

# Inference

Week 7

27 February, 2016

Prof. Andrew Eggers

# What we're trying to understand today

Dependent variable: Nobel Prizes awarded per capita (in log scale)

	(1)	(2)	(3)
Intercept	-1.629* (0.509)	-3.166* (0.511)	-2.982* (0.527)
Chocolate consumption per capita (log scale)	2.092* (0.298)	1.026* (0.326)	0.709 (0.415)
GDP/capita (thousands of USD)		0.105* (0.024)	0.106* (0.024)
NW Europe			0.549 (0.452)
R <sup>2</sup>	0.70	0.85	0.86
N	34	34	34

- What do the stars mean on regression tables? Numbers in parentheses?
- What is the “margin of error” of a poll?
- What statistical findings are reliable? Which might be just a fluke?

Standard errors in parentheses. \* Indicates  $p < 0.05$

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TABLE 15.2

Multivariate regression analyses of the effect of consensus democracy (executives-parties dimension) on five indicators of violence, with controls for the effects of the level of economic development, logged population size, and degree of societal division, and with extreme outliers removed

Performance variables	Estimated regression coefficient	Absolute t-value	Countries (N)
Political stability and absence of violence (1996–2009)	0.189***	3.360	34
Internal conflict risk (1990–2004)	0.346**	2.097	32
Weighted domestic conflict index (1981–2009)	-105.0*	1.611	30
Weighted domestic conflict index (1990–2009)	-119.7**	2.177	33
Deaths from domestic terrorism (1985–2010)	-2.357**	1.728	33

\* Statistically significant at the 10 percent level (one-tailed test)

\*\* Statistically significant at the 5 percent level (one-tailed test)

\*\*\* Statistically significant at the 1 percent level (one-tailed test)

Source: Based on data in Kaufmann, Kraay, and Mastruzzi 2010; PRS Group 2004; Banks, 2010; and GTD Team 2010

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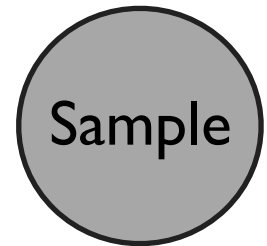
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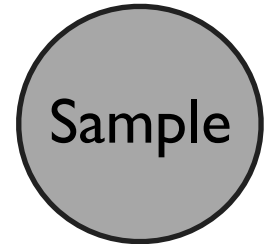
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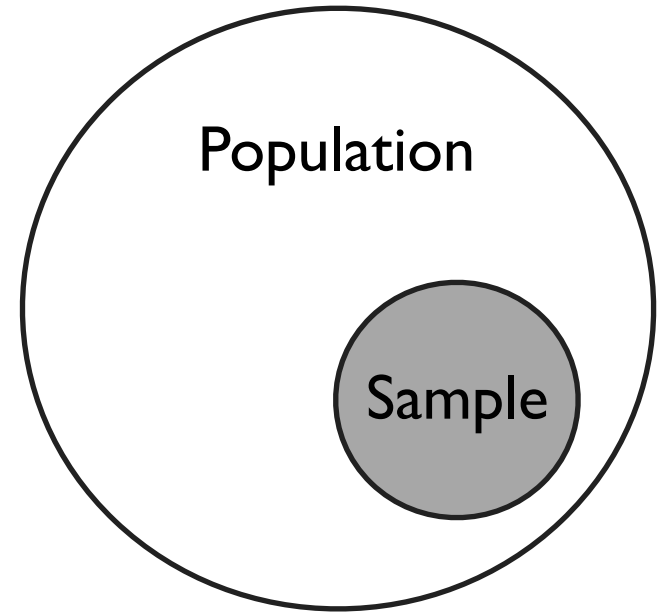


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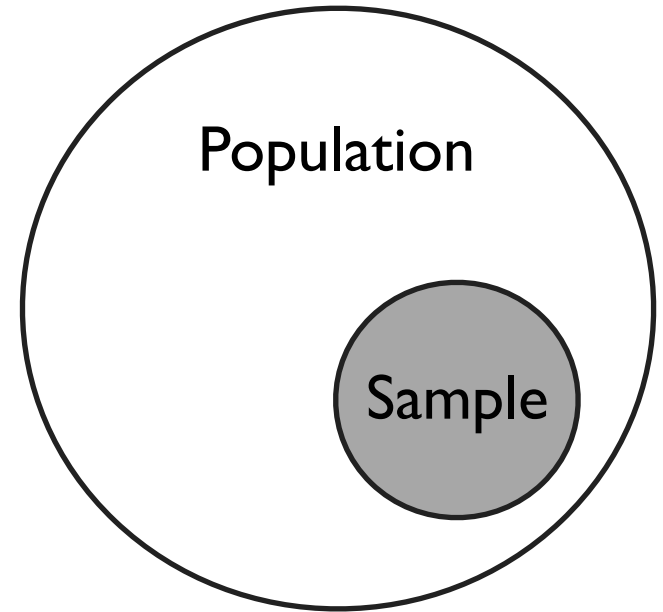
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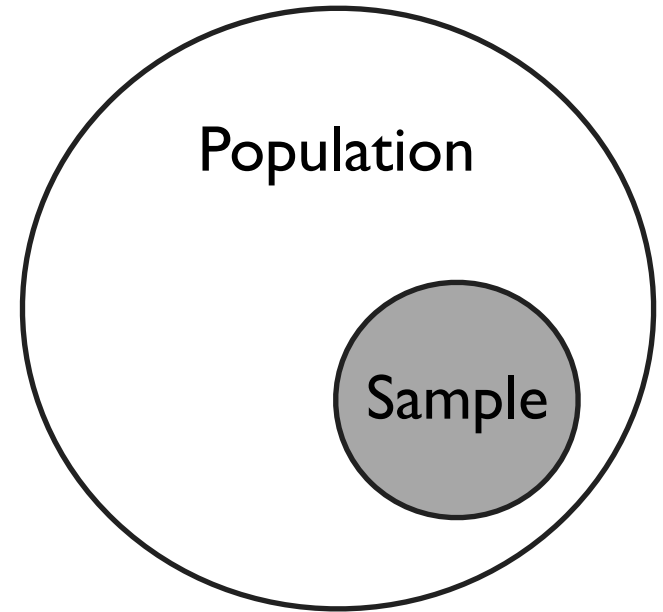
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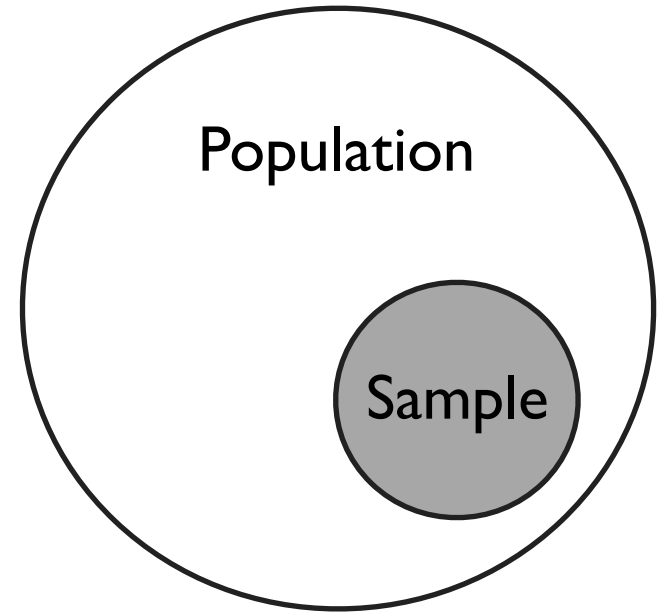


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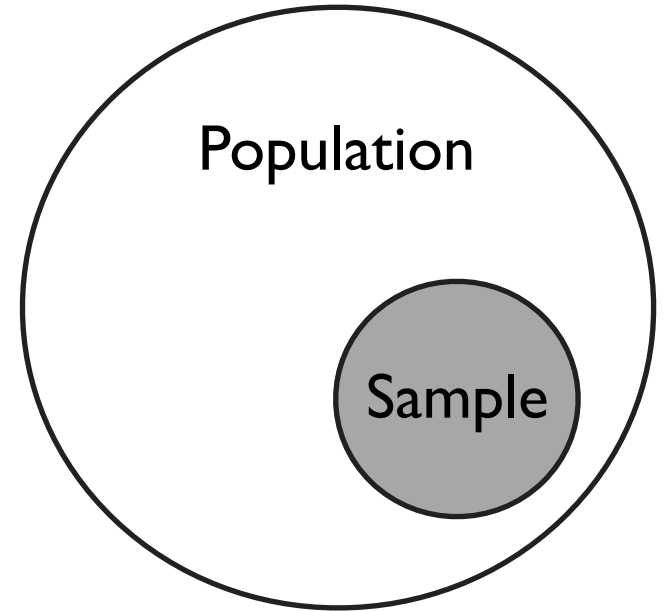
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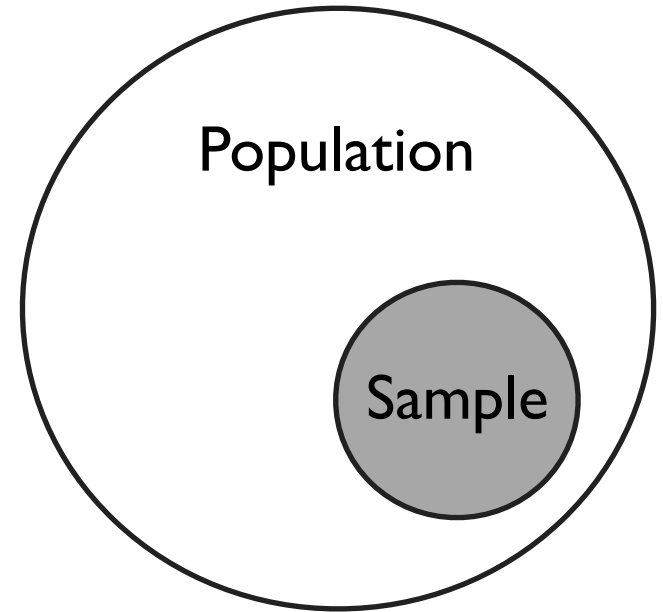
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Because the sample is not the population:

- polls have a **margin of error**
- regression coefficients have **standard errors**
- our conclusions in hypothesis testing are guesses, with confidence summarized by **p-values**

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“Margin of error” tries to summarize the **magnitude of random error due to sampling.**

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- size of sample (1,006 GB adults vs. 10,000,000)
- true level of support (what if 100% supported remaining in EU?)











