### Regression discontinuity designs: an introduction

Oxford spring school in methods 27 March 2017) Andy Eggers







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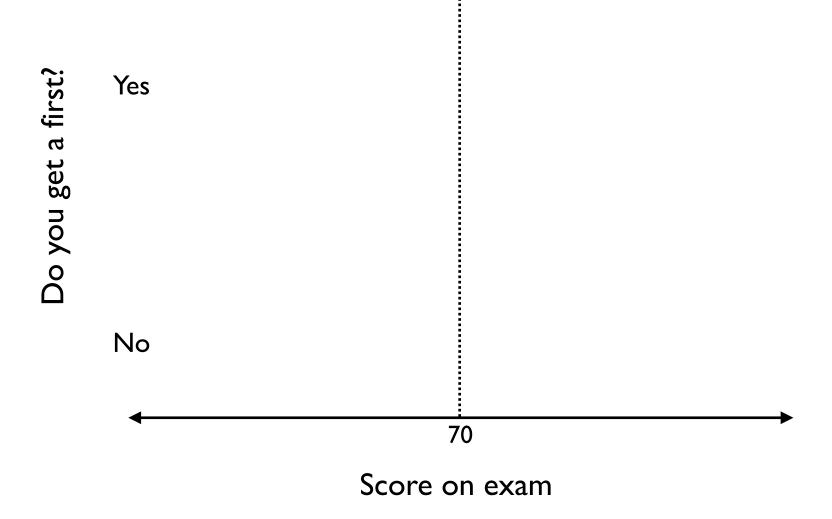


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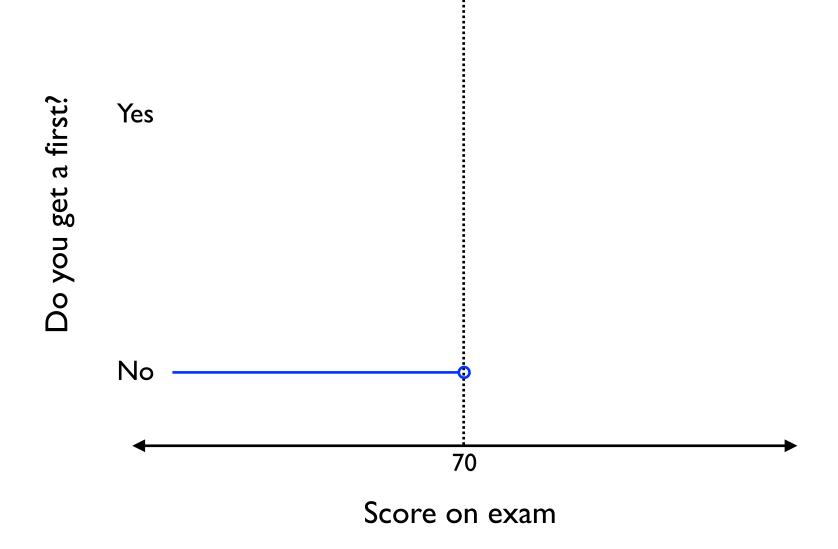


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- Journalists report a recession if the economy shrinks for two consecutive quarters

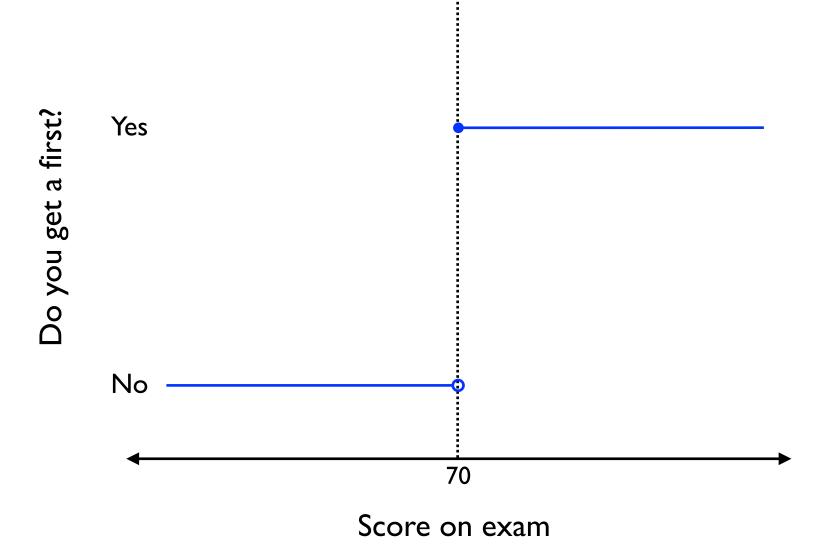
#### "Policies" make jumps



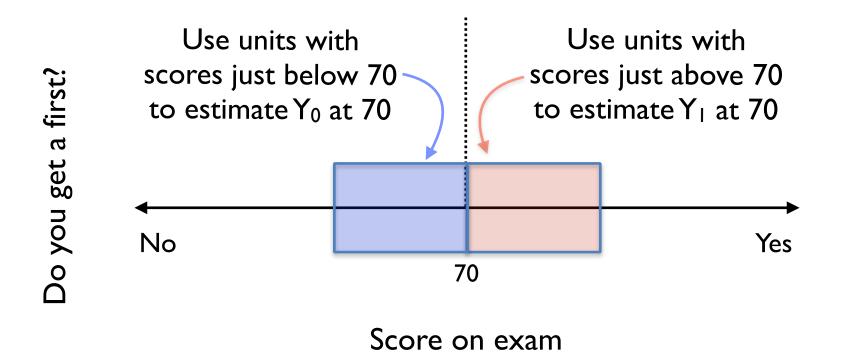
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If potential outcomes don't jump but treatment does, then near the threshold we have comparable groups getting different treatment.



**Goal**: estimate ATE at threshold (70), i.e. Local Average Treatment Effect (LATE), using units just above and below the threshold.

