Multivariate relationships: more regression

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OCCASIONAL NOTES

Chocolate Consumption, Cognitive Function, and Nobel Laureates

Franz H. Messerli, M.D.

Dietary flavonoids, abundant in plant-based foods, have been shown to improve cognitive function. Specifically, a reduction in the risk of dementia, enhanced performance on some cognitive tests, and improved cognitive function in elderly patients with mild impairment have been associated with a regular intake of flavonoids.[1,2] A subclass of flavonoids called flavanols, which are widely present in cocoa, green tea, red wine, and some fruits, seems to be effective in slowing down or even reversing the reductions in cognitive performance that occur with aging. Dietary flavanols have also been shown to improve endothelial

cause the population of a country is substantially higher than its number of Nobel laureates, the numbers had to be multiplied by 10 million. Thus, the numbers must be read as the number of Nobel laureates for every 10 million persons in a given country.

All Nobel Prizes that were awarded through October 10, 2011, were included. Data on per capita yearly chocolate consumption in 22 countries was obtained from Chocosuisse (www.chocosuisse.ch/web/chocosuisse/en/home), Theobroma-cacao (www.theobroma-cacao.de/wissen/wirtschaft/international/konsum), and

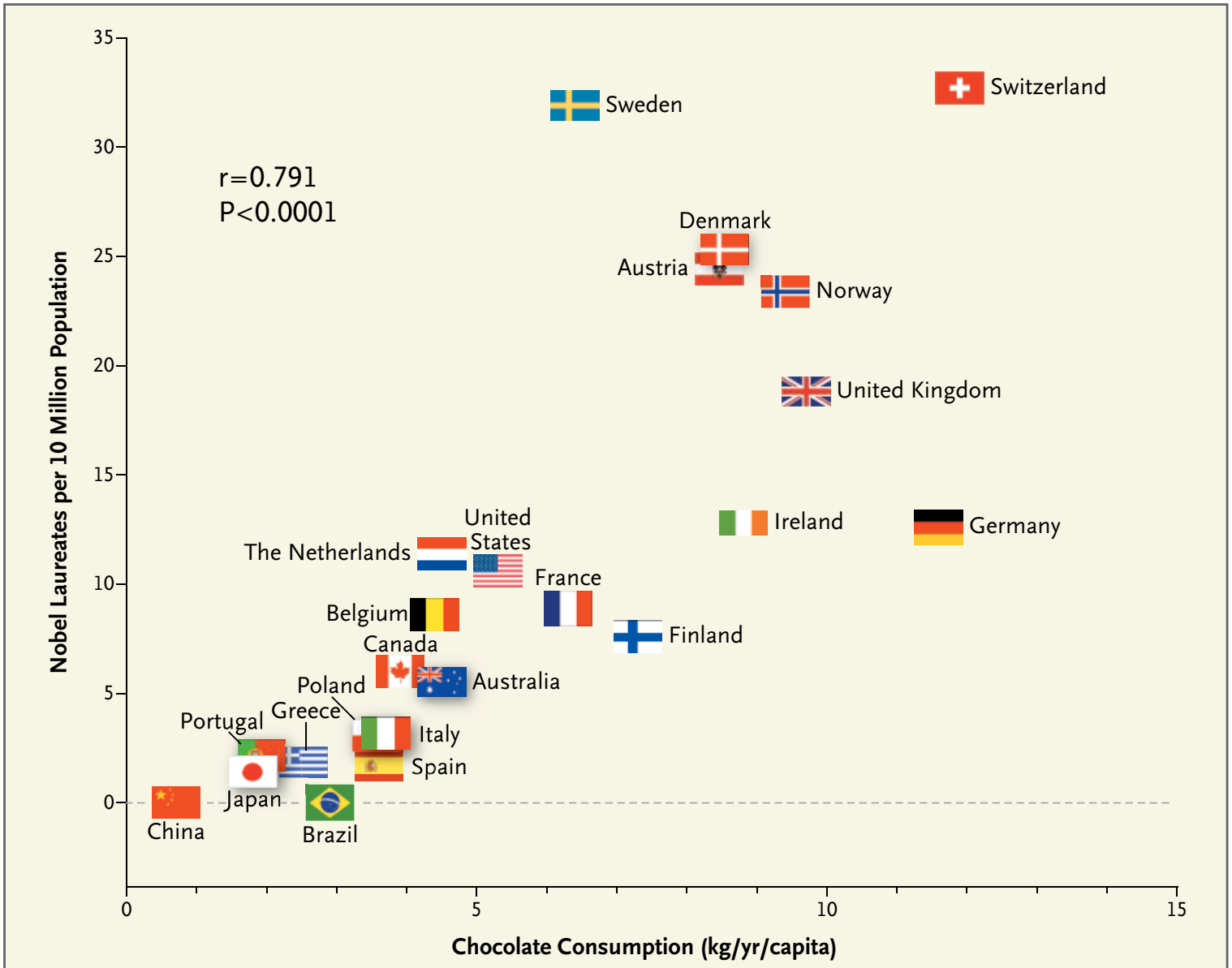


Figure 1. Correlation between Countries' Annual Per Capita Chocolate Consumption and the Number of Nobel Laureates per 10 Million Population.

We want you to understand:

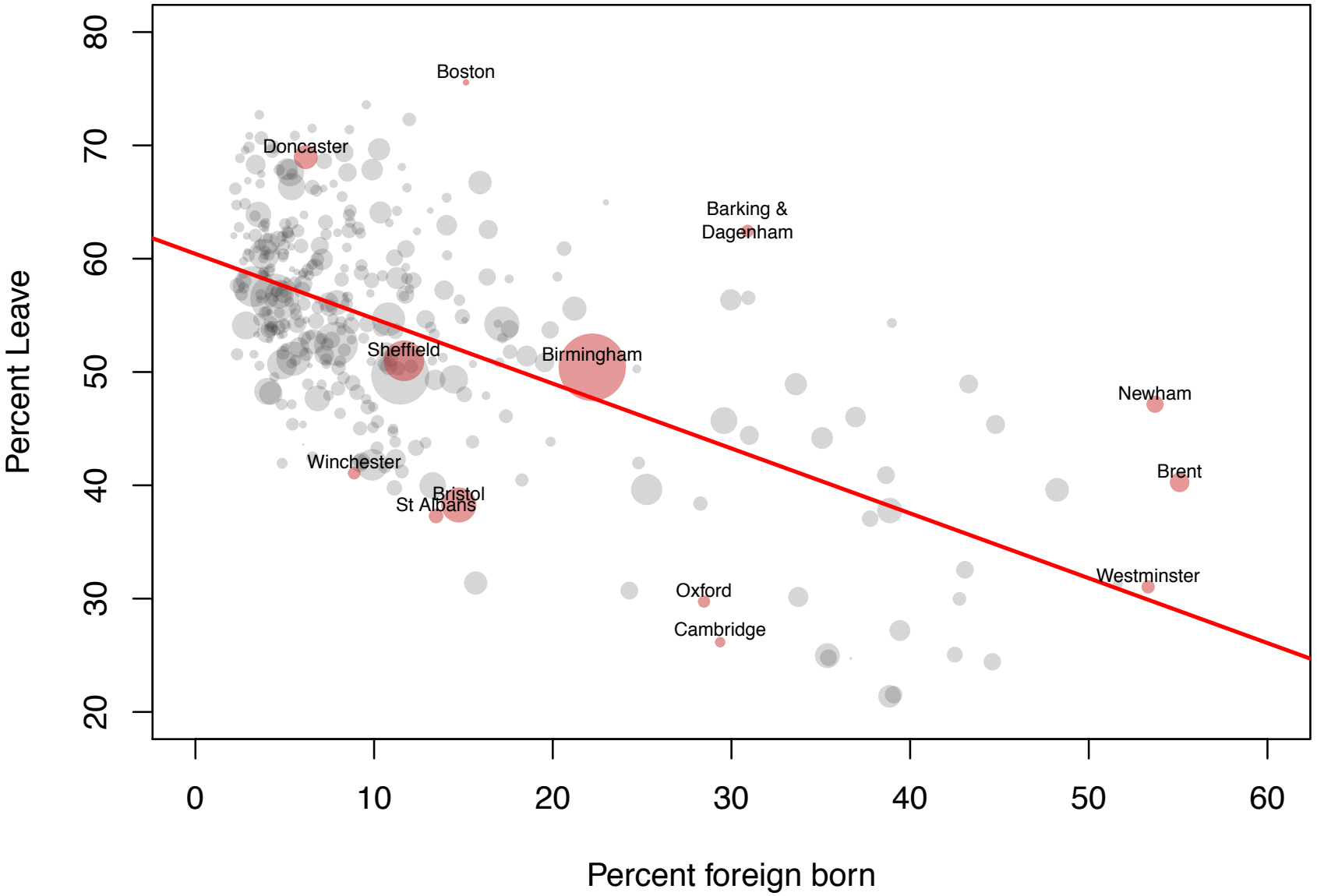
Dependent variable: Nobel Prizes awarded per capita (in log scale)