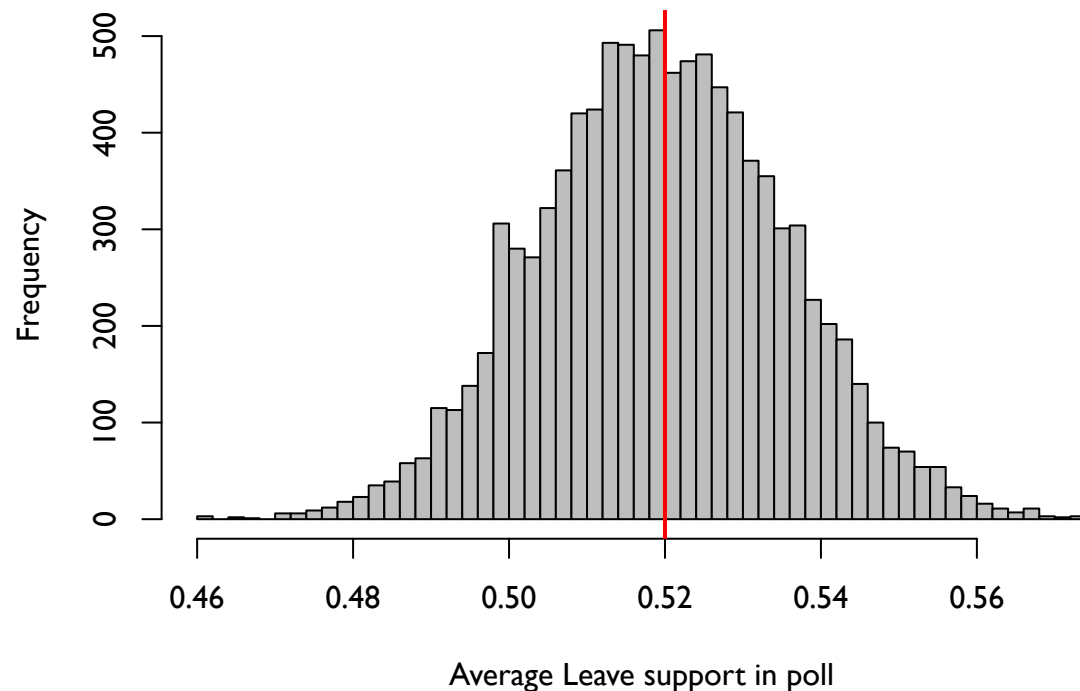




# Simulating the thought experiment (2)

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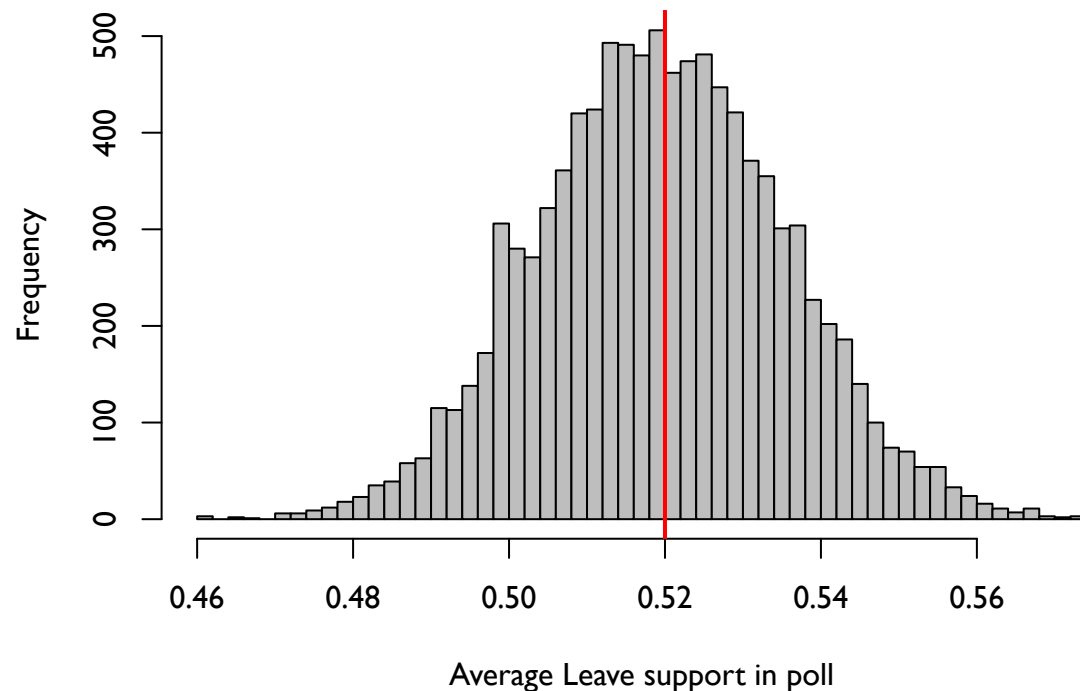


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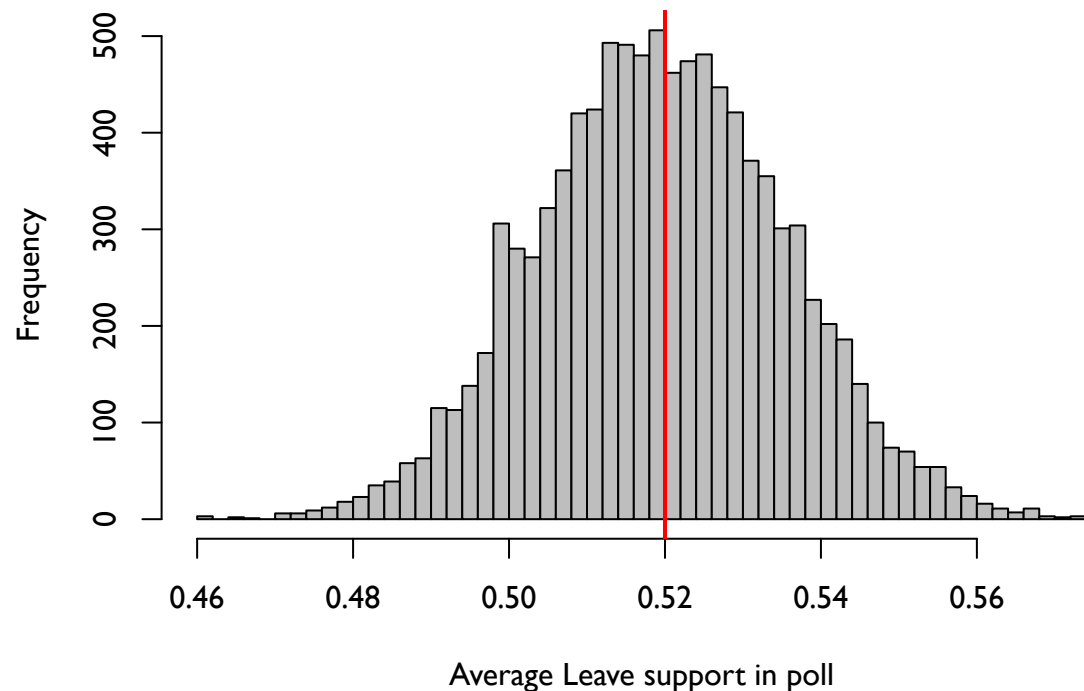
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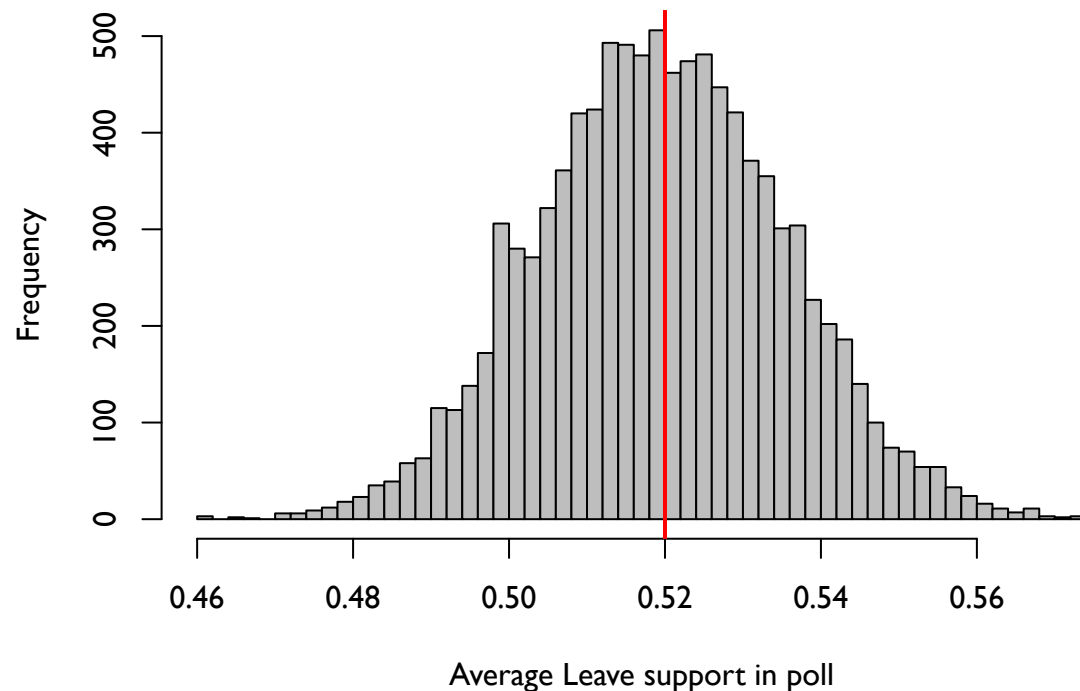
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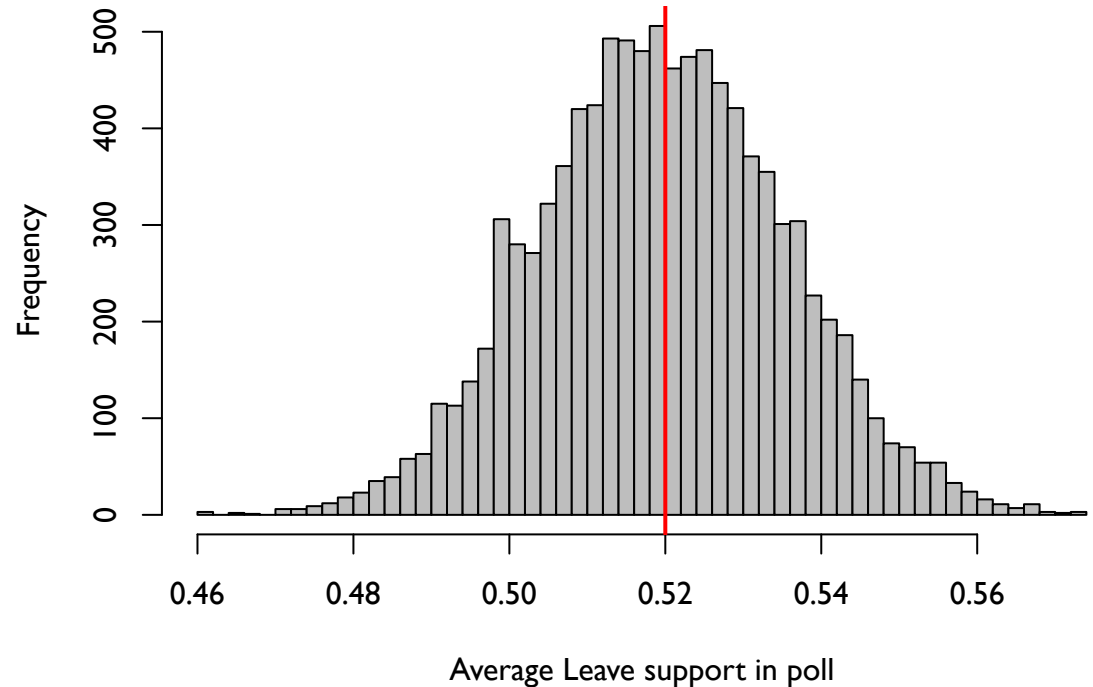
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I can do it  
10,000 times  
and look at  
the histogram  
of support:



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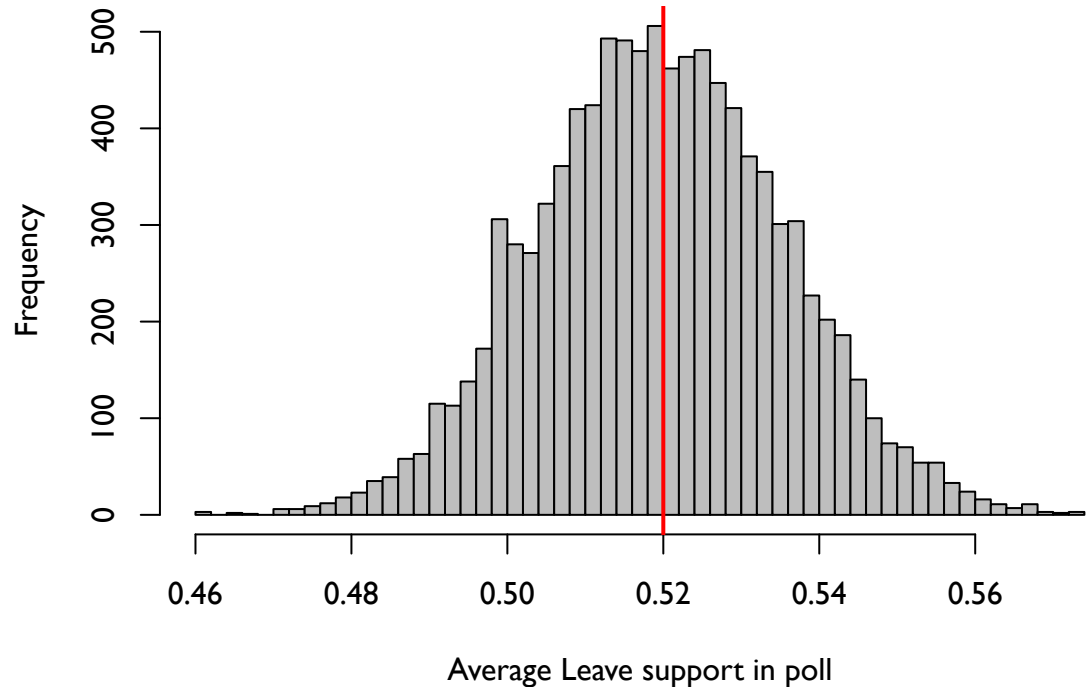