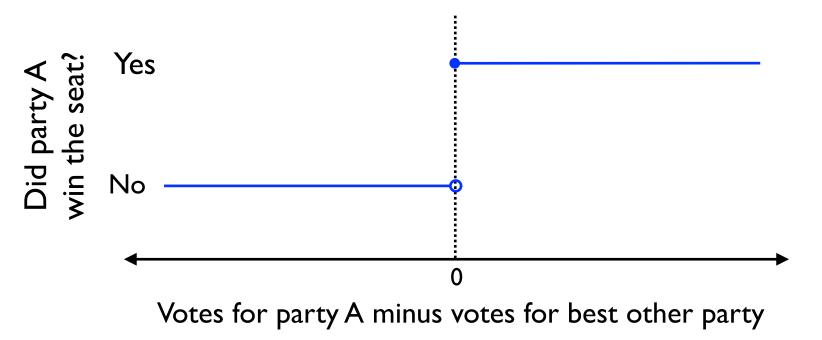
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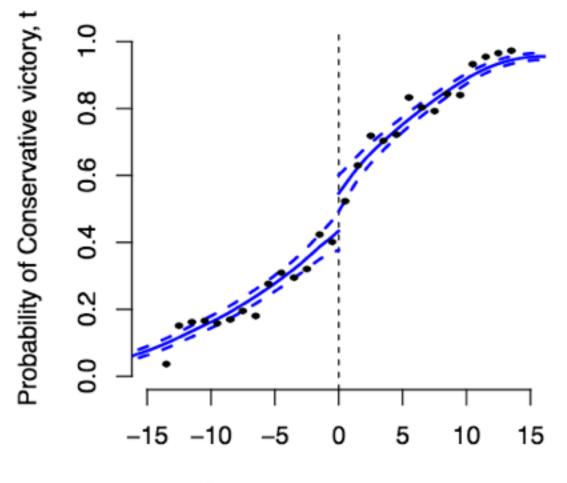
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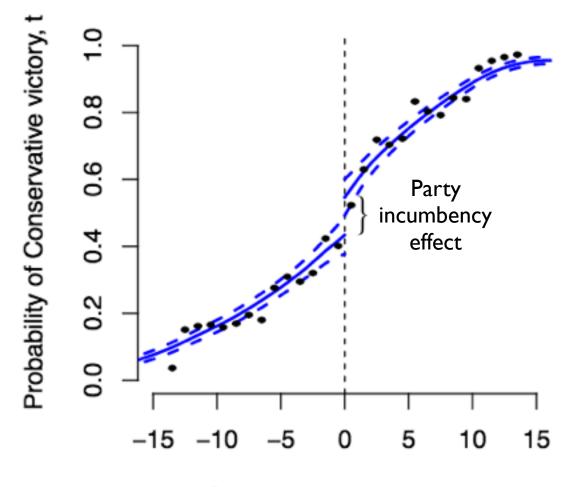


#### **Conservative vs. Labour**

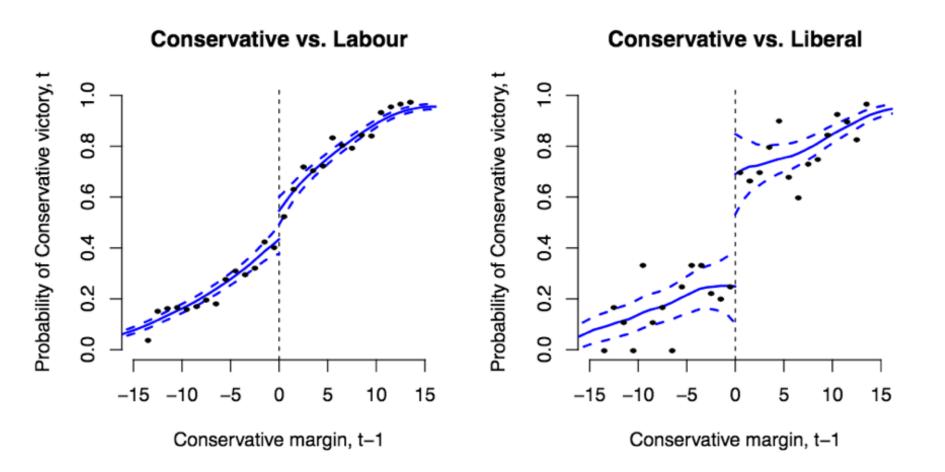


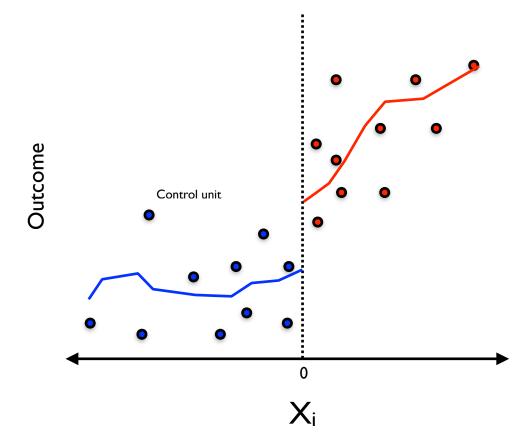
Conservative margin, t-1

#### Conservative vs. Labour

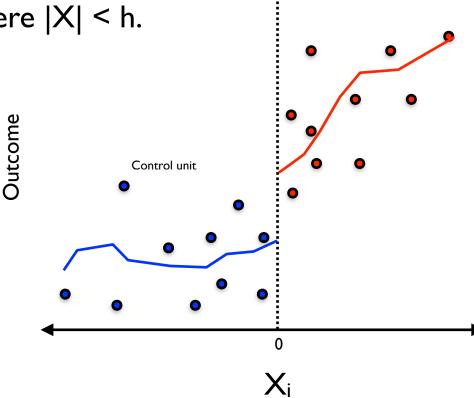


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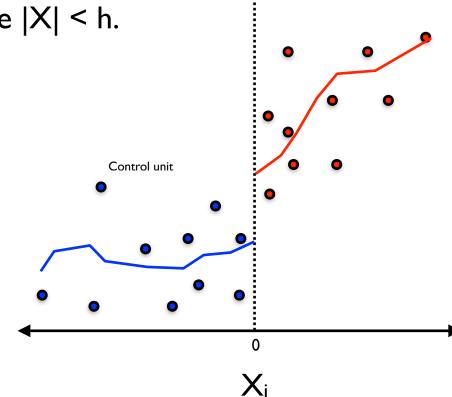
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Outcome

