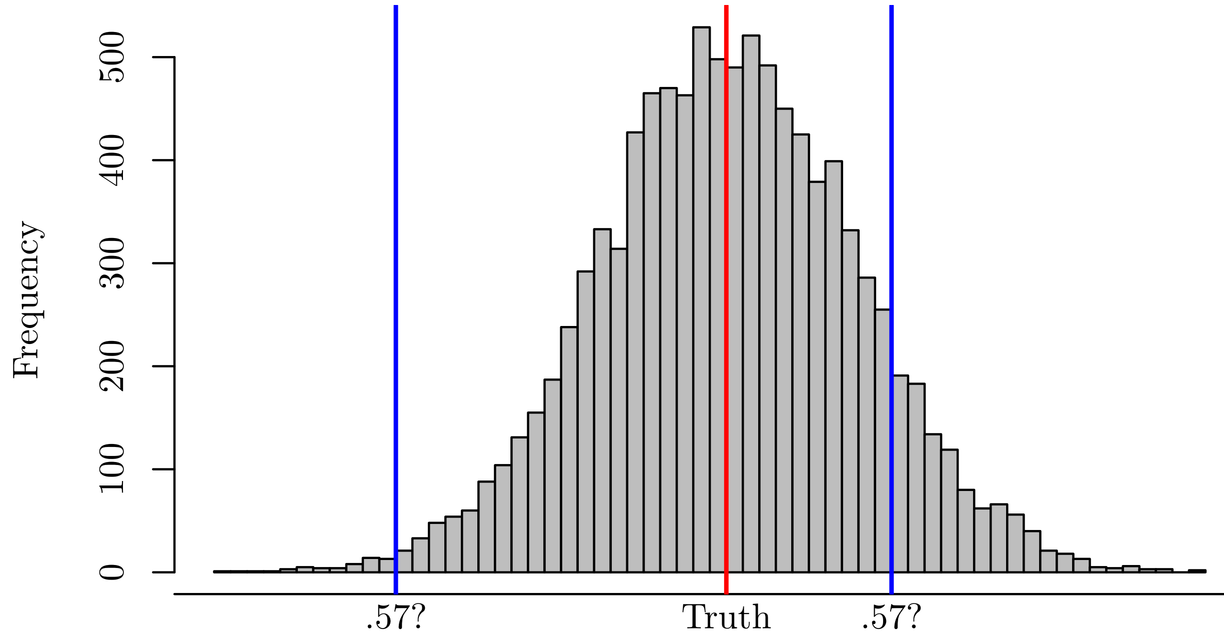

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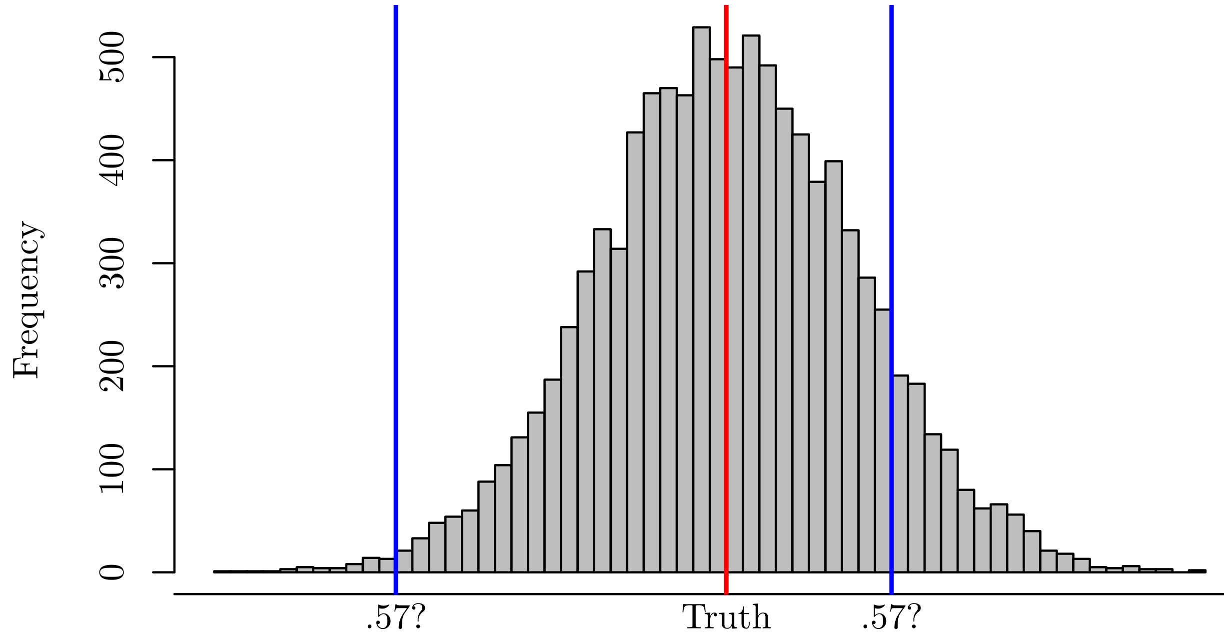