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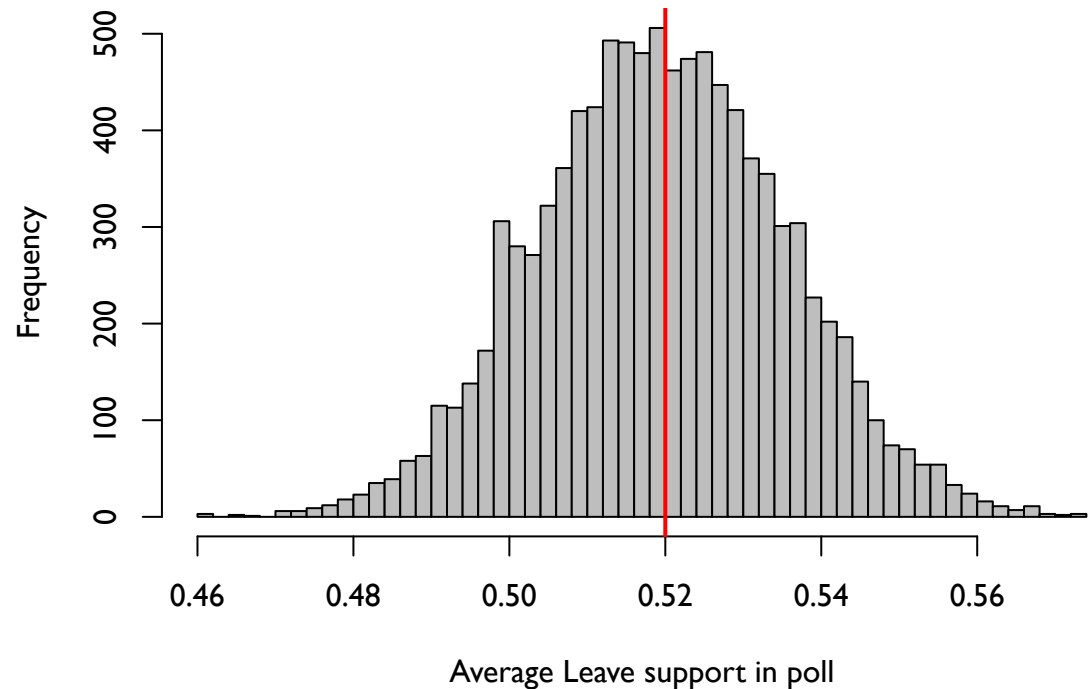
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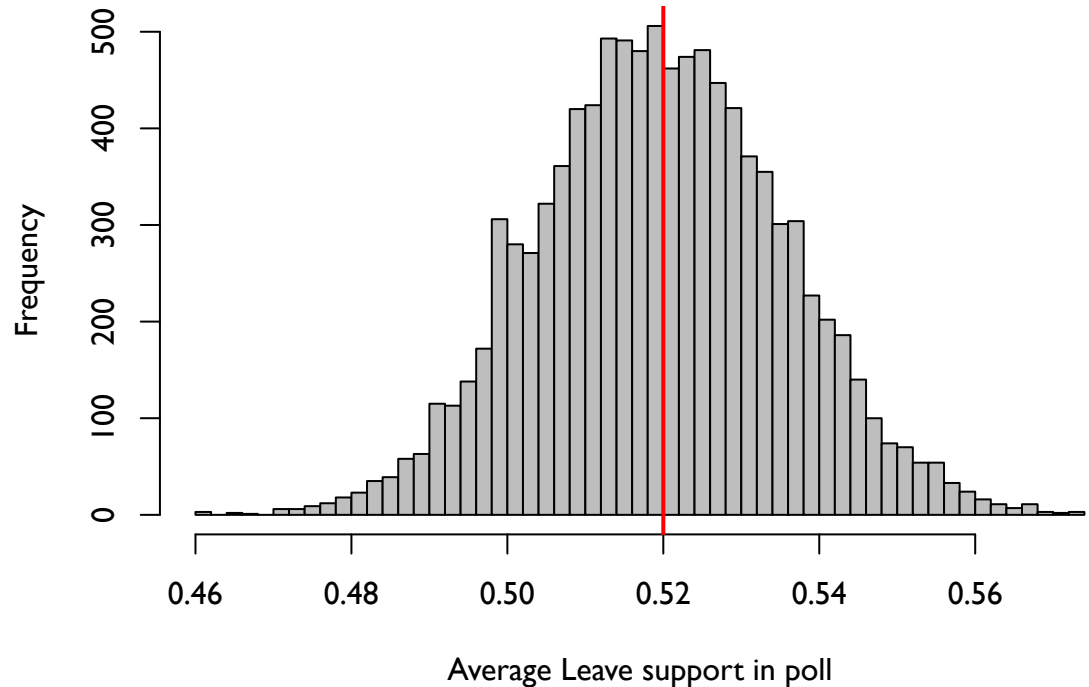
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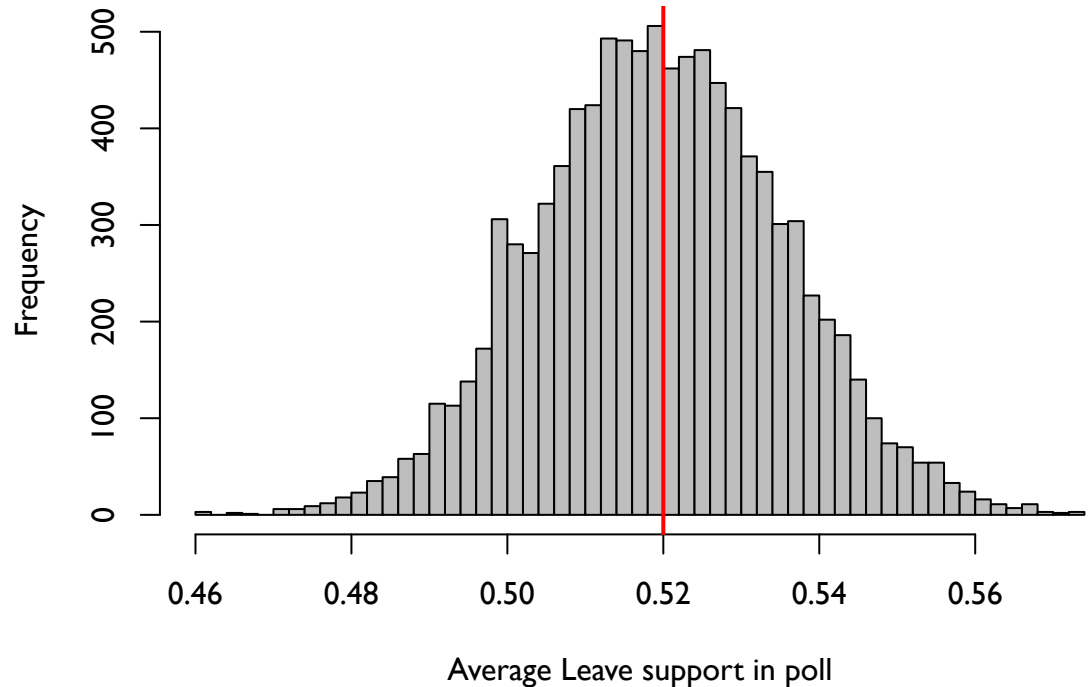
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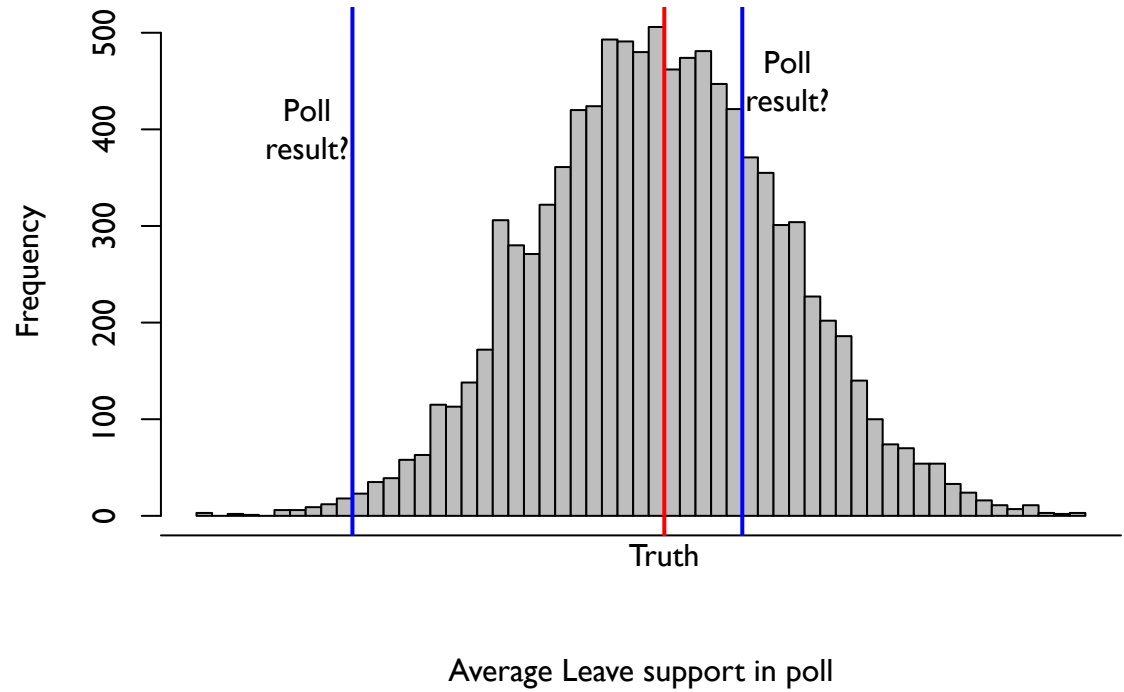
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95% of the samples had a mean between 0.49 and 0.55:

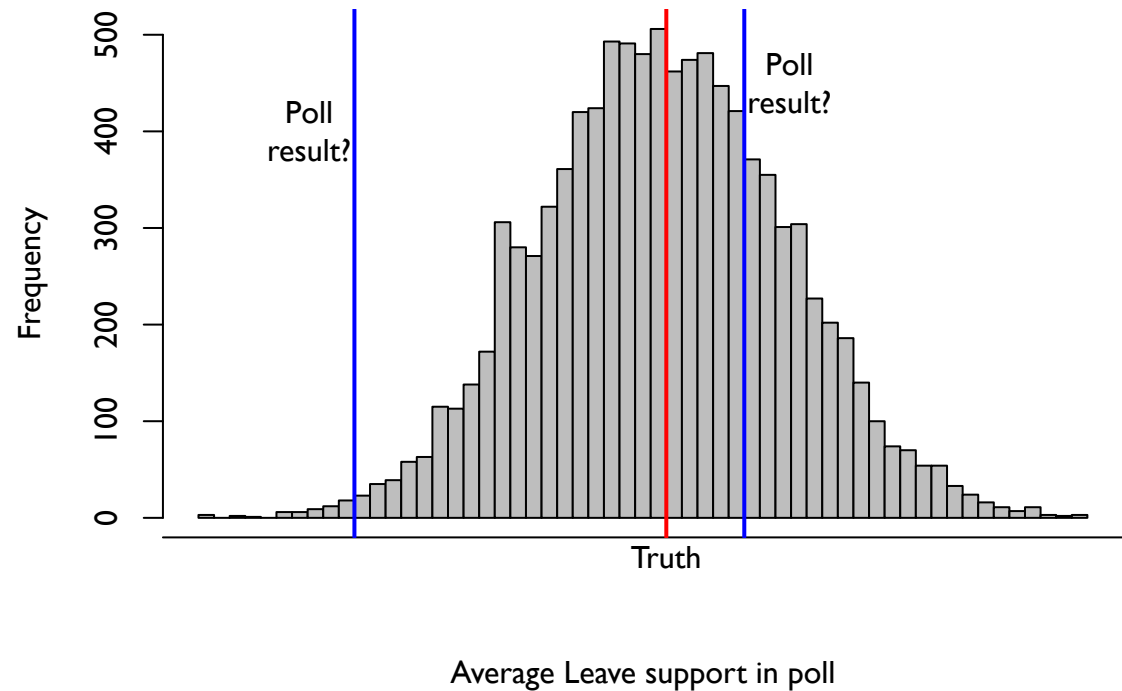
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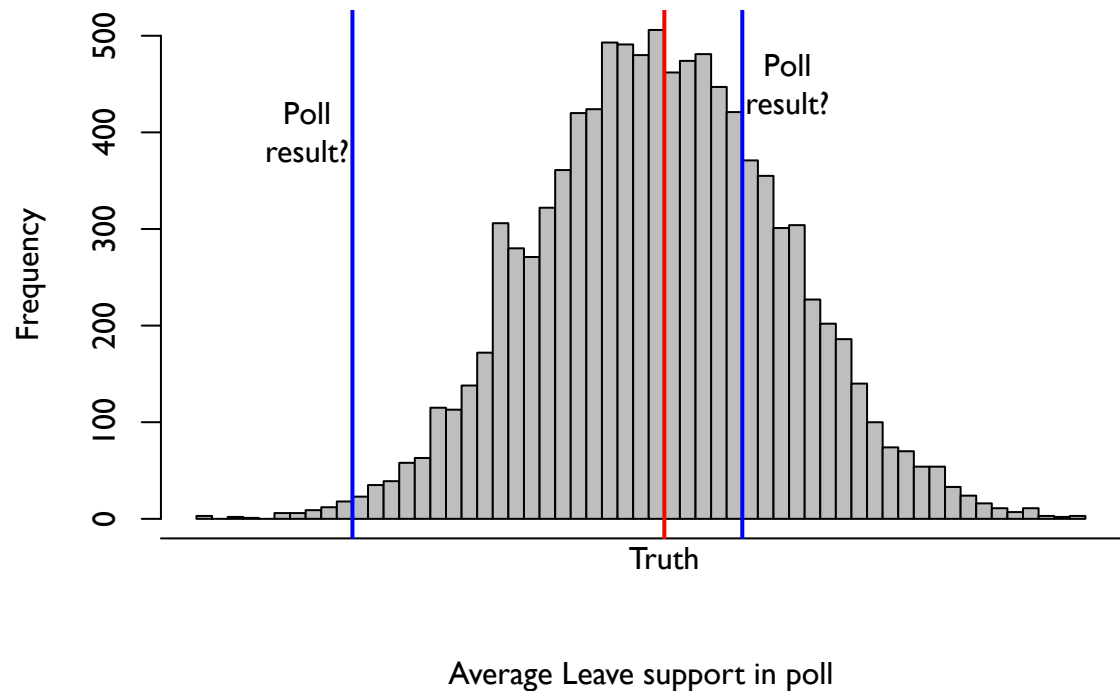