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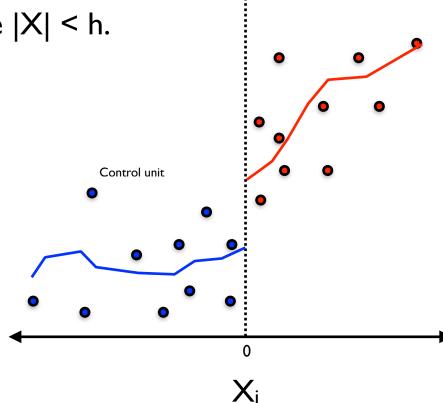
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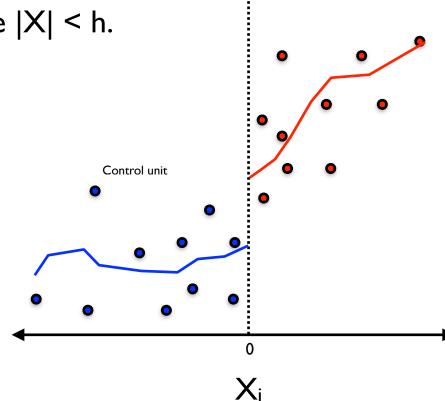
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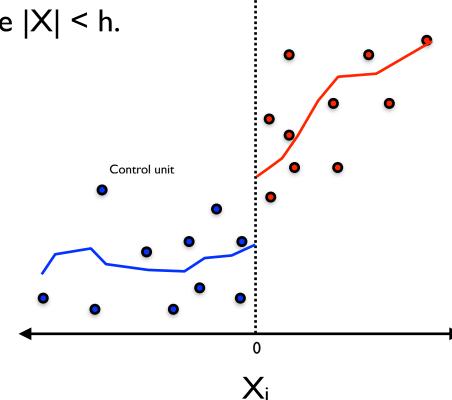
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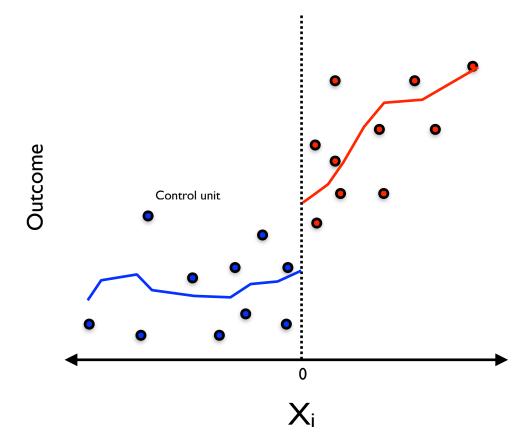
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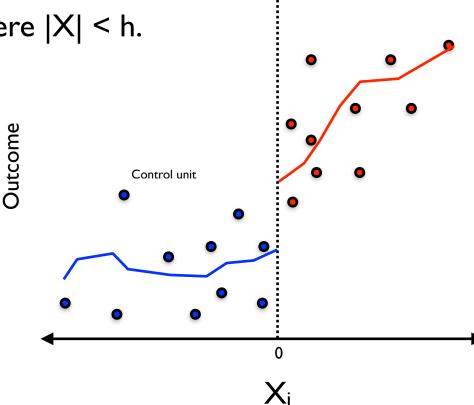
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There is a **bias-variance tradeoff**.





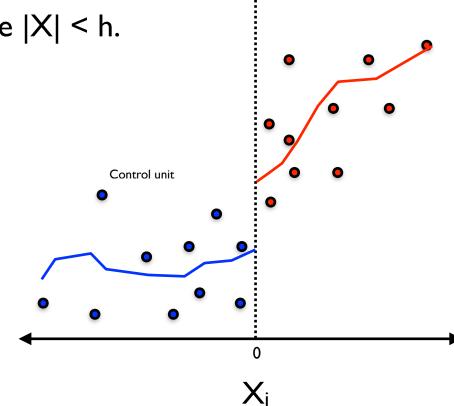
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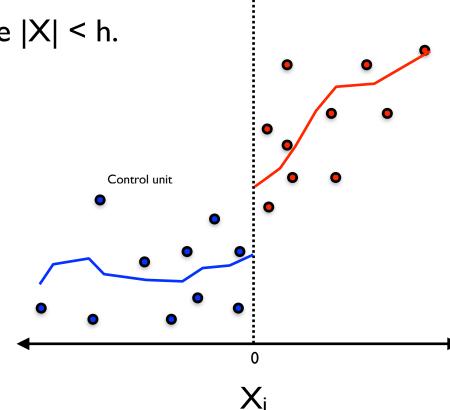


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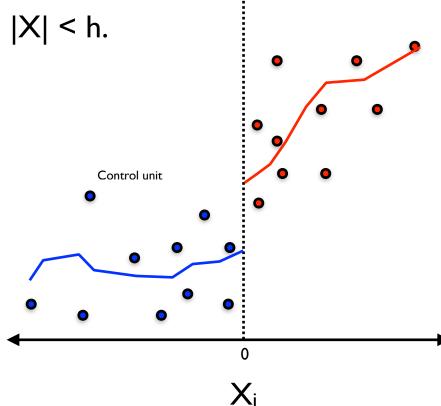


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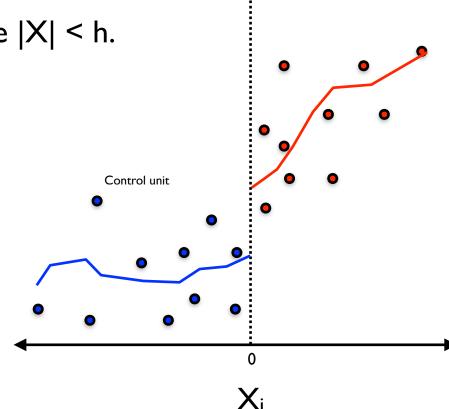


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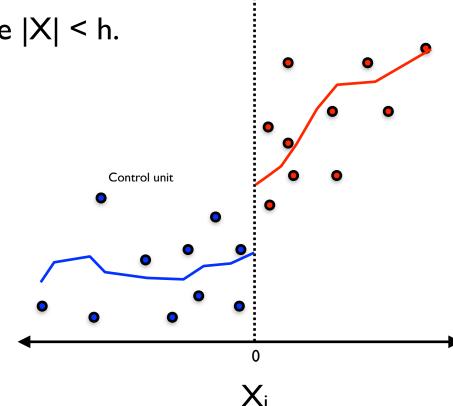
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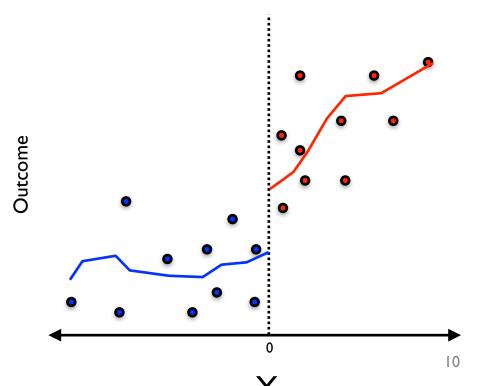
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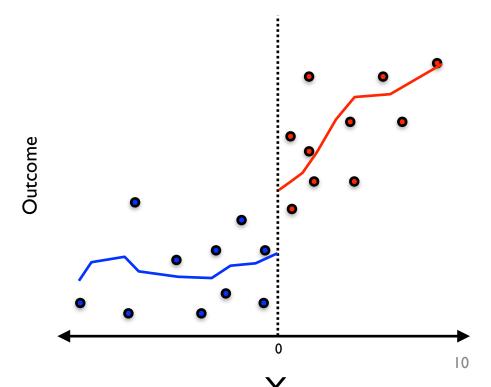