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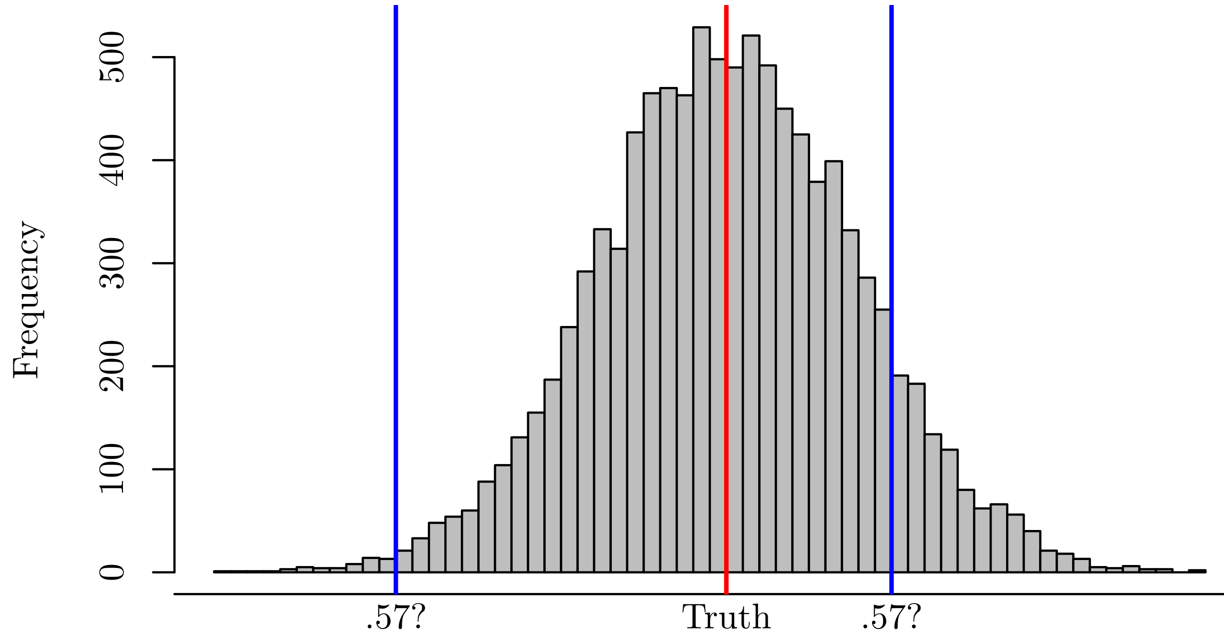

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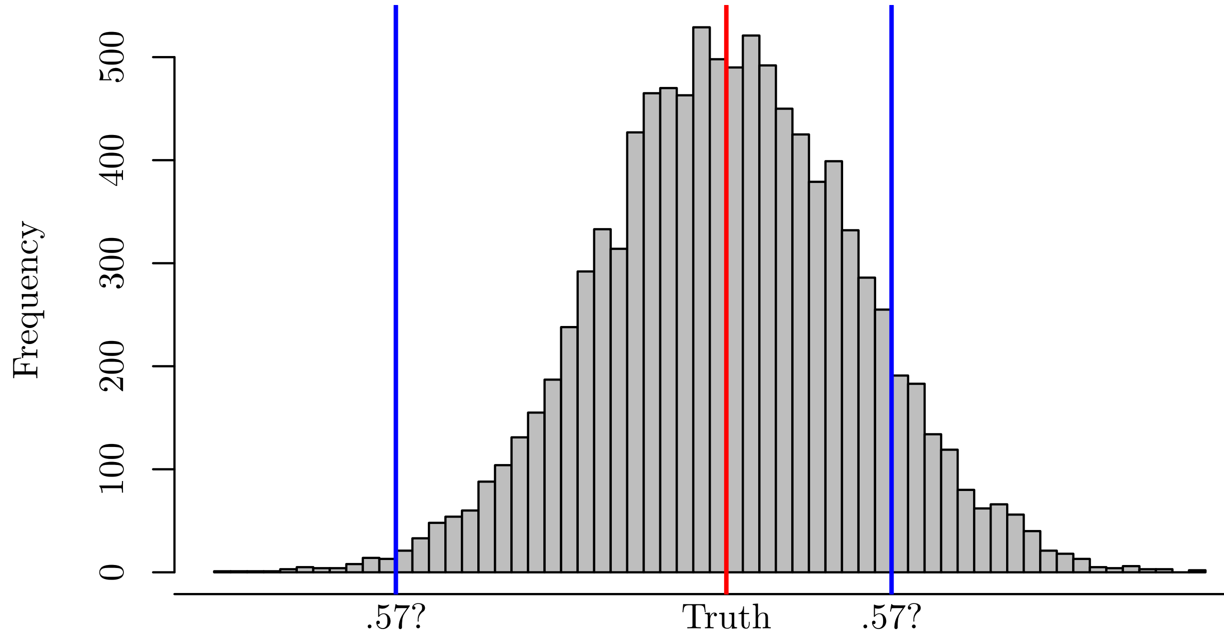

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Compare: the standard deviation of our simulations was 0.0155

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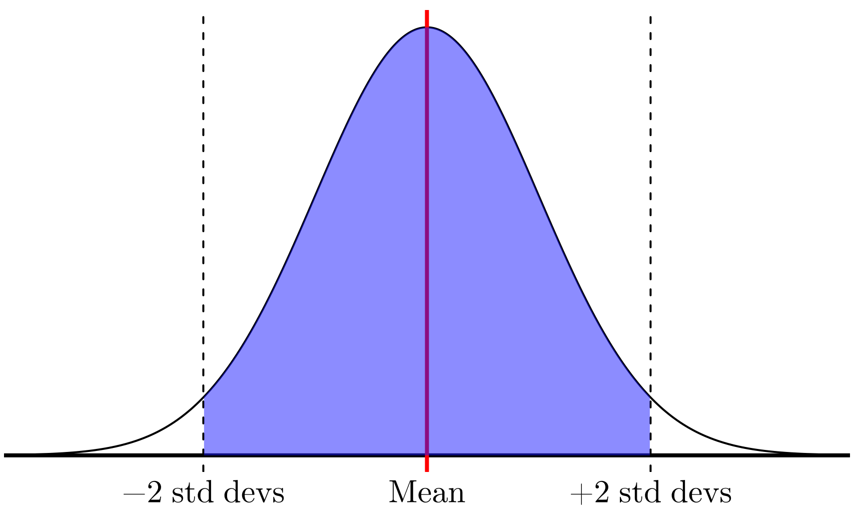

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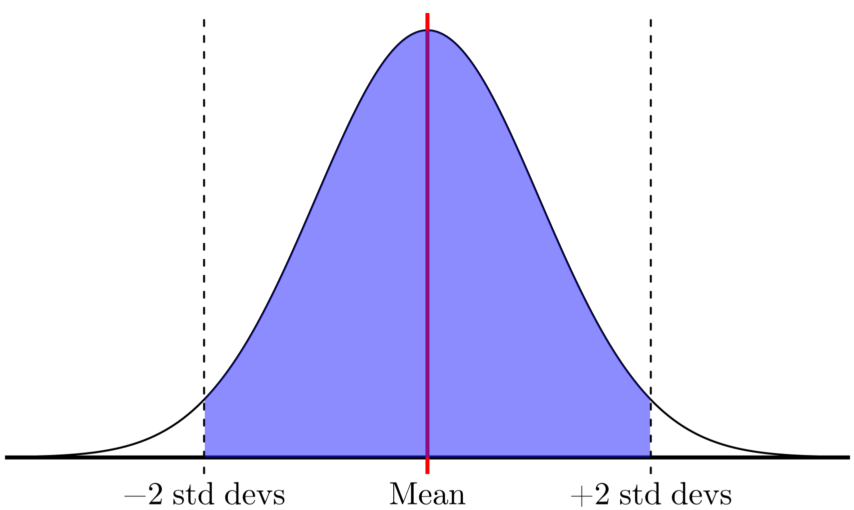