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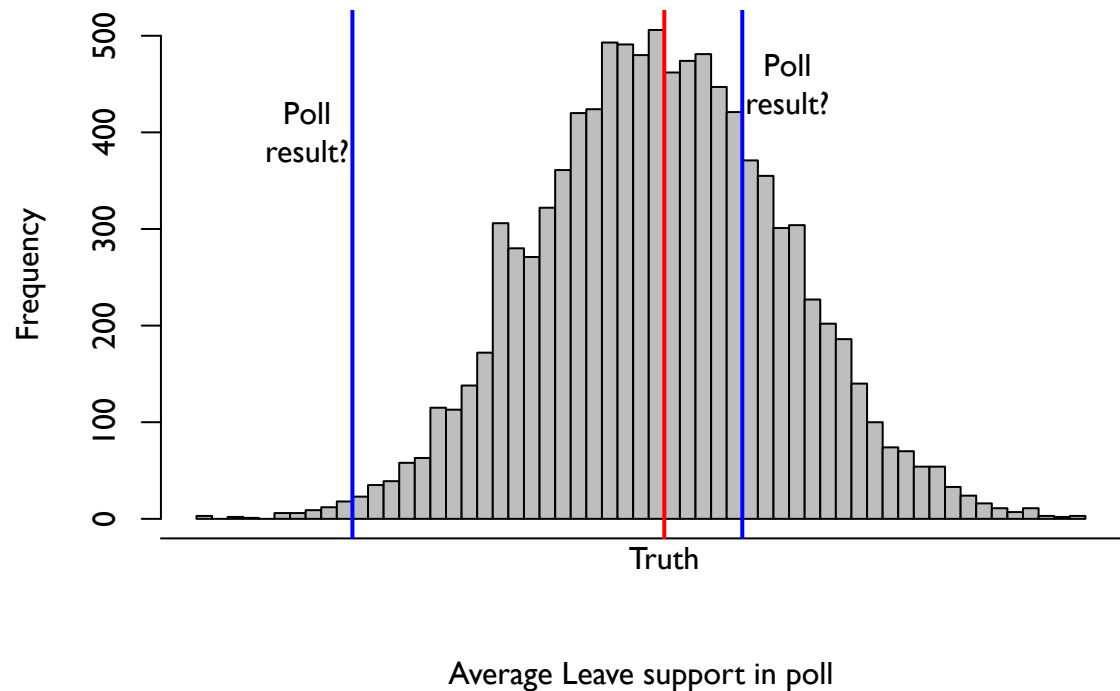
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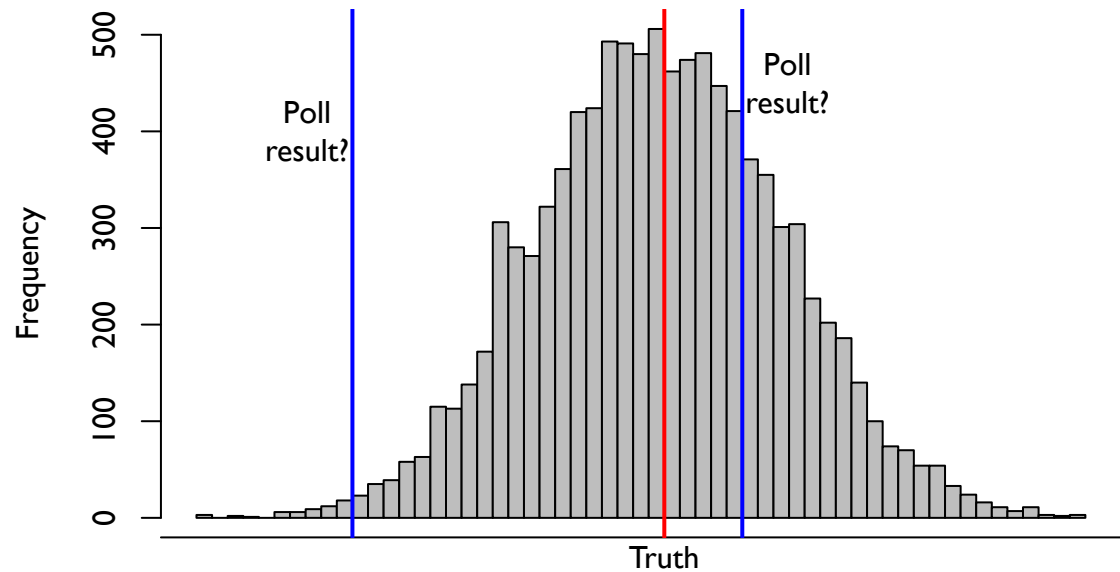


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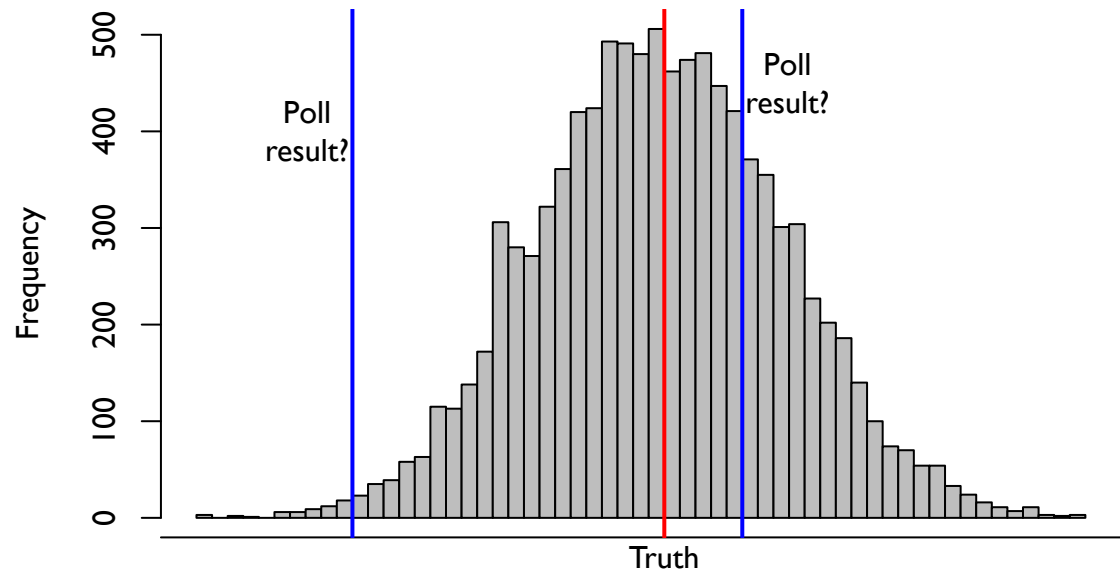
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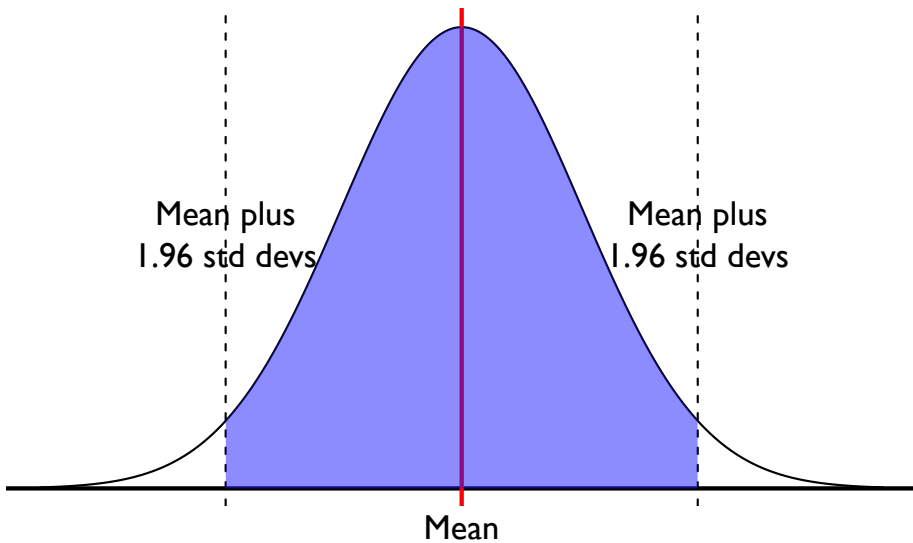
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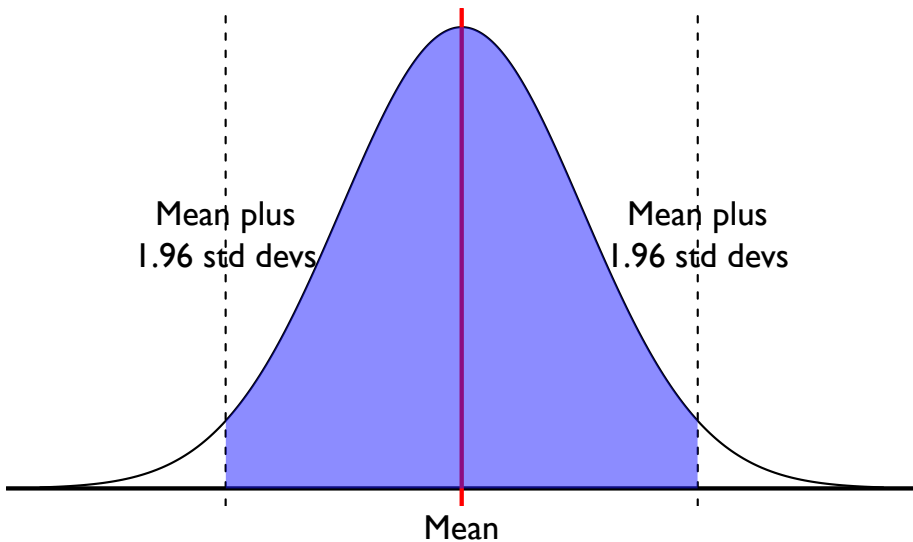
Compare: the standard deviation of our simulations was 0.0157