#### What people do now



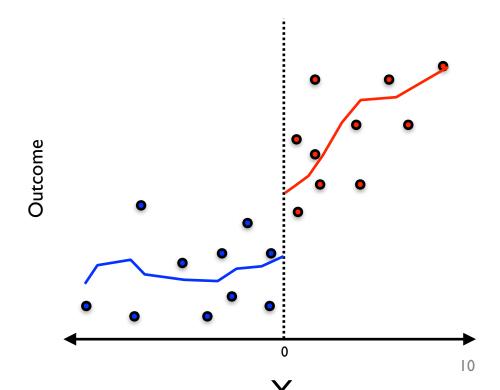
#### What people do now

Most common approach: local regression, as on previous slide, with:

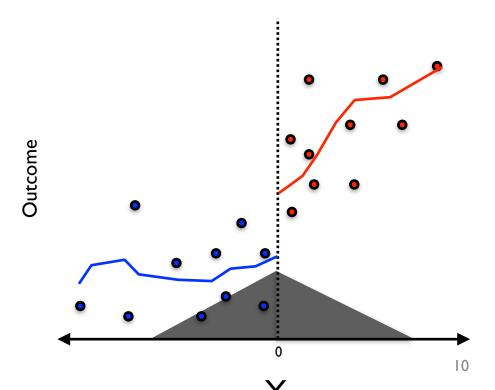


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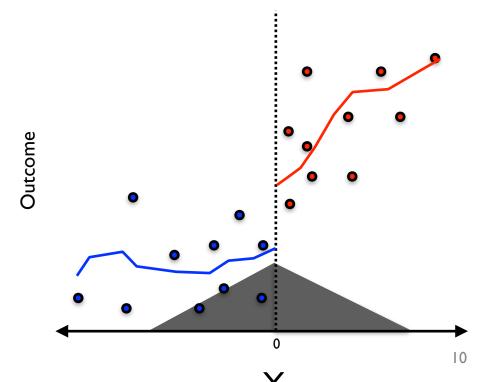
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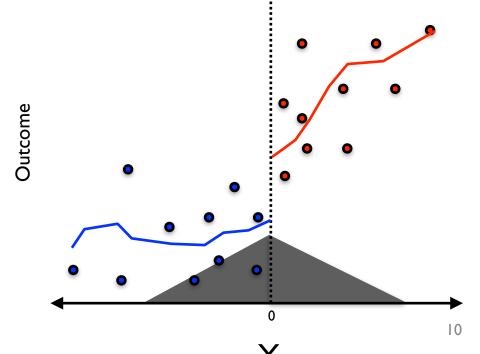
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- weights that are larger closer to the threshold (triangular kernel)



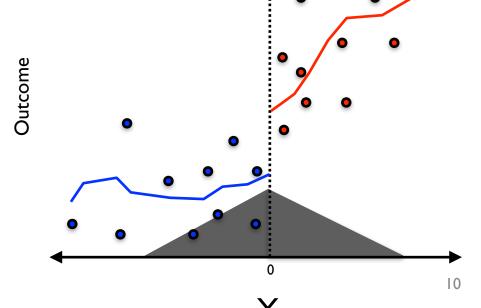
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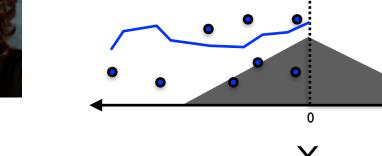
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State of the art: rdrobust package (Stata, R), other work by Calonico, Cattaneo, Titiunik



**Rocío** Titiunik



10

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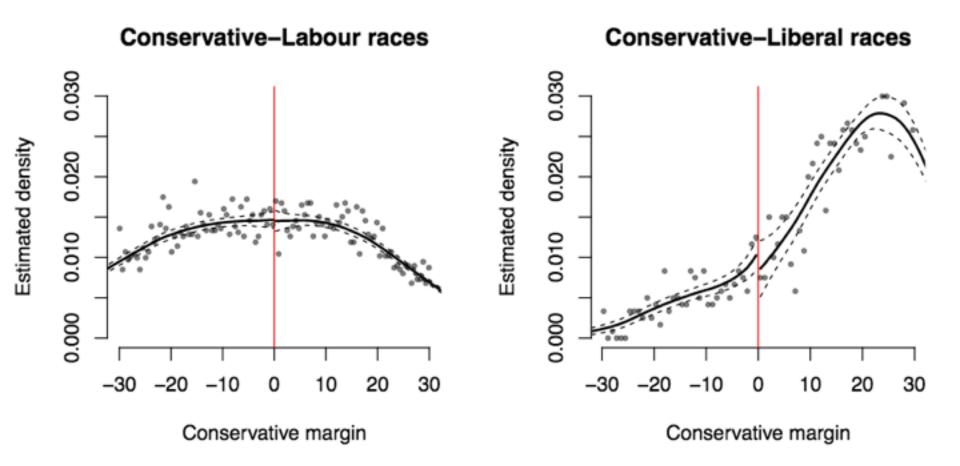
- Bias from misspecification (see previous slides depends on algorithm, also curvature of CEF, amount of data)
- Sorting: units just above the threshold may differ substantially from those just below if some units choose their X (and thus treatment), and the ability to choose depends on variables that affect outcome.

#### So how do we check for sorting?

- Is the density of the running variable continuous across the threshold? (McCrary 2008)
- Are covariates (e.g. incumbency status, mayor characteristics, lagged outcomes) continuous across the threshold?

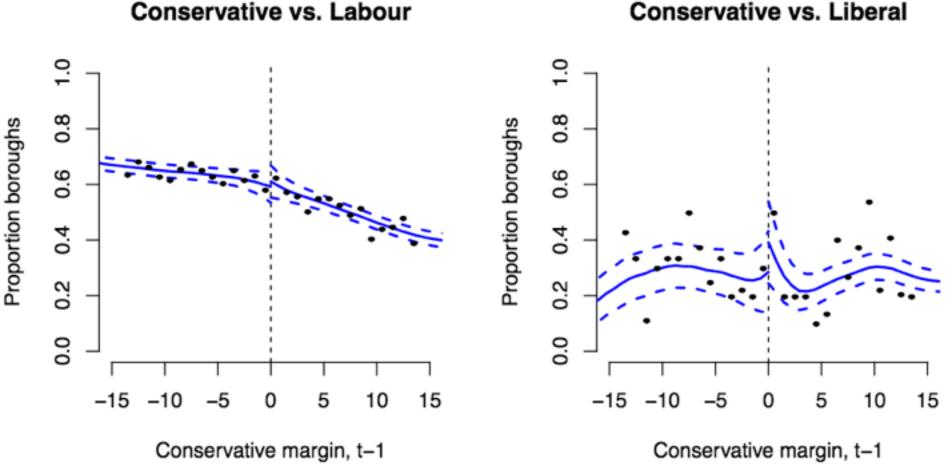
#### Example I: tests of the continuity assumption (I)