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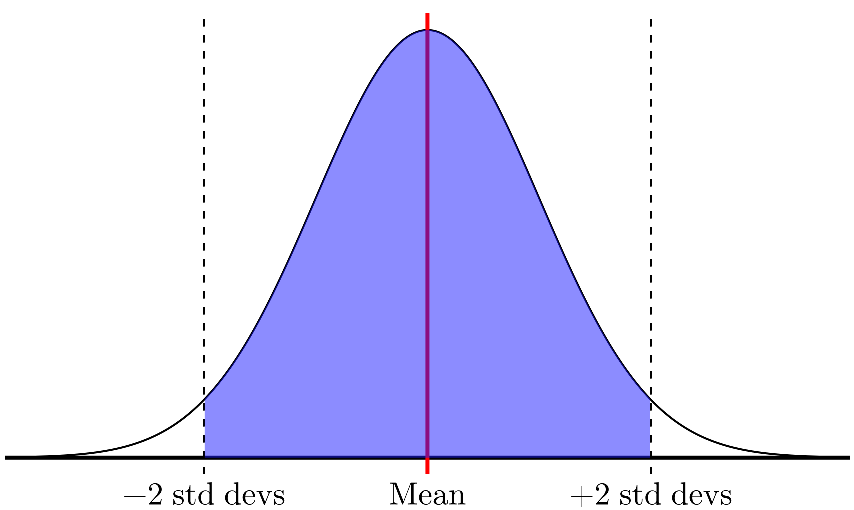

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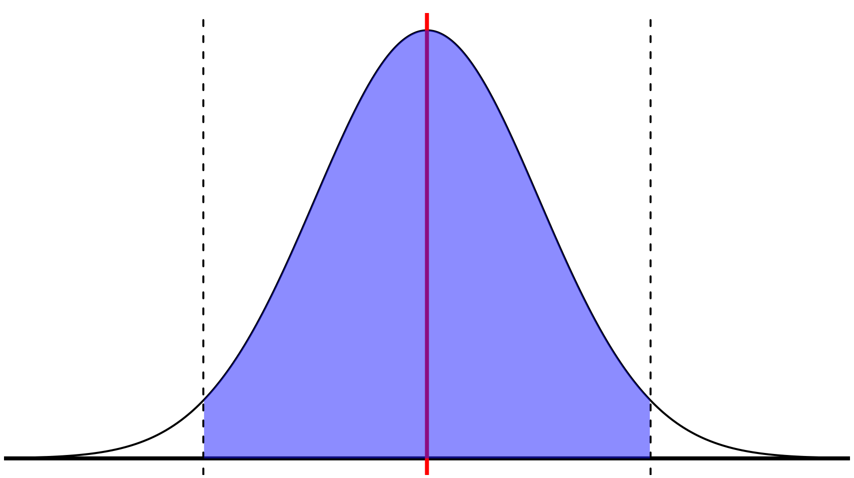

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Compare: our simulations implied a margin of error of 0.031.

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Important: Margin of error captures random error (i.e. sampling error), not bias.

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  - **Standard error**: our estimate of the standard deviation of the result across many surveys

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3. Calculate the **p-value**: probability of getting a statistic as large as yours if the null hypothesis were true (e.g. p=0.2, p=.002)
4. If p-value is low enough, reject null hypothesis, and say the correlation or regression coefficient is "statistically significant"