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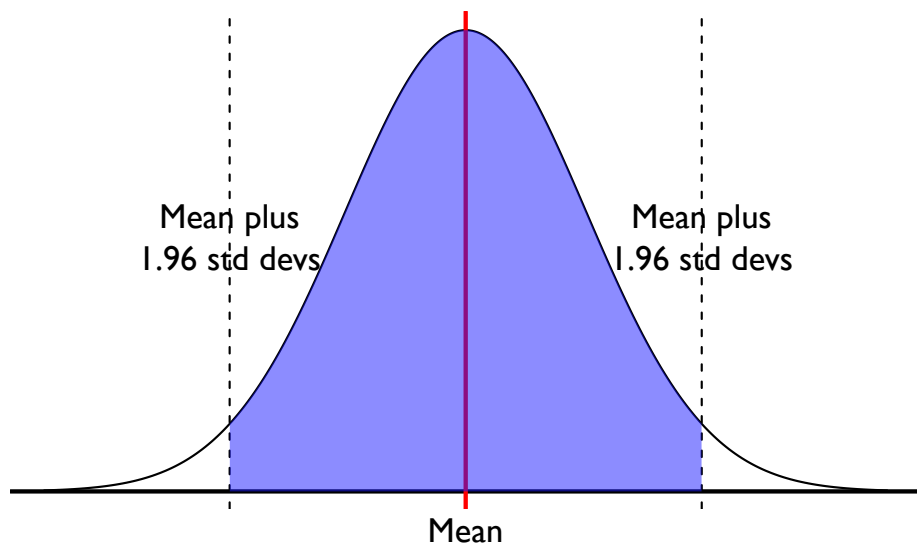
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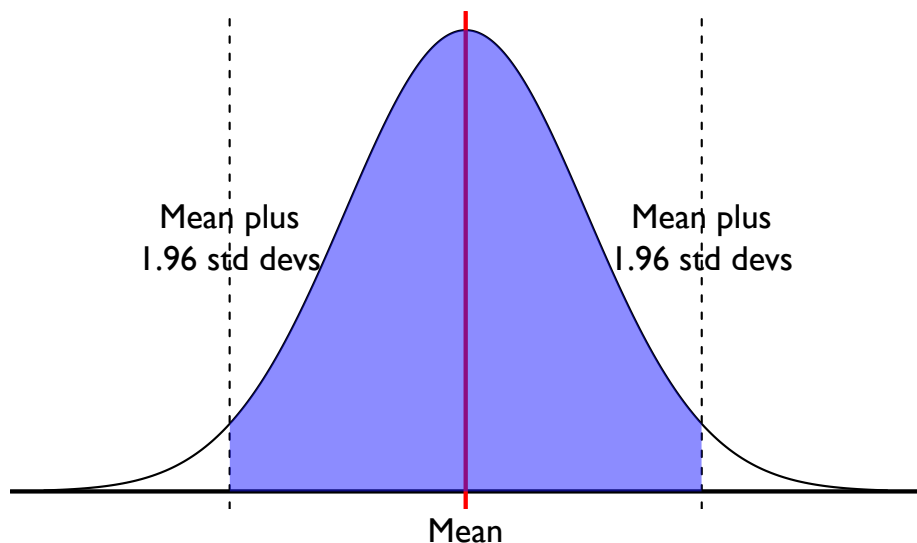
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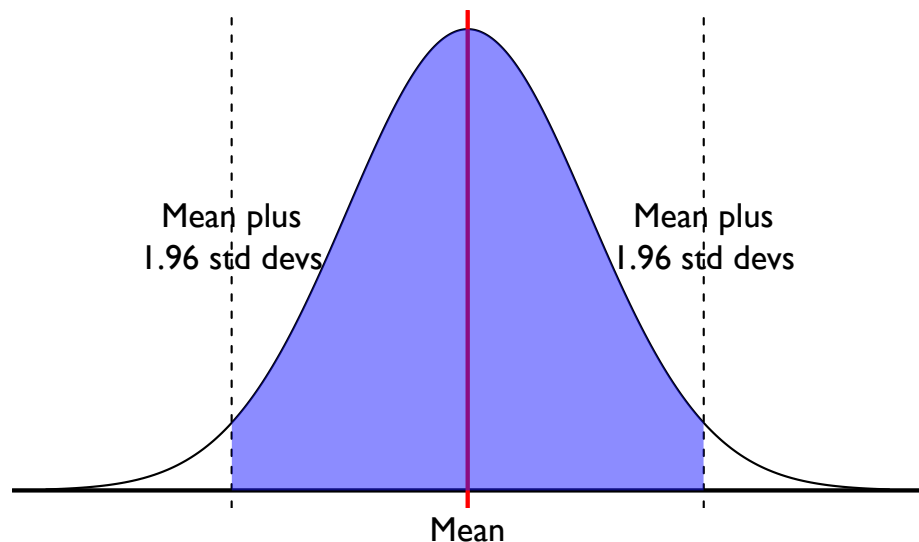


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  - **Simulation** in R of 10,000 random samples of size 1,006 given a known level of support for “Leave”
  - **Central limit theorem:** approximation to a normal distribution

# Inference in regression

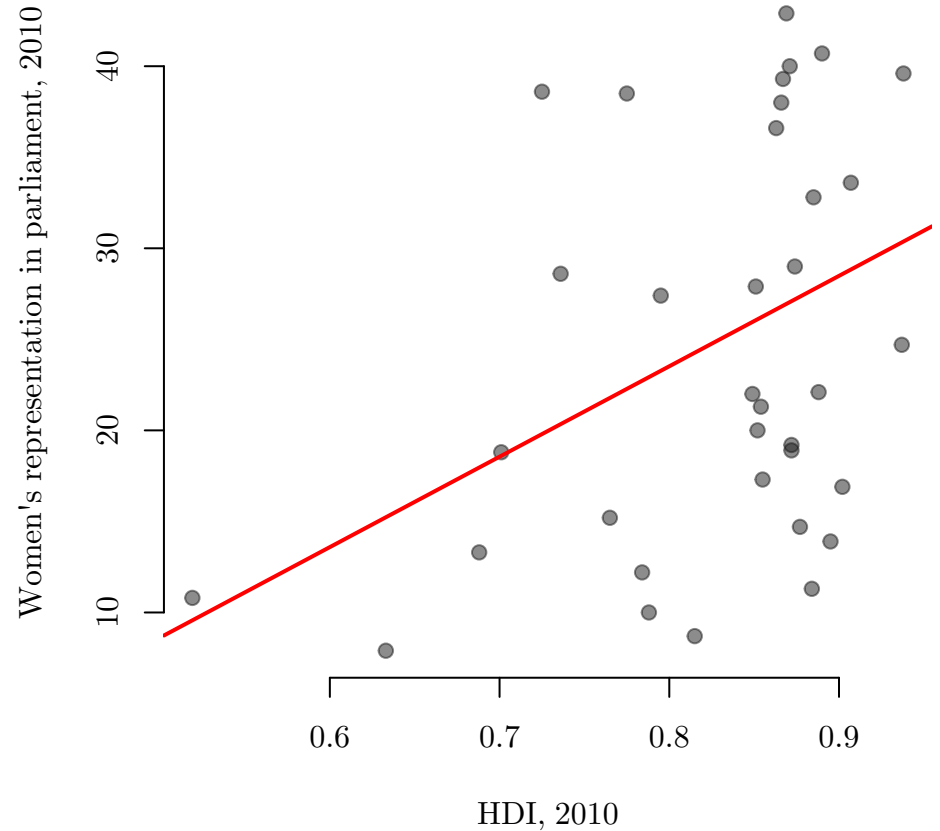
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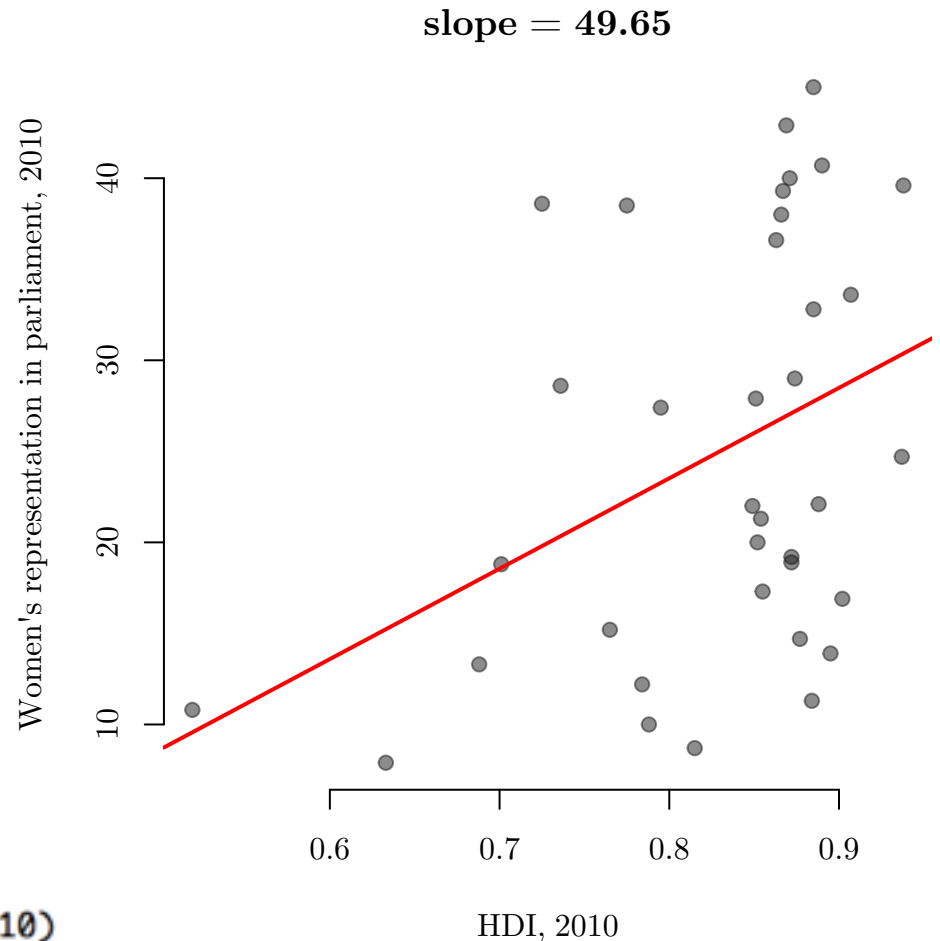
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Recall from Lab 2: in Lijphart's data, positive relationship between development and women's representation in parliament:

```
> lm(data$women2010 ~ data$hdi_2010)
```

```
Call:  
lm(formula = data$women2010 ~ data$hdi_2010)
```

```
Coefficients:  
(Intercept)  data$hdi_2010  
-16.20      49.65
```





# Inference in regression (2)

Looking at `summary()`, R gives us standard errors for the coefficients:

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Residuals:
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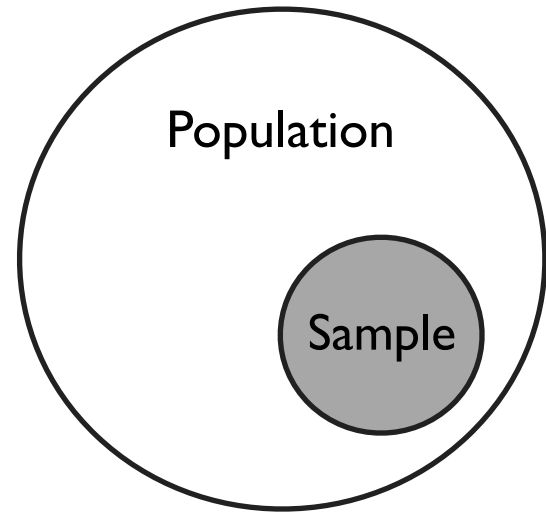
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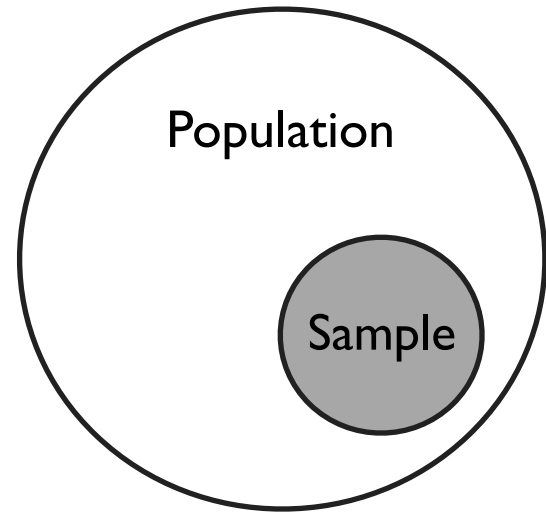
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Two ways to think  
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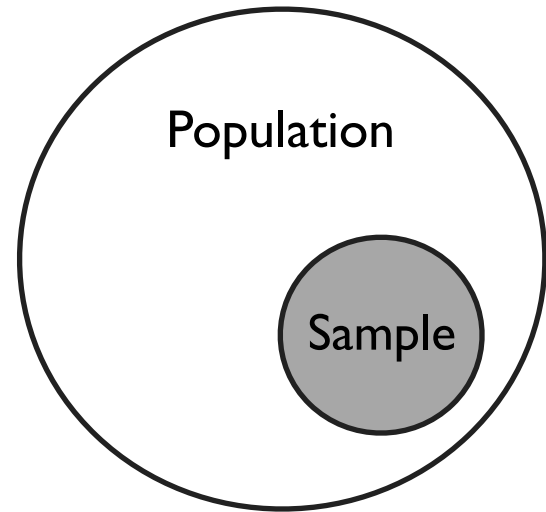


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Our estimates have standard errors because we view our data as a sample, and we want to characterize the population.