McCrary test for continuity in the density



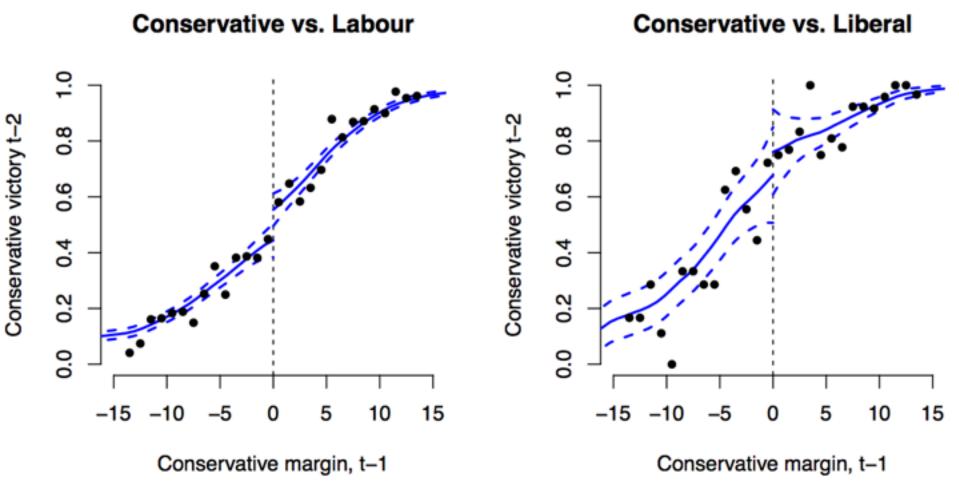
### Example I: tests of the continuity assumption (2)

Tests for continuity in covariate: whether or not the election took place in a borough (vs county) constituency



#### Example I: tests of the continuity assumption (3)

Tests for continuity in covariate: whether or not the Conservatives won the *previous* election



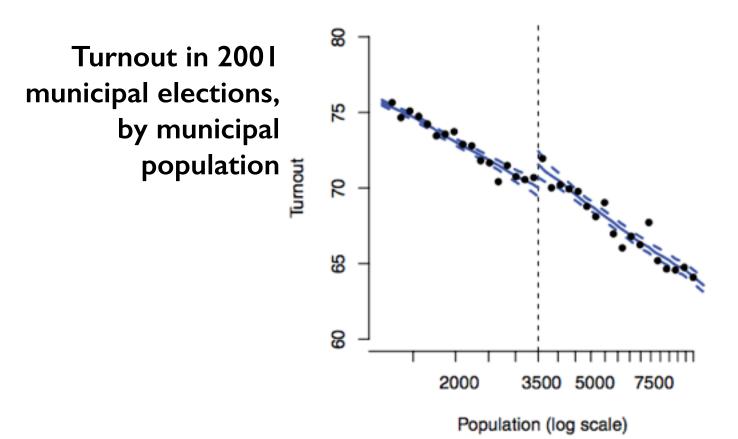
**Research question:** Does a more proportional electoral system lead to more voter turnout on average?

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another. One standard way of checking the validity of the RDD, due to McCrary (2008), involves testing for a jump in the density of the forcing variable at the threshold; in this case, McCrary (2008)'s test fails to reject the null (p = .127). Another standard validity check is to carry out RDD analysis in which pre-treatment covariates serve as outcome variables. Table A1 in the appendix reports RDD effect estimates at varying population windows (25%, 50%, and 75%), showing that there is (as one would expect) no "effect" of crossing the 3,500 population threshold on the vast majority of placebo outcomes. These tests suggest that cities just above and below the population threshold are indeed comparable in not just observed but also unobservable features (e.g., local political culture). (Page 144)

### Example 2: testing the continuity assumption (2)

Estimated effect of crossing 3,500 on turnout in municipal elections and higher-level elections

Mean (I) Outcome (2) (3) turnout 1.525\*\*\* Municipal, 2001 70.73 0.989 1.537\*\* (0.778) (0.538) (0.433) 1.476\*\*\* Municipal, 2008 69.14 0.763 0.929 (0.765) (0.523) (0.423)1.242\*\* 1.502\*\*\* Municipal, 2001 & 2008 0.878 69.96 (0.71)(0.481)(0.385) 74.95 Presidential, 2002 -0.04 -0.189-0.038(0.413)(0.241)(0.29) Regional, 2004 -0.7† -0.241 63.38 -0.448(0.583) (0.414) (0.341)-0.248 -0.439 -0.253 Presidential, 2007 86.33 (0.326)(0.224)(0.185)25% 50% 75% 25% Window:

Effect

### Example 2: testing the continuity assumption (3)

#### Table A1. RDD Analysis: The Effect of Crossing the 3,500 Population Threshold on Placebo (Pre-Treatment) Outcomes.

Outcome	Mean	Effect estimates							
		(1)	(2)	(3)	Pct. for Chirac, 1995 pres. elections	51.87	0.633 (0.927)	0.772 (0.674)	0.585 (0.541)
Pct. retired, 1999	19.07	0.711	0.313	0.24	Region: Center	0.08	-0.033	-0.006	-0.005
		(0.667)	(0.455)	(0.361)			(0.028)	(0.019)	(0.016)
Pct. working in agriculture, 1999	5.34	-0.023	-0.068	0.775*	Region: West	0.22	-0.005	-0.076*	-0.084***
		(0.638)	(0.461)	(0.38)	-		(0.044)	(0.031)	(0.025)
Pct. with "bac" degree, 1999	36.45	-0.713	-0.758	-0.187	Region: South	0.22	0.018	0.042	0.052†
		(0.985)	(0.718)	(0.585)			(0.045)	(0.032)	(0.026)
Pct. unemployed, 2001	12.33	-0.764	-0.456	0.064	Region: East	0.13	0.046	0.044	0.013
		(0.762)	(0.542)	(0.441)			(0.037)	(0.027)	(0.022)
Log pop., 1990	8.03	0.025†	0.013	0.011	Region: North	0.16	-0.019	0.004	0.018
		(0.013)	(0.009)	(0.007)			(0.039)	(0.028)	(0.023)
Area (sq. km.)	21.37	-3.087	-1.883	-1.803	Region: SW	0.13	0.013	0.003	0.015
		(2.021)	(1.406)	(1.123)			(0.037)	(0.026)	(0.021)
Local tax revenue, 2000 (in thous. euros)	1,142.57	172.027	125.215	109.848	Region: Paris	0.06	-0.021	-0.01	-0.01
		(106.377)	(75.952)	(66.66)			(0.026)	(0.019)	(0.015)
VAT compensation fund (FCTVA), 2000 (in thous. euros)	101.36	5.422	4.661	6.406	Window	25%	25%	50%	75%
		(8.829)	(6.544)	(5.628)					
Turnout, 1995 pres. elections	81.87	-0.157	-0.057	-0.109					
		(0.341)	(0.236)	(0.192)					

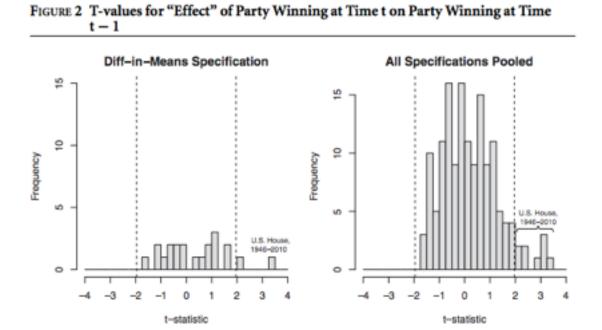
**Caughey and Sekhon (2011):** RDD might not work for close elections — evidence that incumbents disproportionately win very close U.S. congressional elections post 1950. Imbalance (i.e. discontinuity) in incumbency, amount of money raised, predicted winner, many other pre-treatment characteristics.