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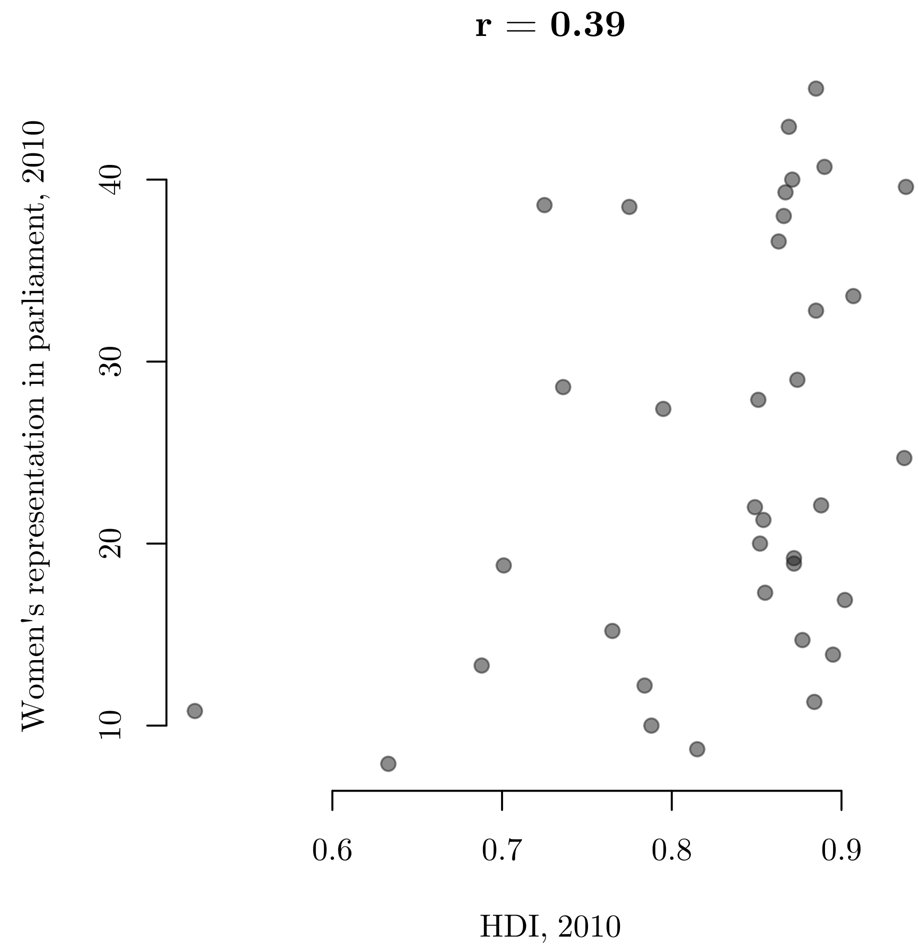
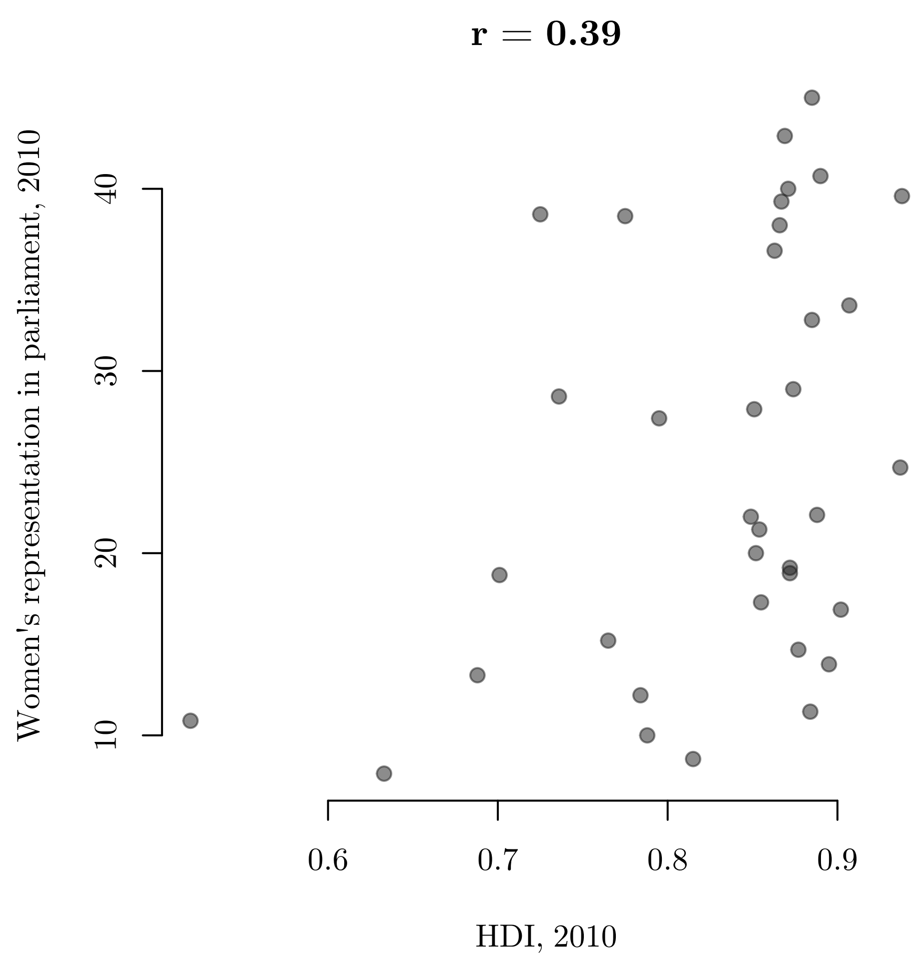


Example: correlation

Recall from Lab 2: in Lijphart's data there is a positive correlation across countries between the level of development and the proportion of women in parliament:

```
> cor(data$hdi_2010, data$women2010)
[1] 0.3869576
```



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```
> cor(data$hdi_2010, data$women2010)
[1] 0.3869576
```

The actual data

Example: correlation (2)

Our question: How likely are we to observe a correlation this large if there is actually no relationship?

Our approach: repeatedly reshuffle the data (so there actually *is* no relationship) and see how often we get a correlation as large as 0.39.

```
> cor(data$hdi_2010, data$women2010)
[1] 0.3869576
```

The actual data

```
> cor(data$hdi_2010, sample(data$women2010))
[1] 0.2154723
```

First reshuffle

Example: correlation (2)

Our question: How likely are we to observe a correlation this large if there is actually no relationship?

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[1] 0.3869576
```

The actual data

```
> cor(data$hdi_2010, sample(data$women2010))
[1] 0.2154723
```

First reshuffle

```
> cor(data$hdi_2010, sample(data$women2010))
[1] -0.09618724
```

Second reshuffle

Example: correlation (3)

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Our question: How likely are we to observe a correlation this large if there is actually no relationship?

Example: correlation (3)