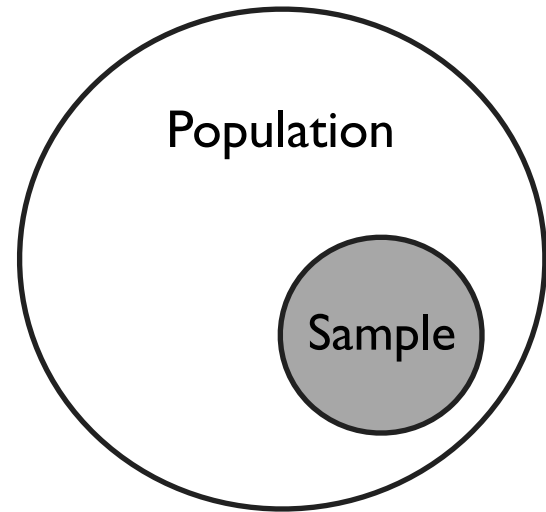
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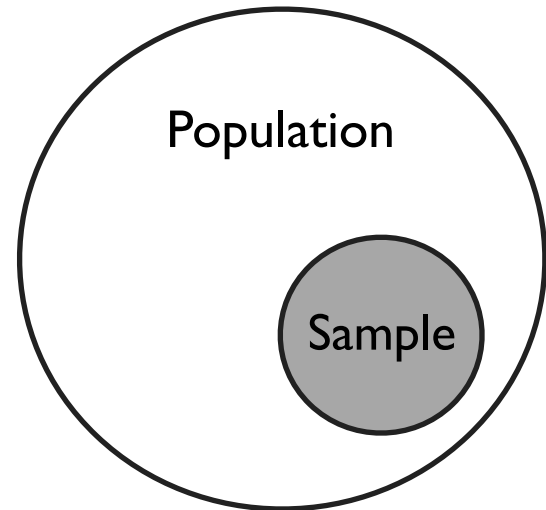


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1. **Just like in the survey:** Our *units* are a random sample from a population, so the coefficients vary across samples.

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Two ways of thinking about this:

1. **Just like in the survey:** Our *units* are a random sample from a population, so the coefficients vary across samples.
2. **Slightly differently:** Our *residuals* are a random sample from a population, so the coefficients vary across samples.

# Illustration via “re-sampling”



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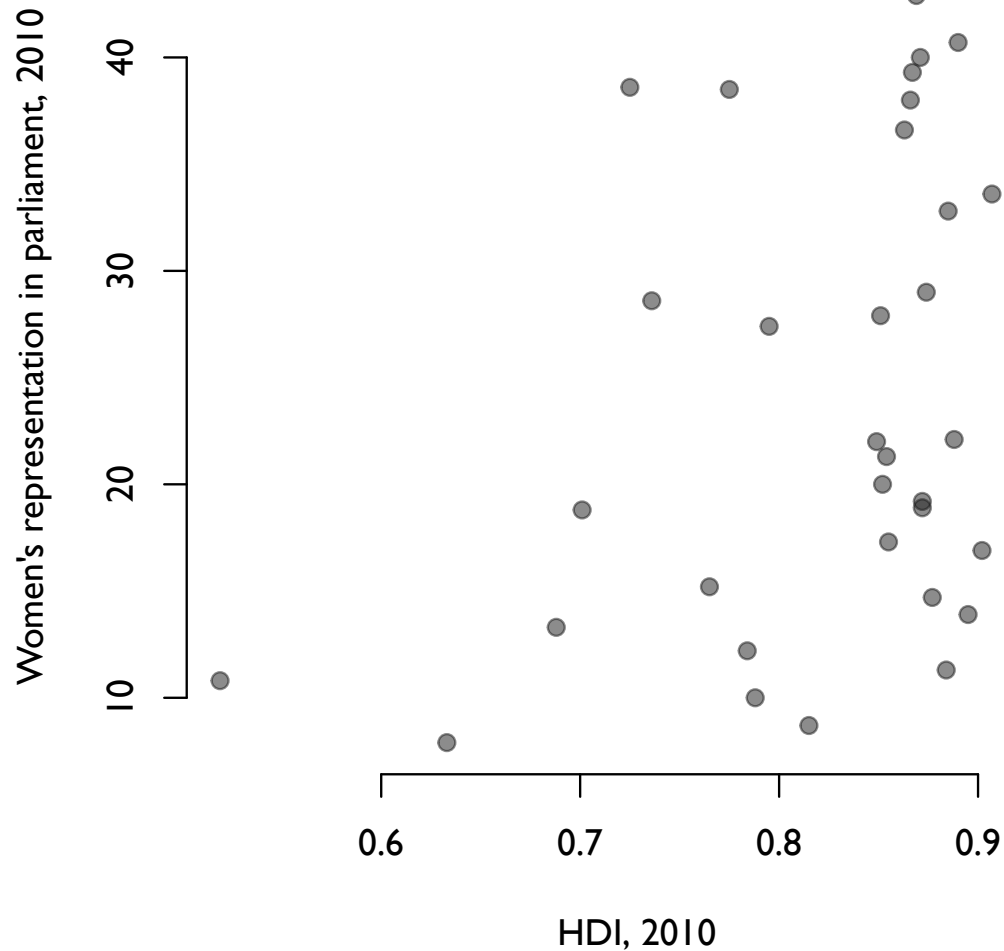
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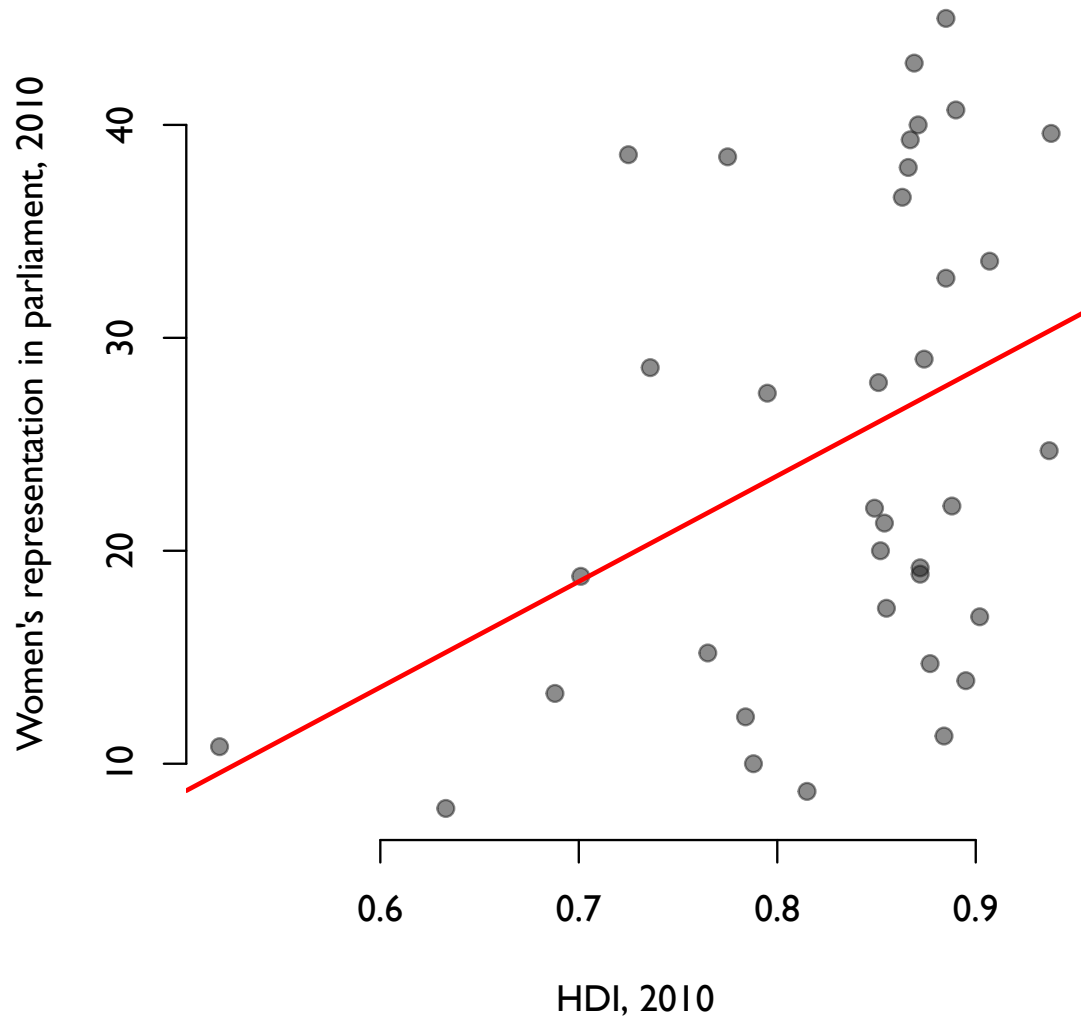
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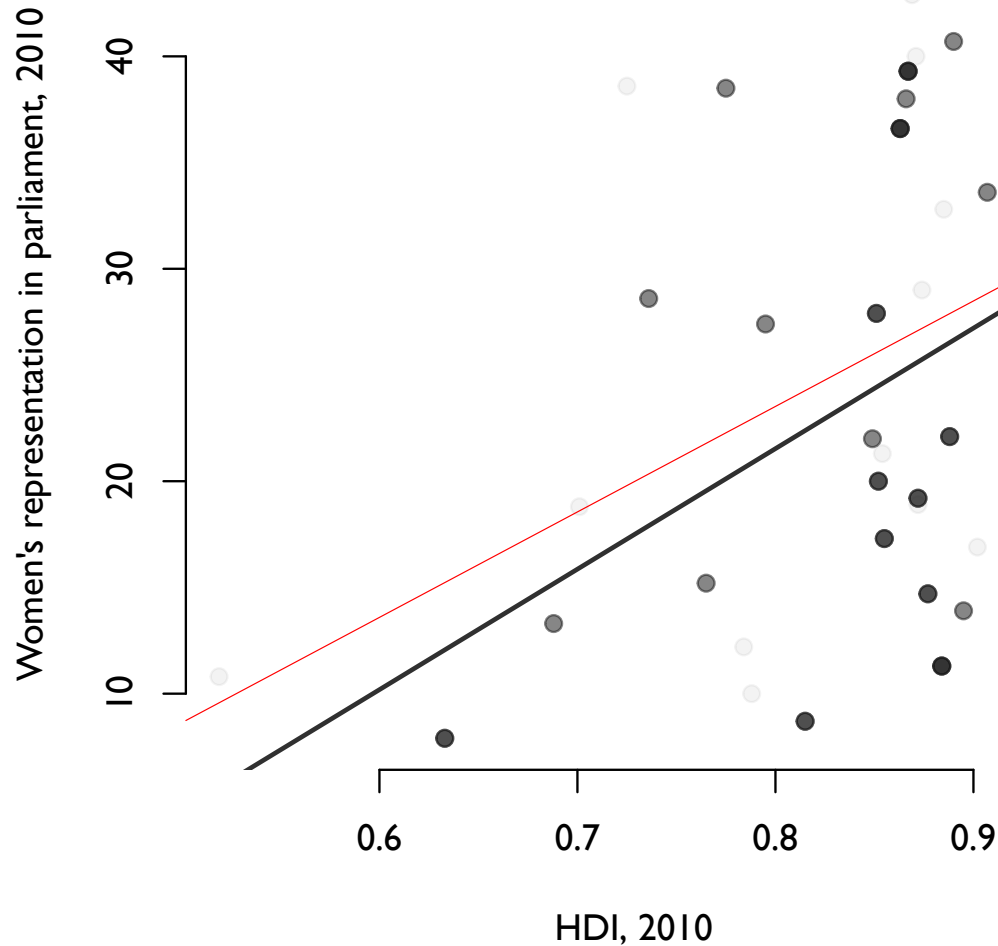
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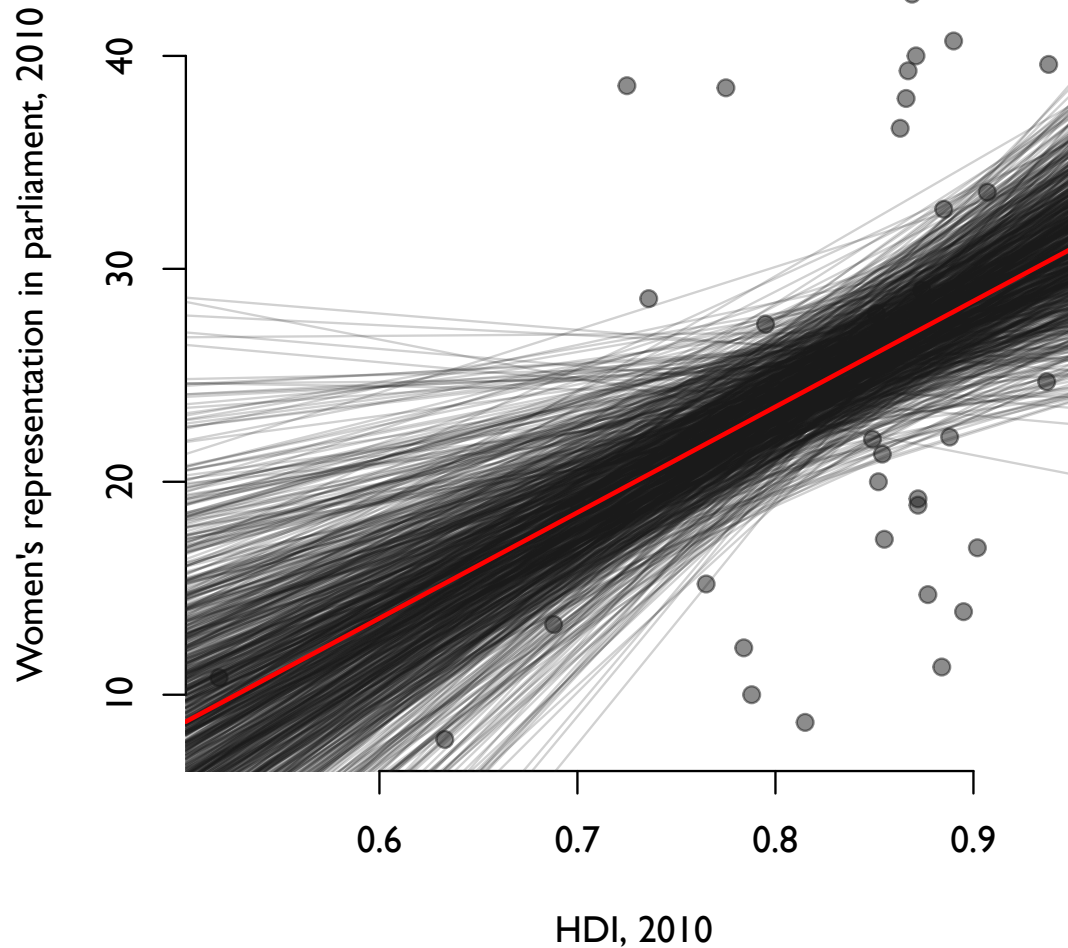
# Illustration via “re-sampling”: regression in original sample