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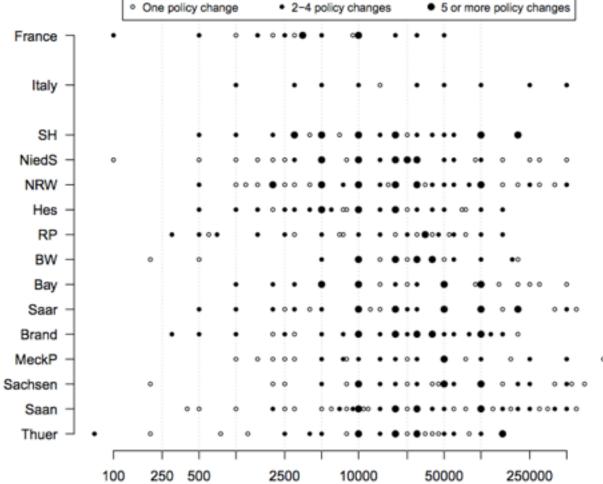
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**Eggers, Freier, Grembi, and Nannicini** (forthcoming): There may be more reason to doubt RDDs based on population thresholds.

First problem: same threshold often used to determine more than one treatment.

See Eggers, Freier, Grembi, Nannichini (forthcoming) and Eggers (2015) for ideas about handling this.



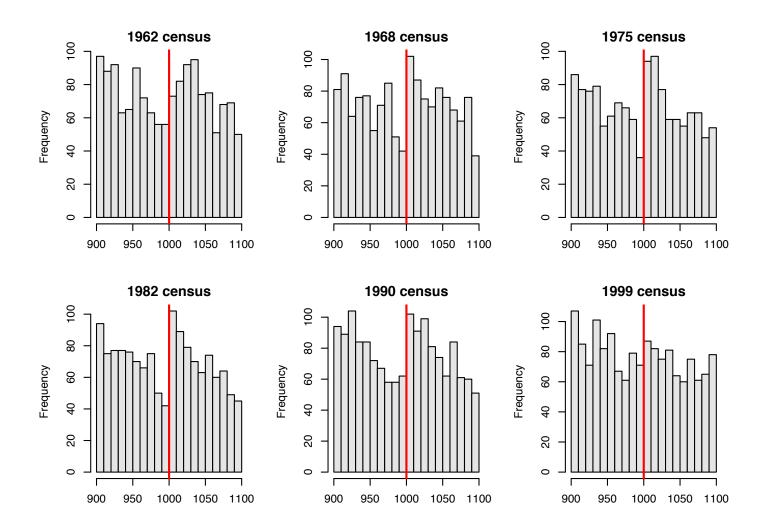
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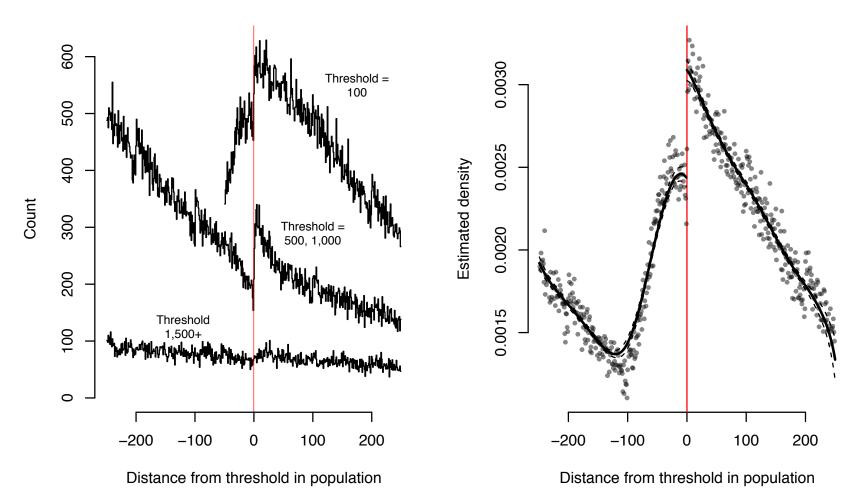


# Does RDD work for political science applications? The case of population thresholds (3)

Pooling all thresholds, censuses from France:

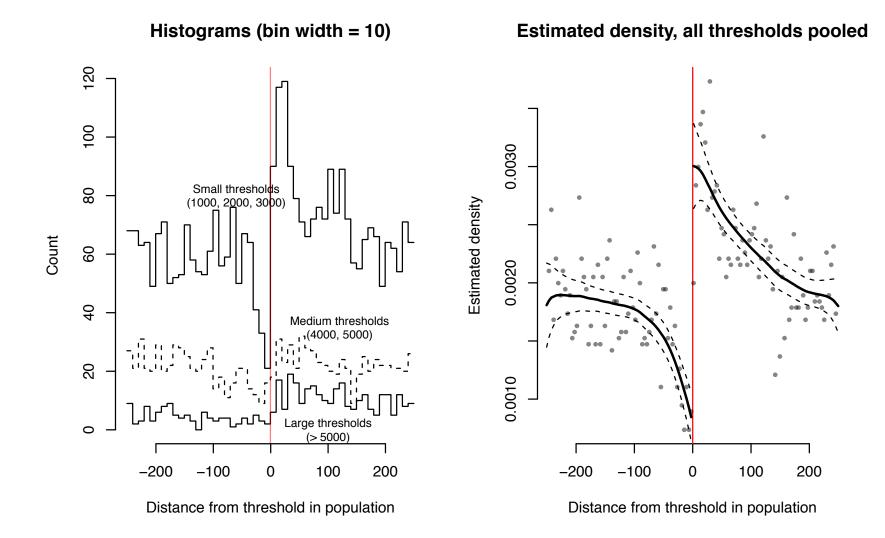
Histograms (bin width = 1)

Estimated density, all thresholds pooled



# Does RDD work for political science applications? The case of population thresholds (3)

Even worse in Italy:



#### Does RDD work for political science applications? General questions about sorting

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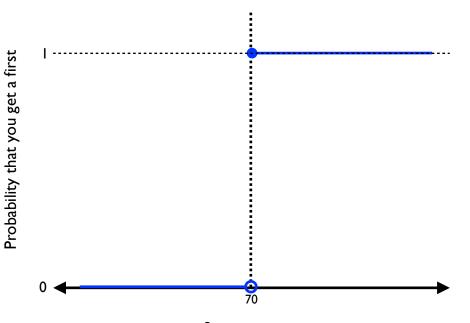
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#### Does RDD work for political science applications? General questions about sorting

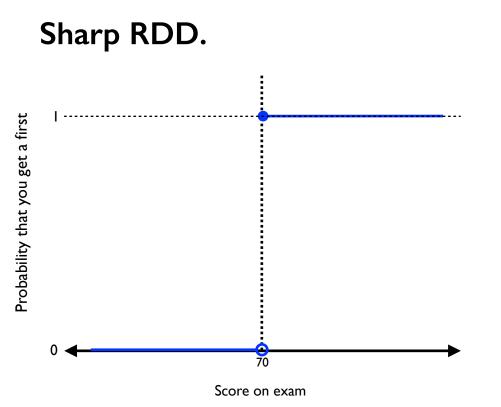
(1) Why do you think there is sorting in the municipal population case but not (apparently) close elections?(2) If there is sorting, is the RDD ruined?

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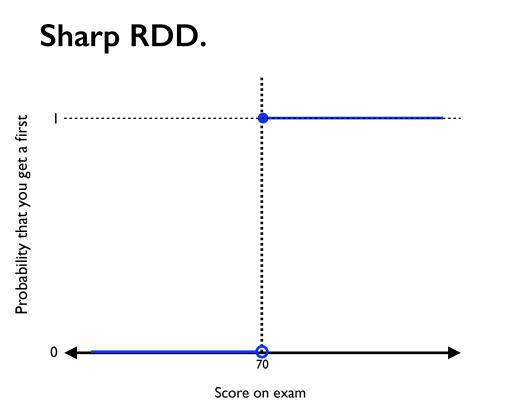
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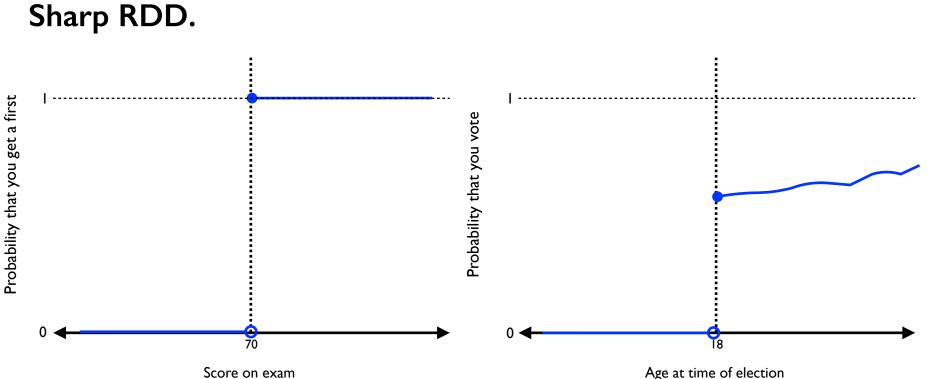


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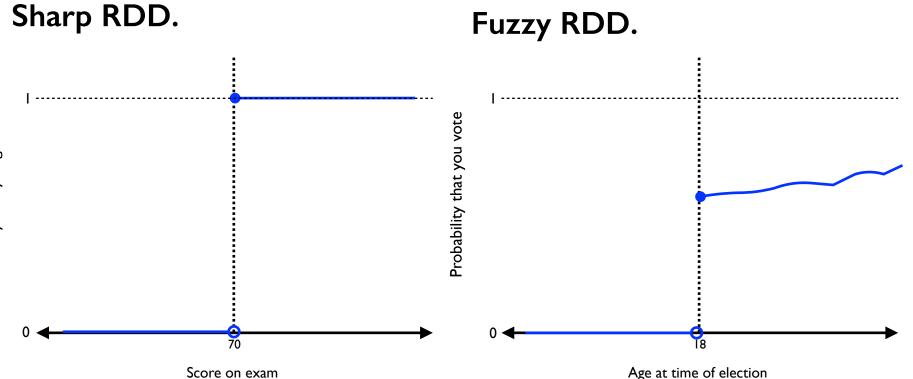


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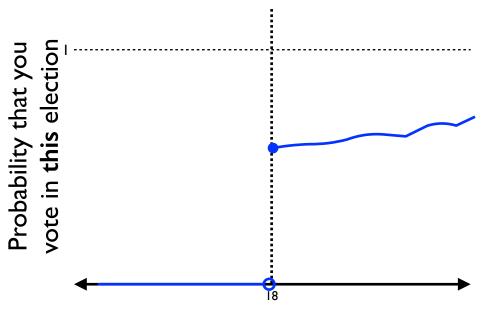


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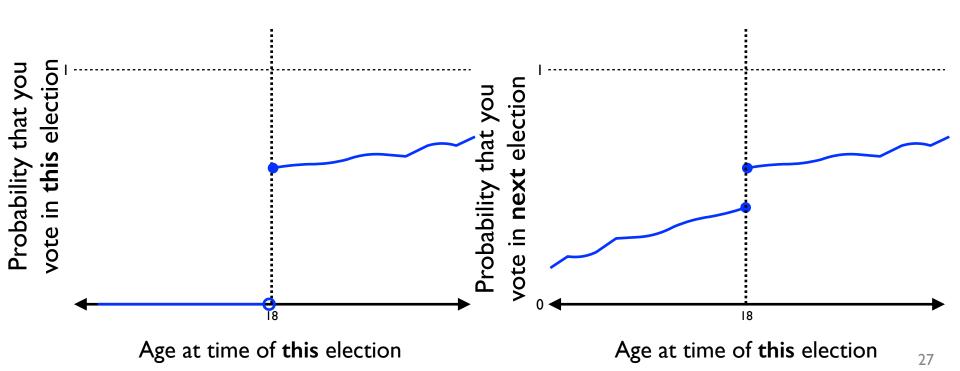
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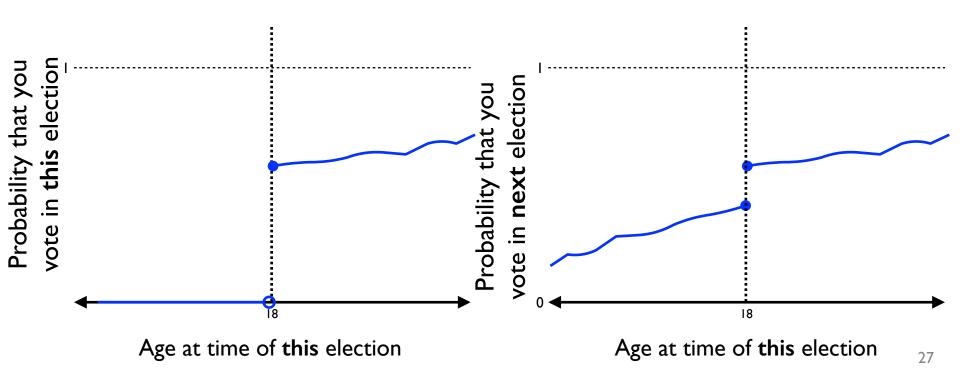