	(1)	(2)	(3)
Intercept	-1.629* (0.509)	-3.166* (0.511)	-2.982* (0.527)
Chocolate consumption per capita (log scale)	2.092* (0.298)	1.026* (0.326)	0.709 (0.415)
GDP/capita (thousands of USD)		0.105* (0.024)	0.106* (0.024)
NW Europe			0.549 (0.452)
R^2	0.70	0.85	0.86
N	34	34	34

Standard errors in parentheses. * Indicates $p<0.05$

- what a dependent variable is
- what an independent variable is
- what the coefficients mean (intercept, slopes)
- what the stars mean (i.e. what $p<0.05$ means)
- what the standard errors mean

Did this pattern arise because contact with immigrants makes people less opposed to immigration?



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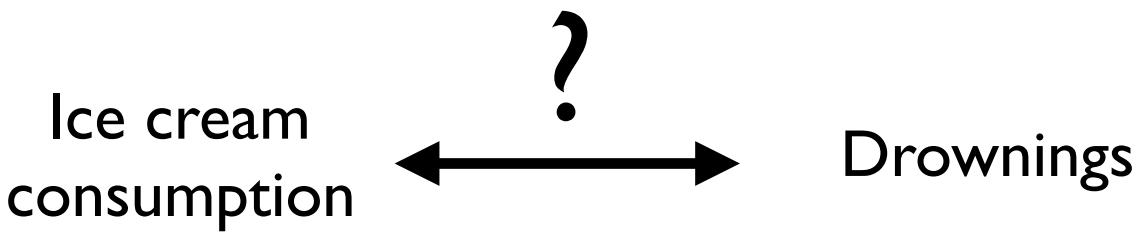

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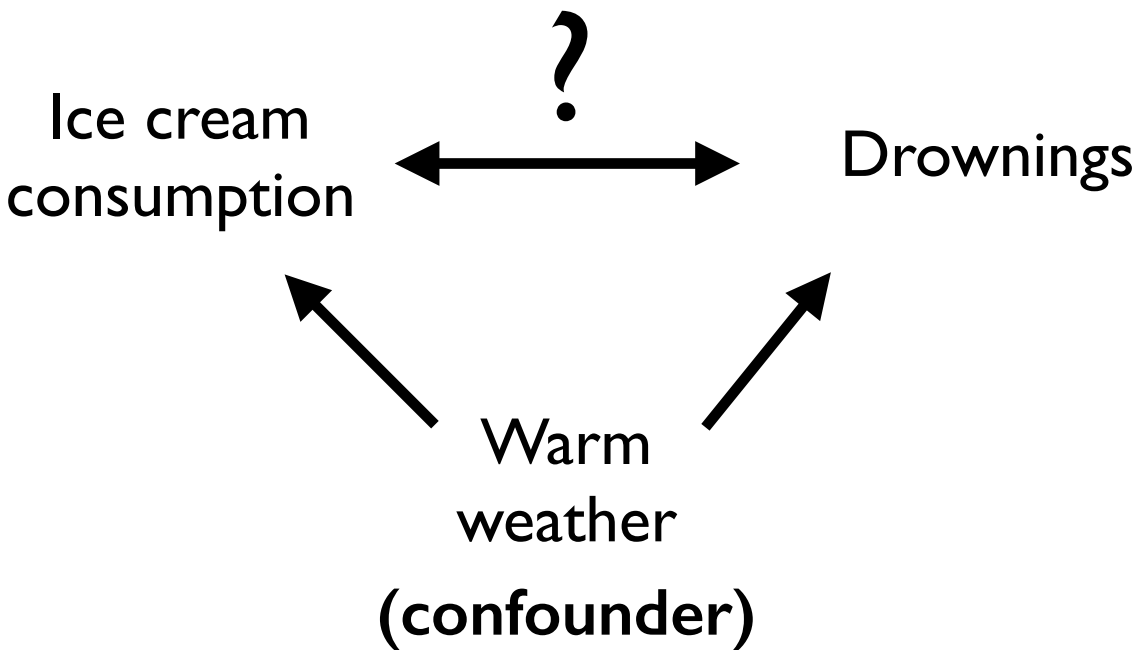