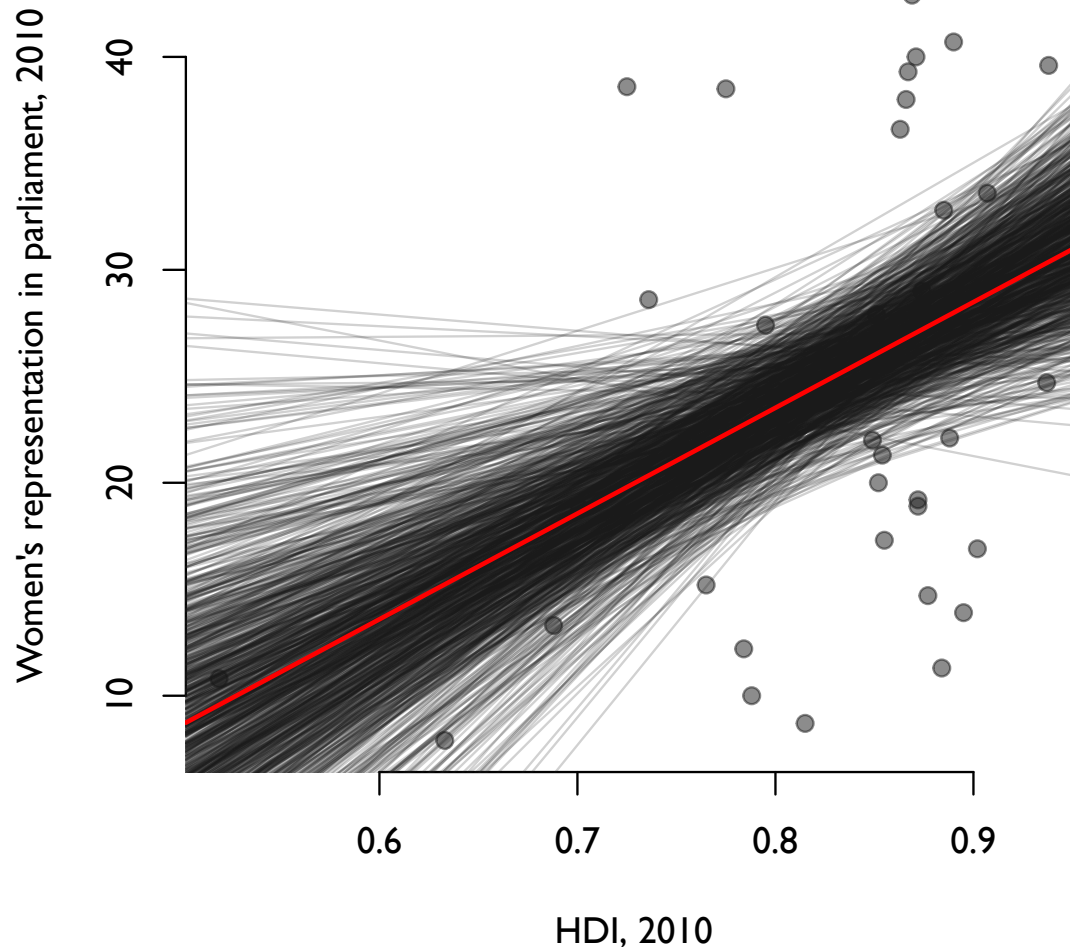
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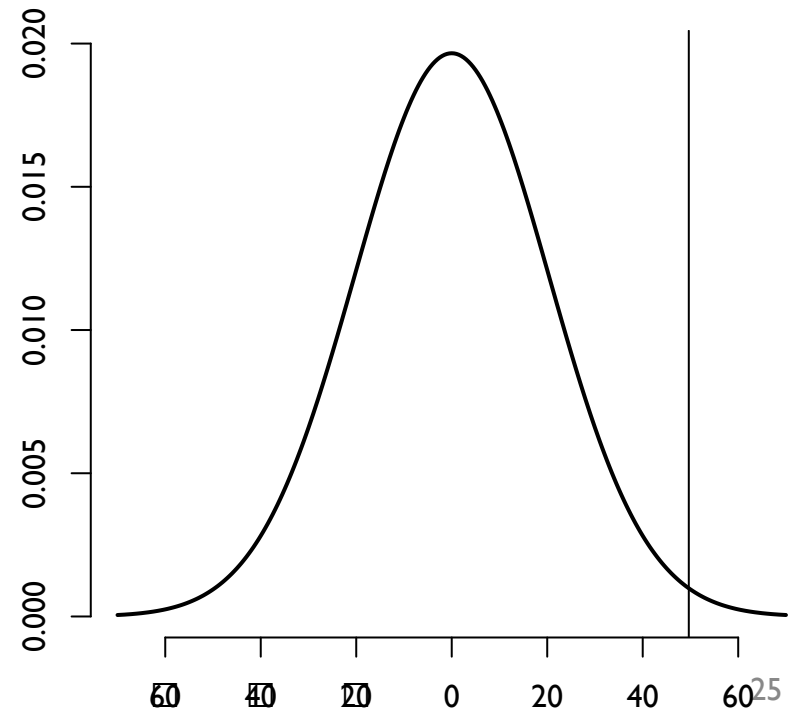
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We use standard errors to do hypothesis testing.