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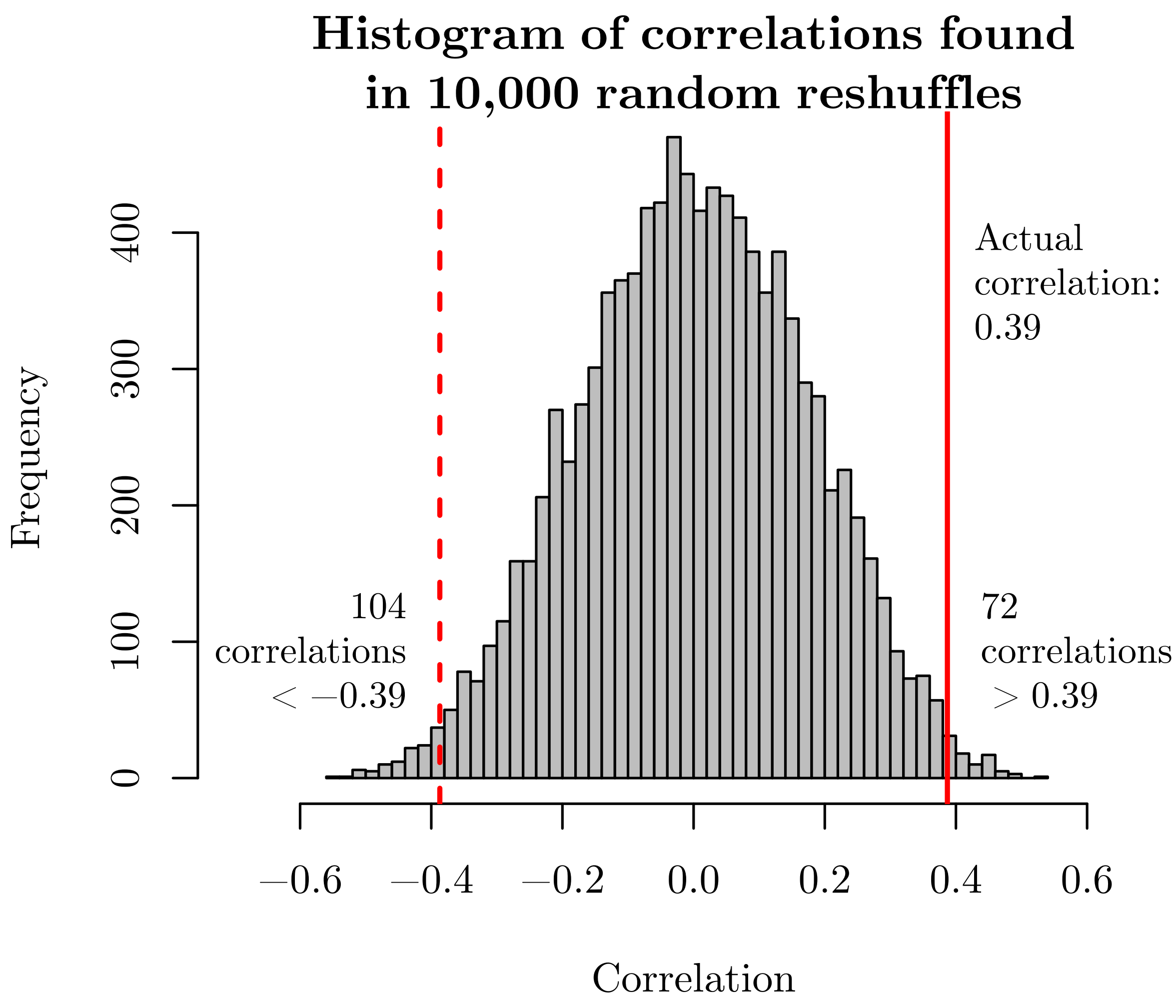
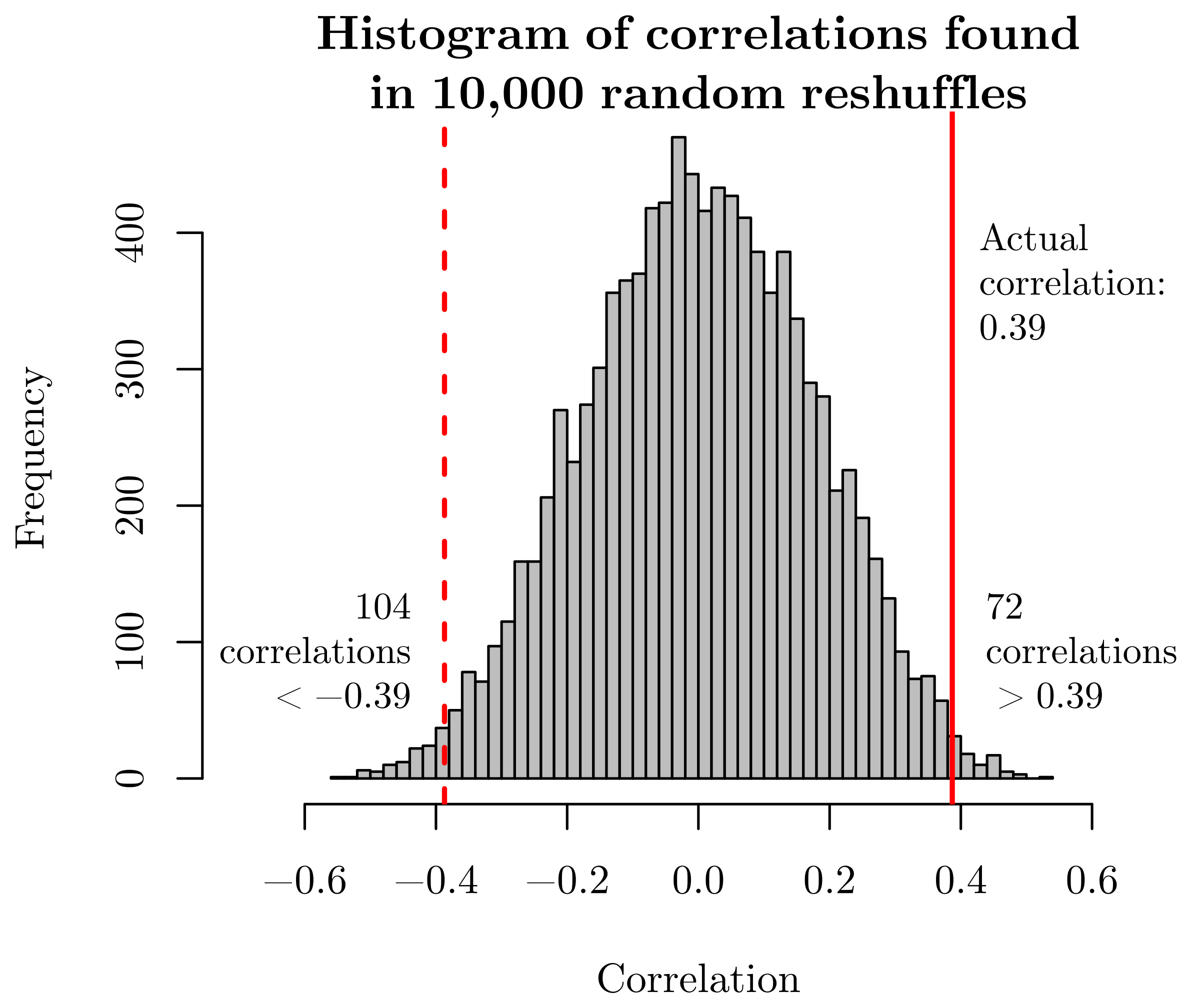


Example: correlation (3)

Our question: How likely are we to observe a correlation this large if there is actually no relationship?

Our answer: p = 0.0176. In 10,000 reshuffles, 176 had correlations larger than 0.39 or smaller than -0.39.