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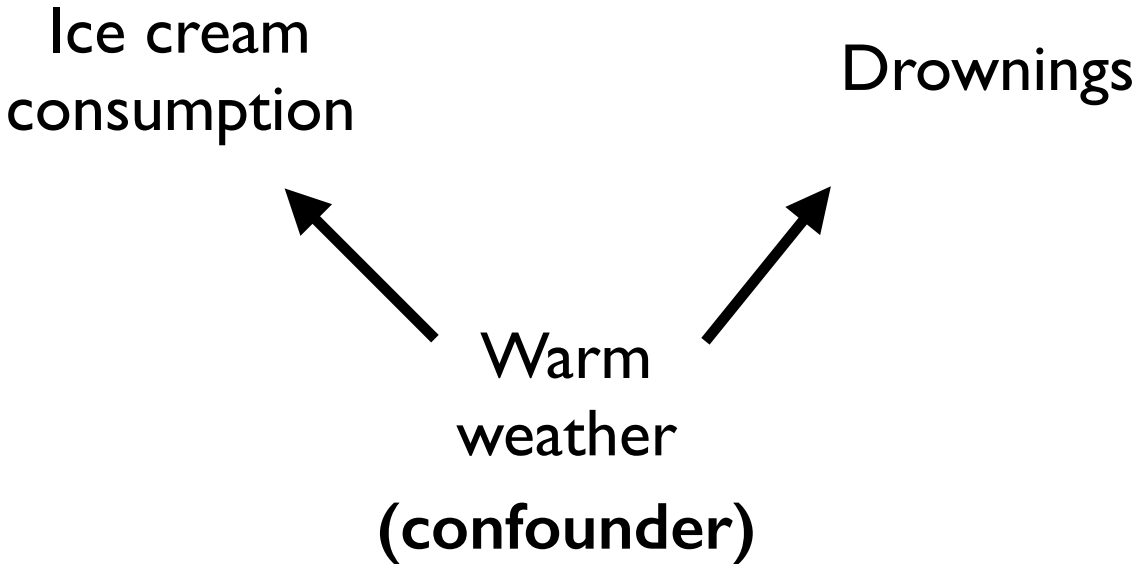

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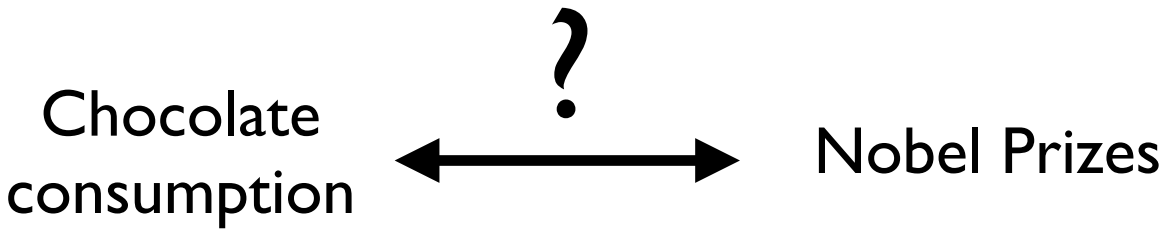

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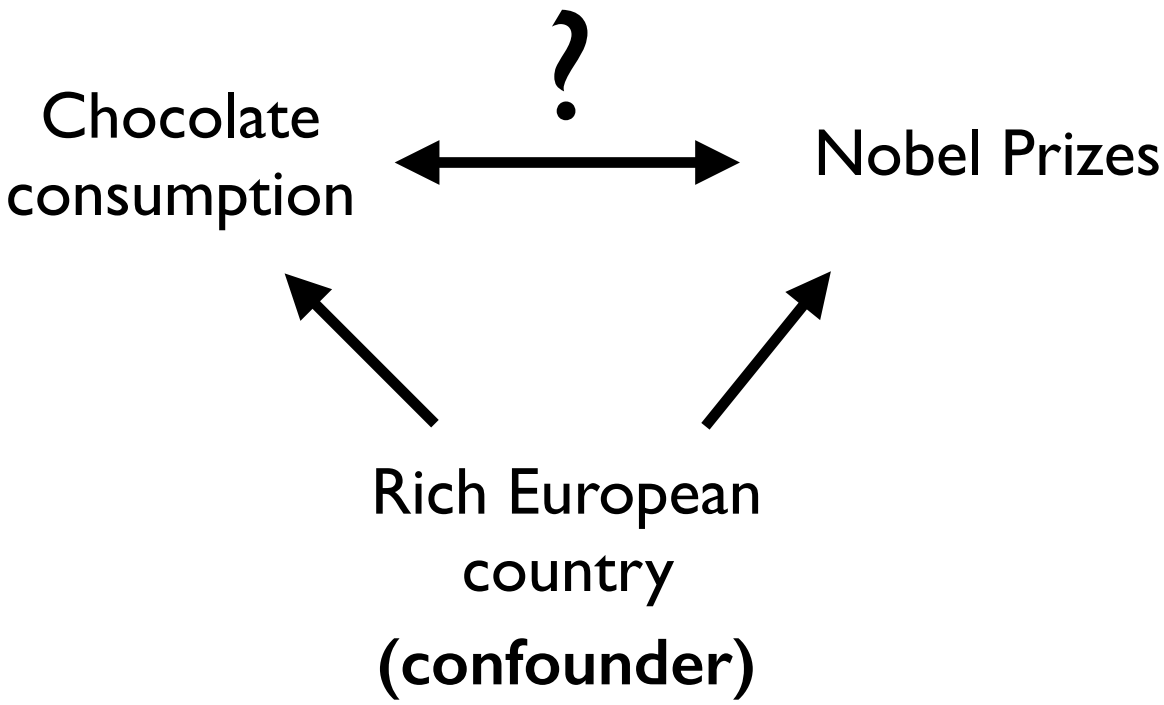