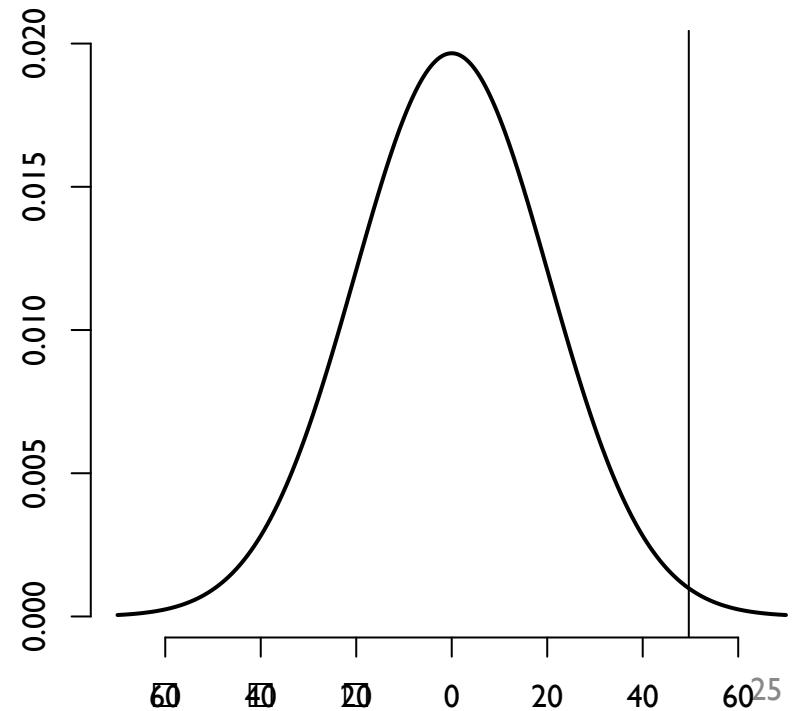


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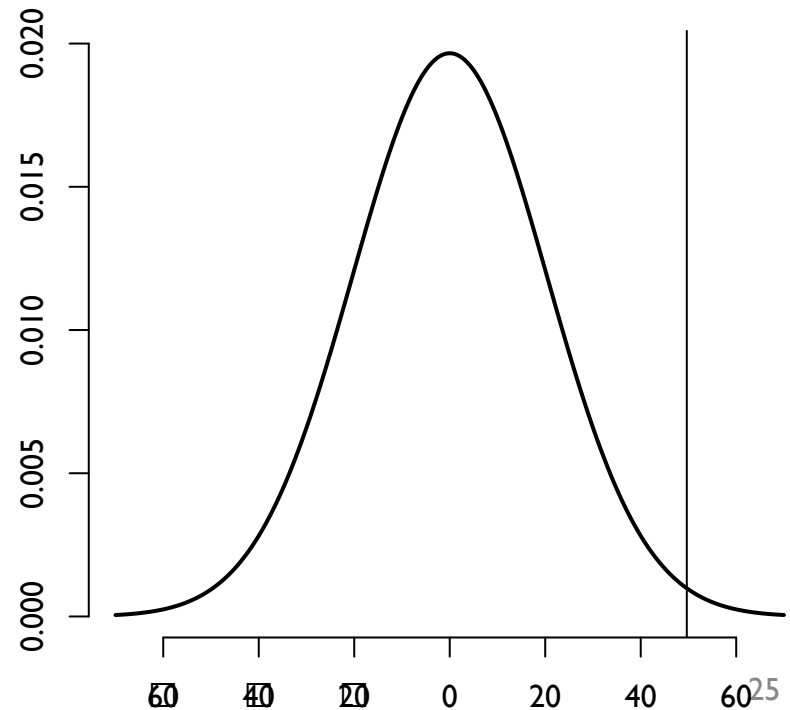
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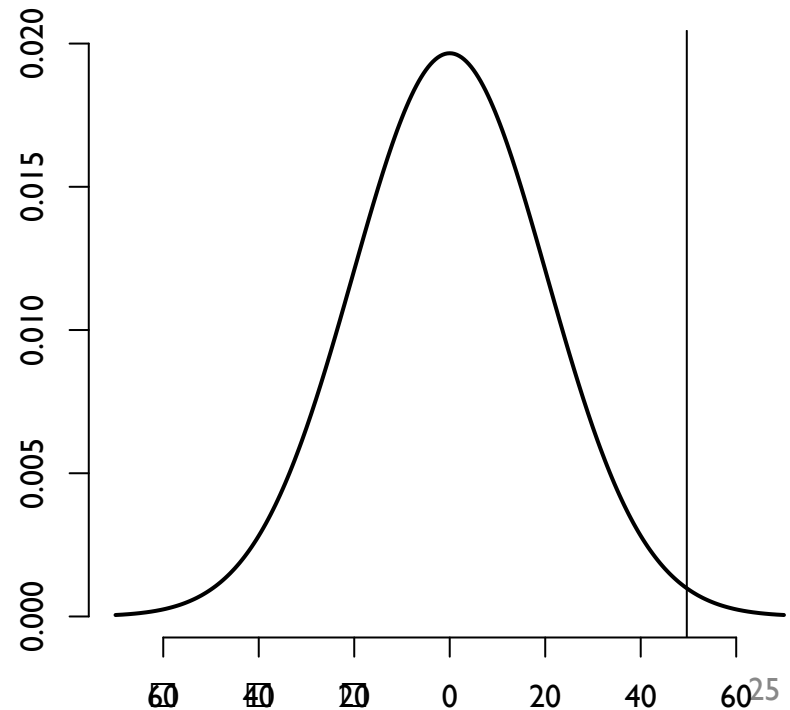
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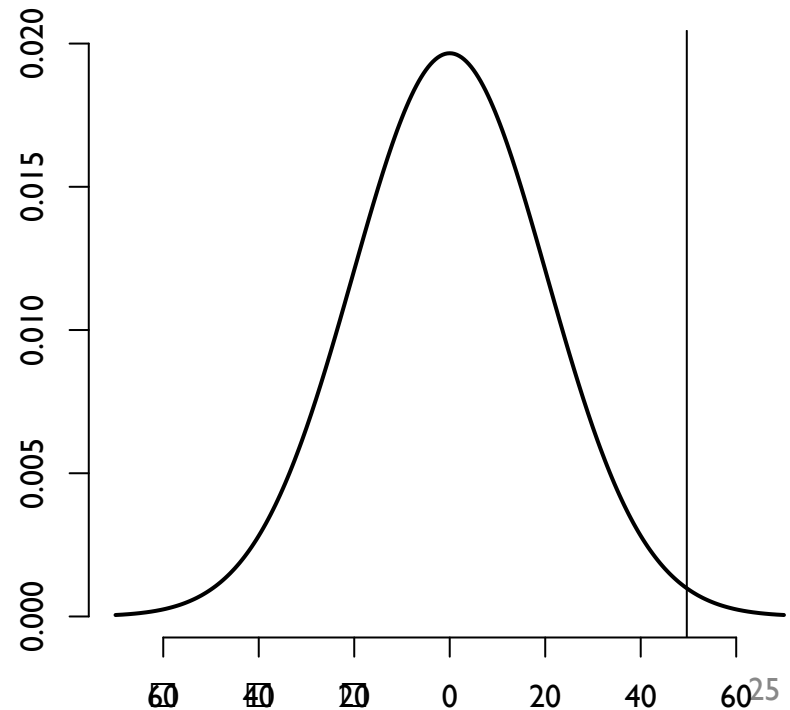


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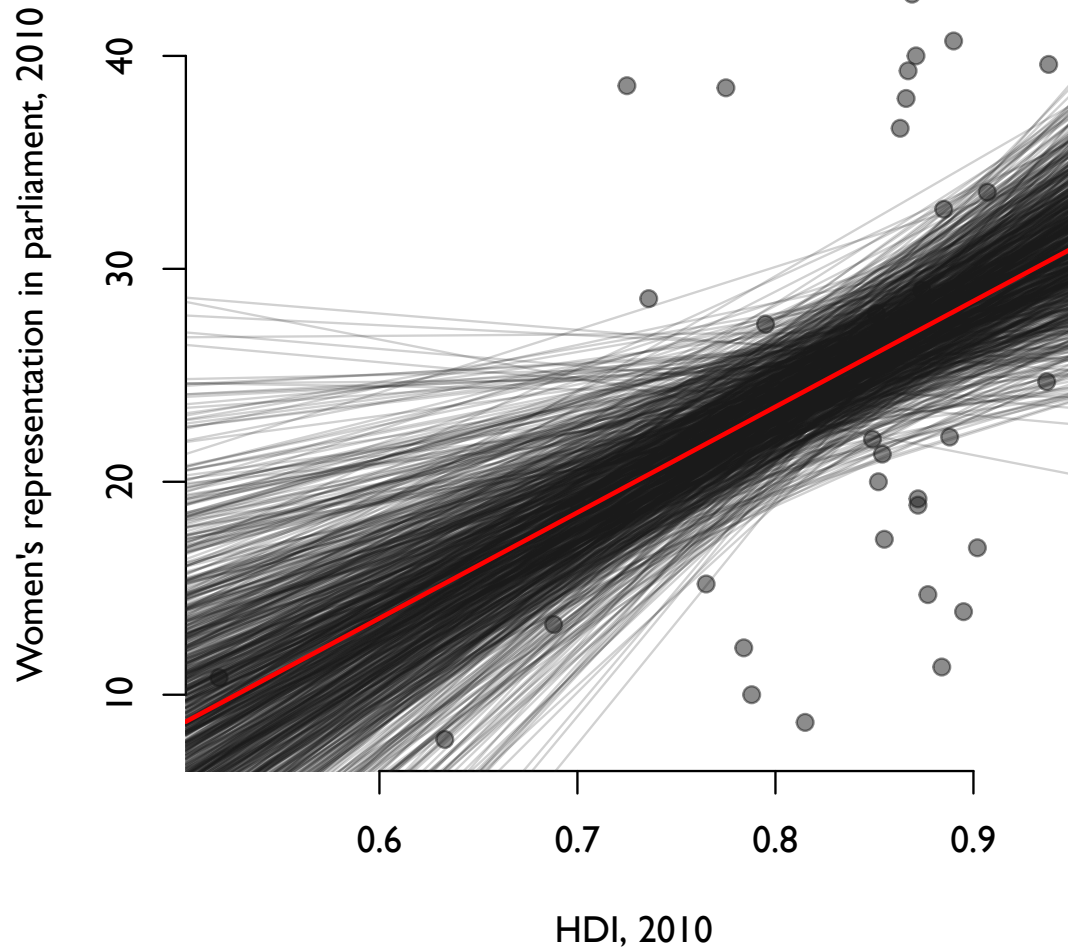
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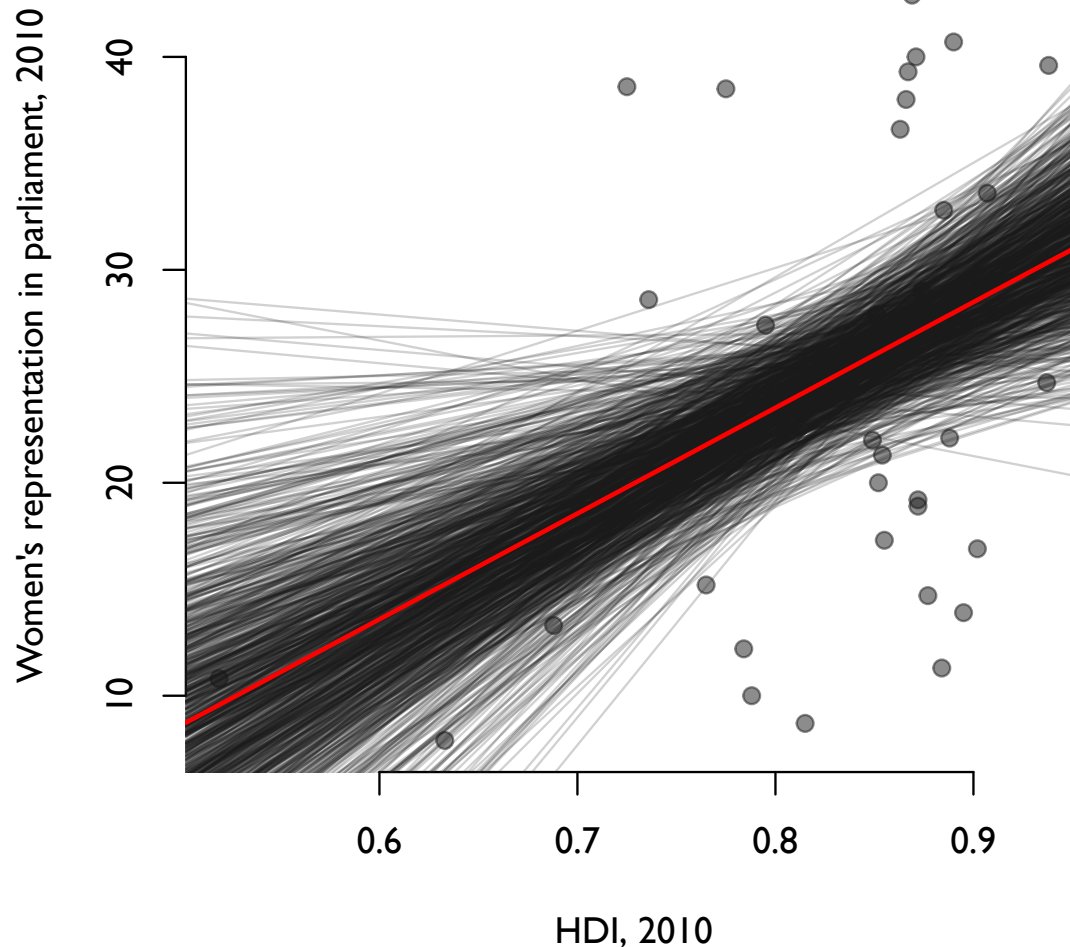
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4. If p-value is low enough, reject null hypothesis, and say the correlation or regression coefficient is “**statistically significant**”

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**Extremely difficult** to get truly representative random sample.

**Important:** Margin of error captures random error (i.e. sampling error), not bias.

# Now you should understand:

Dependent variable: Nobel Prizes awarded per capita (in log scale)

	(1)	(2)	(3)
Intercept	-1.629* (0.509)	-3.166* (0.511)	-2.982* (0.527)
Chocolate consumption per capita (log scale)	2.092* (0.298)	1.026* (0.326)	0.709 (0.415)
GDP/capita (thousands of USD)		0.105* (0.024)	0.106* (0.024)
NW Europe			0.549 (0.452)
R <sup>2</sup>	0.70	0.85	0.86
N	34	34	34

- what a dependent variable is
- what an independent variable is
- what the coefficients mean (intercept, slopes)
- what the stars mean (i.e. what  $p < 0.05$  means)
- what the standard errors mean

Standard errors in parentheses. \* Indicates  $p < 0.05$

# And this too!

TABLE 15.2

Multivariate regression analyses of the effect of consensus democracy (executives-parties dimension) on five indicators of violence, with controls for the effects of the level of economic development, logged population size, and degree of societal division, and with extreme outliers removed

Performance variables	Estimated regression coefficient	Absolute t-value	Countries (N)
Political stability and absence of violence (1996–2009)	0.189***	3.360	34
Internal conflict risk (1990–2004)	0.346**	2.097	32
Weighted domestic conflict index (1981–2009)	-105.0*	1.611	30
Weighted domestic conflict index (1990–2009)	-119.7**	2.177	33
Deaths from domestic terrorism (1985–2010)	-2.357**	1.728	33

\* Statistically significant at the 10 percent level (one-tailed test)

\*\* Statistically significant at the 5 percent level (one-tailed test)

\*\*\* Statistically significant at the 1 percent level (one-tailed test)

Source: Based on data in Kaufmann, Kraay, and Mastruzzi 2010; PRS Group 2004; Banks, 2010; and GTD Team 2010

- what the dependent and independent variables are
- what Lijphart means by “controlling for” three other variables
- what the stars mean
- t-values: estimate divided by standard error