Example: correlation (4)

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How you did this in Lab 2:

Example: correlation (4)

How you did this in Lab 2:

```
> cor.test(data$hdi_2010, data$women2010)

	Pearson's product-moment correlation

data:  data$hdi_2010 and data$women2010
t = 2.447, df = 34, p-value = 0.01973
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
 0.06693071 0.63479253
sample estimates:
      cor
0.3869576
```

The hypothesis testing recipe applied

The hypothesis testing recipe applied

1. Calculate your statistic

Correlation is 0.39

The hypothesis testing recipe applied

1. Calculate your statistic
2. Define a null hypothesis

Correlation is 0.39
No relationship

The hypothesis testing recipe applied

1. Calculate your statistic	Correlation is 0.39
2. Define a null hypothesis	No relationship
3. Calculate the **p-value**	0.0176

The hypothesis testing recipe applied

1. Calculate your statistic	Correlation is 0.39
2. Define a null hypothesis	No relationship
3. Calculate the **p-value**	0.0176
4. If p-value is low enough, reject null hypothesis	Null hypothesis rejected!

Example: bivariate regression

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Recall from Lab 2: in Lijphart's data, positive relationship between development and women's representation in parliament:

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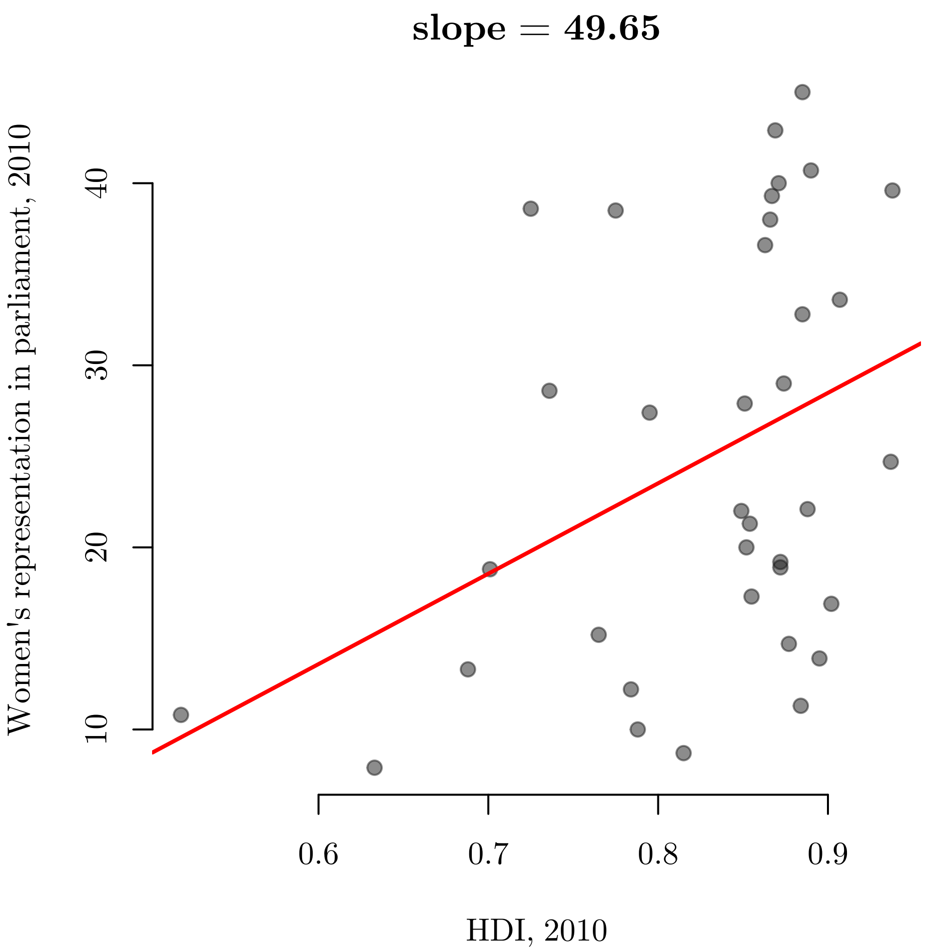