Age at time of this election

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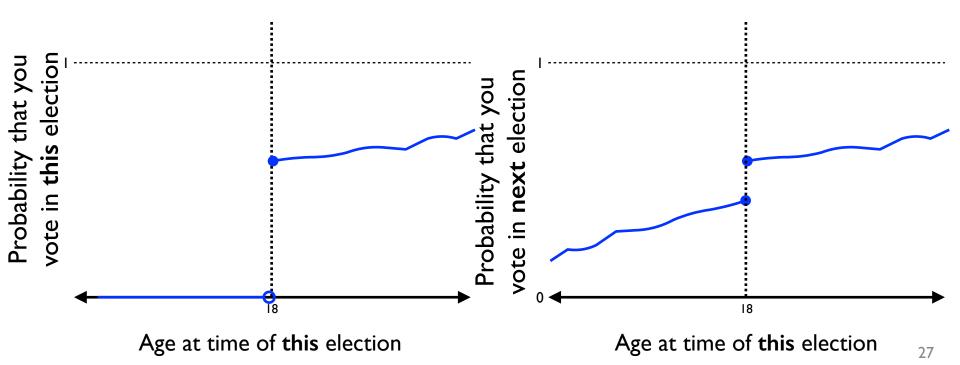


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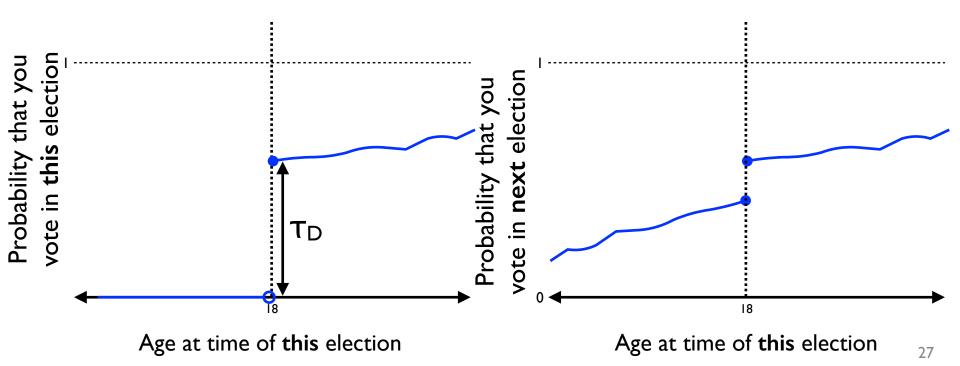
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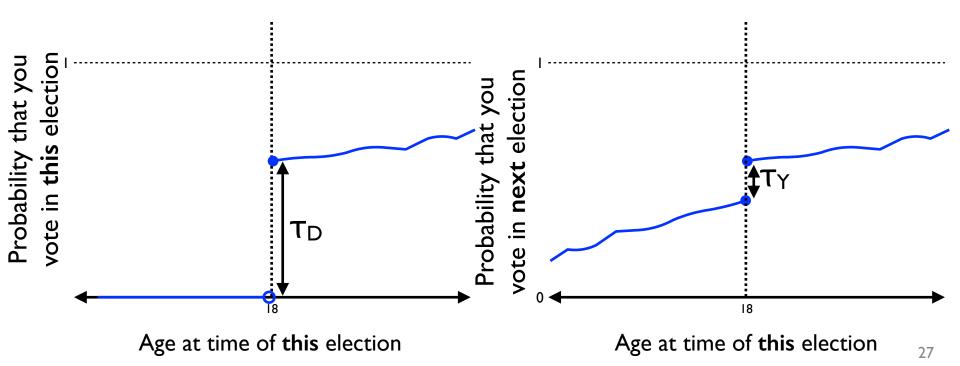
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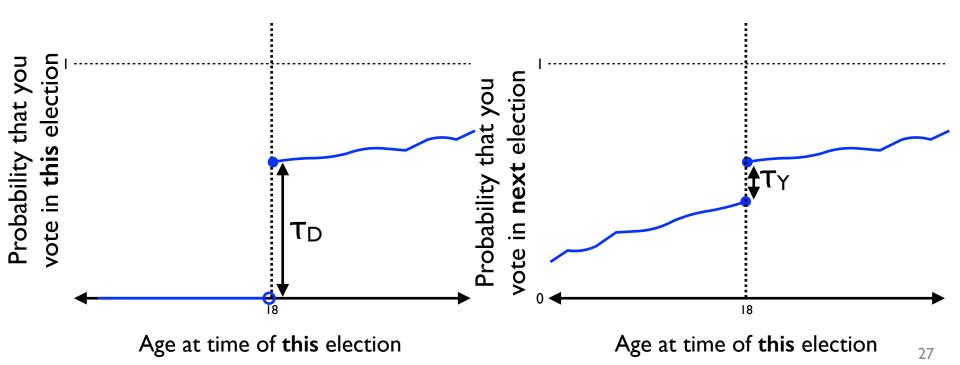
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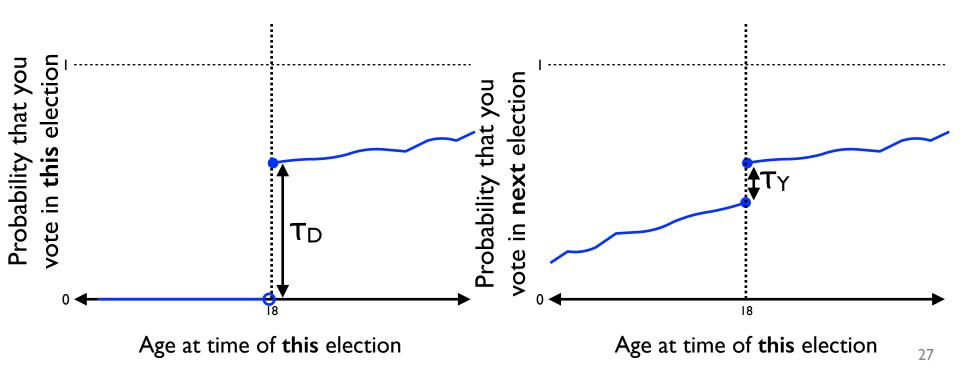
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- Other suggestions?

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You may find other cases where a transparent rule treats similar units differently — keep an eye out!