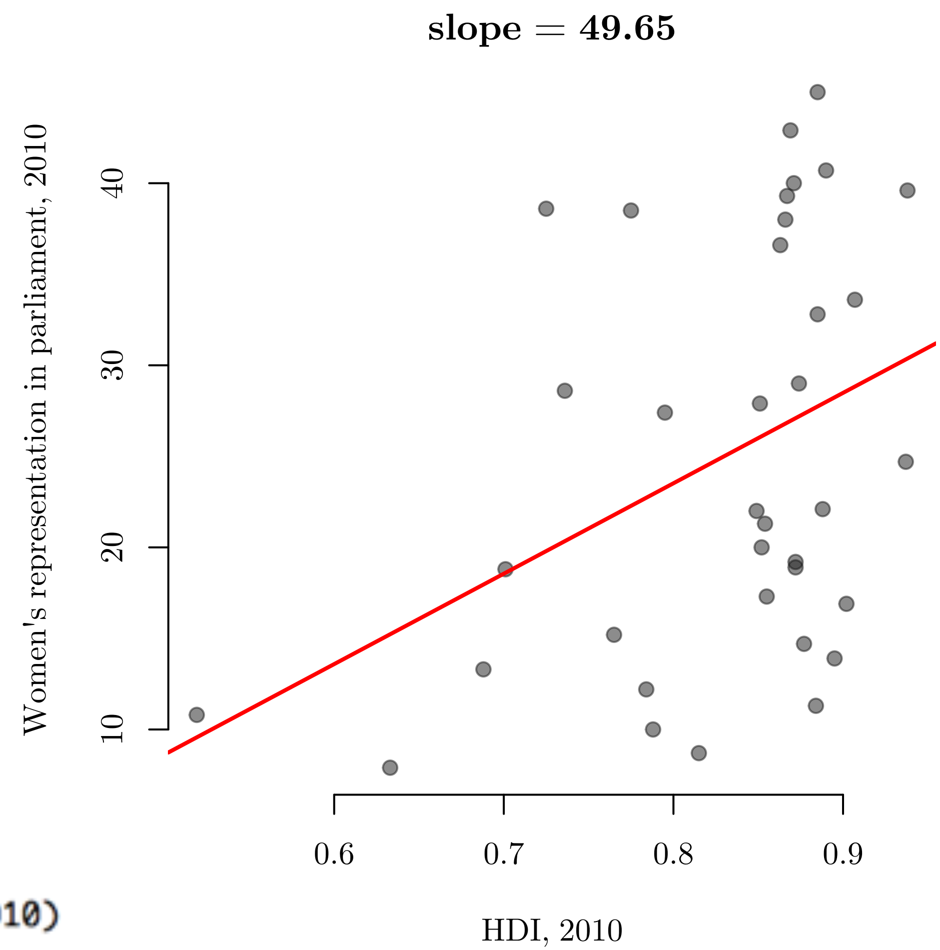
Example: bivariate regression

Recall from Lab 2: in Lijphart's data, positive relationship between development and women's representation in parliament:

```
> lm(data$women2010 ~ data$hdi_2010)

Call:
lm(formula = data$women2010 ~ data$hdi_2010)

Coefficients:
  (Intercept)  data$hdi_2010
       -16.20          49.65
```



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Our approach: repeatedly reshuffle one of the variables (so there actually *is* no relationship) and see how often we get a slope as large as 49.65.

Example: bivariate regression (2)

Our question: How likely are we to observe a slope this large if there is actually no relationship?

Our approach: repeatedly reshuffle one of the variables (so there actually *is* no relationship) and see how often we get a slope as large as 49.65.

```
> coef(lm(data$women2010 ~ data$hdi_2010))[2]
data$hdi_2010
     49.64976
```

The actual data

Example: bivariate regression (2)

Our question: How likely are we to observe a slope this large if there is actually no relationship?

Our approach: repeatedly reshuffle one of the variables (so there actually *is* no relationship) and see how often we get a slope as large as 49.65.

```
> coef(lm(data$women2010 ~ data$hdi_2010))[2]
data$hdi_2010
     49.64976
```

The actual data

```
> coef(lm(data$women2010 ~ sample(data$hdi_2010)))[2]
sample(data$hdi_2010)
             4.734025
```

First reshuffle

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data$hdi_2010
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The actual data

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> coef(lm(data$women2010 ~ sample(data$hdi_2010)))[2]
sample(data$hdi_2010)
             4.734025
```

First reshuffle

```
> coef(lm(data$women2010 ~ sample(data$hdi_2010)))[2]
sample(data$hdi_2010)
            -12.70981
```

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Example: bivariate regression (3)

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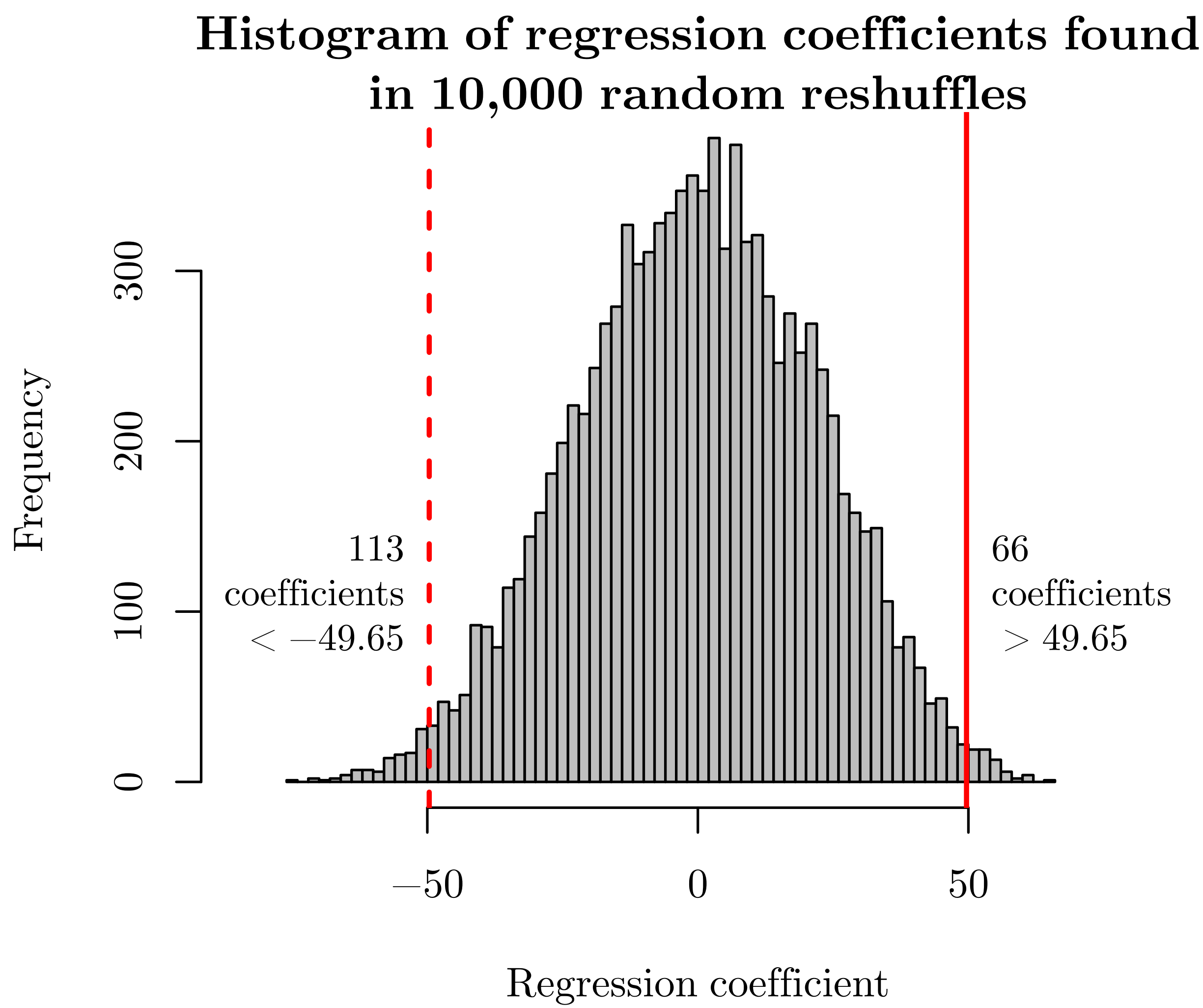
Our question: How likely are we to observe a slope this large if there is actually no relationship?

Example: bivariate regression (3)

Our question: How likely are we to observe a slope this large if there is actually no relationship?

Our answer: p = 0.0179. In 10,000 reshuffles, 179 had slopes larger than 49.65 or smaller than -49.65.



Example: bivariate regression (4)

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```
> model1 = lm(data$women2010 ~ data$hdi_2010)
> summary(model1)

Call:
lm(formula = data$women2010 ~ data$hdi_2010)

Residuals:
    Min      1Q  Median      3Q     Max 
-16.390  -7.970  -1.879   9.410  18.804 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)  
(Intercept)     -16.20      16.90  -0.958   0.3447  
data$hdi_2010    49.65      20.29   2.447   0.0197 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 10.66 on 34 degrees of freedom
Multiple R-squared:  0.1497,	Adjusted R-squared:  0.1247 
F-statistic: 5.988 on 1 and 34 DF,  p-value: 0.01973
```

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```

Compare to our p-value: 0.0179

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1. Calculate your statistic

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The hypothesis testing recipe applied

1. Calculate your statistic
2. Define a null hypothesis

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$\boxed{\text{Slope = 0 (no relationship)}}$

The hypothesis testing recipe applied

1. Calculate your statistic
2. Define a null hypothesis
3. Calculate the **p-value**

Slope is 49.65
Slope = 0 (no relationship)
0.0179

The hypothesis testing recipe applied

1. Calculate your statistic
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4. If p-value is low enough, reject null hypothesis

1	Slope is 49.65
2	Slope = 0 (no relationship)
3	0.0179
4	Null hypothesis rejected!

Extending to multivariate regression

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Multivariate regression is more complicated, but interpretation of the p-values is same: "If this variable were *really not related* to the outcome, how unusual would it be to see a slope this big?"

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```
> summary(lm(data$women2010 ~ data$hdi_2010 + data$eiu_democracy_index_2006_2010))

Call:
lm(formula = data$women2010 ~ data$hdi_2010 + data$eiu_democracy_index_2006_2010)

Residuals:
    Min      1Q  Median      3Q     Max 
-16.456  -7.300  -1.435   7.490  23.730 

Coefficients:
                                   Estimate Std. Error t value Pr(>|t|)  
(Intercept)                         -36.367     19.655  -1.850   0.0738 .
data$hdi_2010                        14.638     24.175   0.606   0.5492  
data$eiu_democracy_index_2006_2010    5.939      2.798   2.122   0.0419 *
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Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 10.02 on 31 degrees of freedom
  (2 observations deleted due to missingness)
Multiple R-squared:  0.2473,	Adjusted R-squared:  0.1988 
F-statistic: 5.094 on 2 and 31 DF,  p-value: 0.01223
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This makes sense in a survey:

- Given a random **sample** of n=1,006 in which 57% said "Remain", “If this variable were *really not related* to the outcome, how unusual would it be to see a slope this big?”

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The null hypothesis is a claim about the **truth** that we test with our **sample**.

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- Given a random **sample** of n=1,006 in which 57% said “Remain”,
- can we reject the null hypothesis that actually only 50% of **all GB adults** support “Remain”?

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The **truth** is the number in the population.

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- Given a random **sample** of n=1,006 in which 57% said “Remain”,
- can we reject the null hypothesis that actually only 50% of **all GB adults** support “Remain”?

The **truth** is the number in the population.

What about when we’re talking about 36 democracies in Lijphart’s data? This isn’t a sample! What is the **truth** there? What do the p-values, standard errors mean?

Three ways you can view standard errors, p-values in Lijphart (and other research not based on analysis of random samples)

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    A. …so we should use Bayesian statistics.

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   B. …but we use the conventional (frequentist) approach anyway.

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Kellstedt and Whitten: "no clear scientific consensus" (141)

Finally, back to margin of error

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Recall the margin of error (= 2 times standard error) gave us a sense of how much the estimate would vary across many surveys.

The standard error in regression output plays the same role: in 95% of surveys/repeated samples, the difference between our estimate and the true value is less than 2 times the standard error.

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Multiple R-squared:  0.2473,	Adjusted R-squared:  0.1988 
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F-statistic: 5.094 on 2 and 31 DF,  p-value: 0.01223
```

Now you should understand:

Dependent variable: Nobel Prizes awarded per capita (in log scale)

	(1)	(2)	(3)
Intercept	-1.629* (0.509)	-3.166* (0.511)	-2.982* (0.527)
Chocolate consumption per capita (log scale)	2.092* (0.298)	1.026* (0.326)	0.709 (0.415)
GDP/capita (thousands of USD)		0.105* (0.024)	0.106* (0.024)
NW Europe			0.549 (0.452)
R^2	0.70	0.85	0.86
N	34	34	34

Standard errors in parentheses. * Indicates $p<0.05$

- what a dependent variable is
- what an independent variable is
- what the coefficients mean (intercept, slopes)
- what the stars mean (i.e. what $p<0.05$ means)
- what the standard errors mean

And this too!

TABLE 15.2

Multivariate regression analyses of the effect of consensus democracy (executives-parties dimension) on five indicators of violence, with controls for the effects of the level of economic development, logged population size, and degree of societal division, and with extreme outliers removed

Performance variables	Estimated regression coefficient	Absolute t-value	Countries (N)
Political stability and absence of violence (1996–2009)	0.189***	3.360	34
Internal conflict risk (1990–2004)	0.346**	2.097	32
Weighted domestic conflict index (1981–2009)	−105.0*	1.611	30
Weighted domestic conflict index (1990–2009)	−119.7**	2.177	33
Deaths from domestic terrorism (1985–2010)	−2.357**	1.728	33

* Statistically significant at the 10 percent level (one-tailed test)
** Statistically significant at the 5 percent level (one-tailed test)
*** Statistically significant at the 1 percent level (one-tailed test)

Source: Based on data in Kaufmann, Kraay, and Mastruzzi 2010; PRS Group 2004; Banks, 2010; and GTD Team 2010

- what the dependent and independent variables are
- what Lijphart means by "controlling for" three other variables
- what the stars mean
- t-values: estimate divided by standard error