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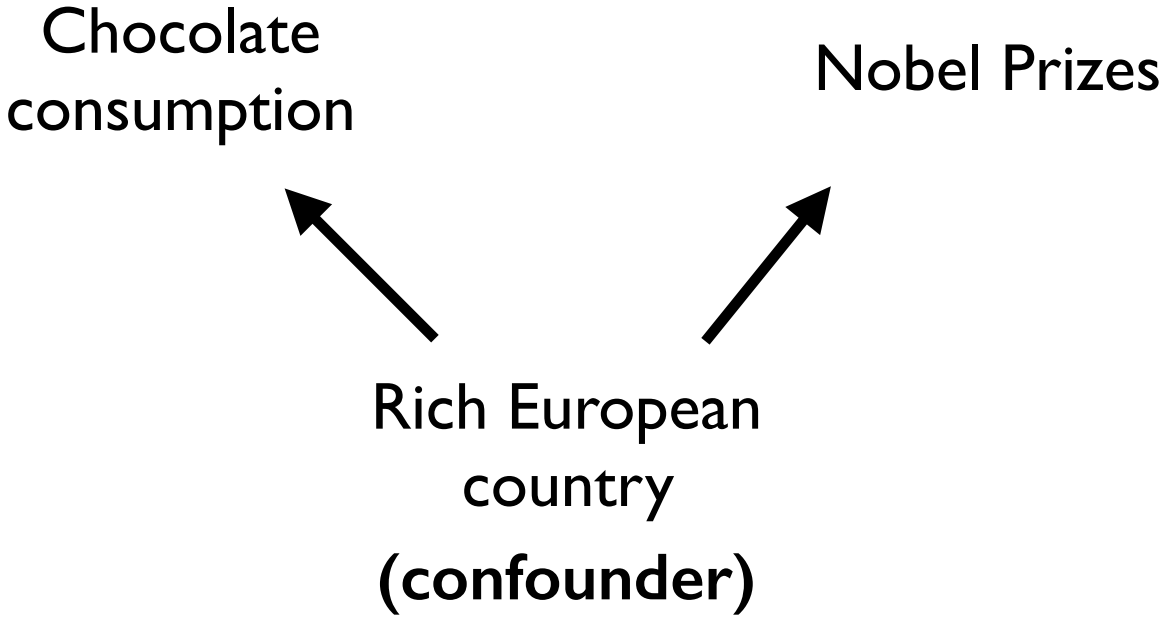

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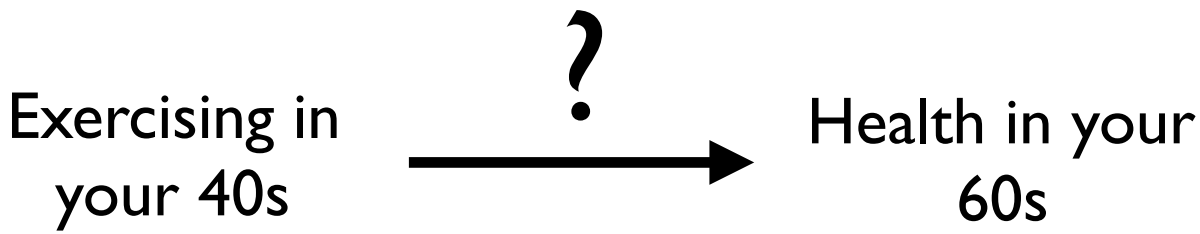

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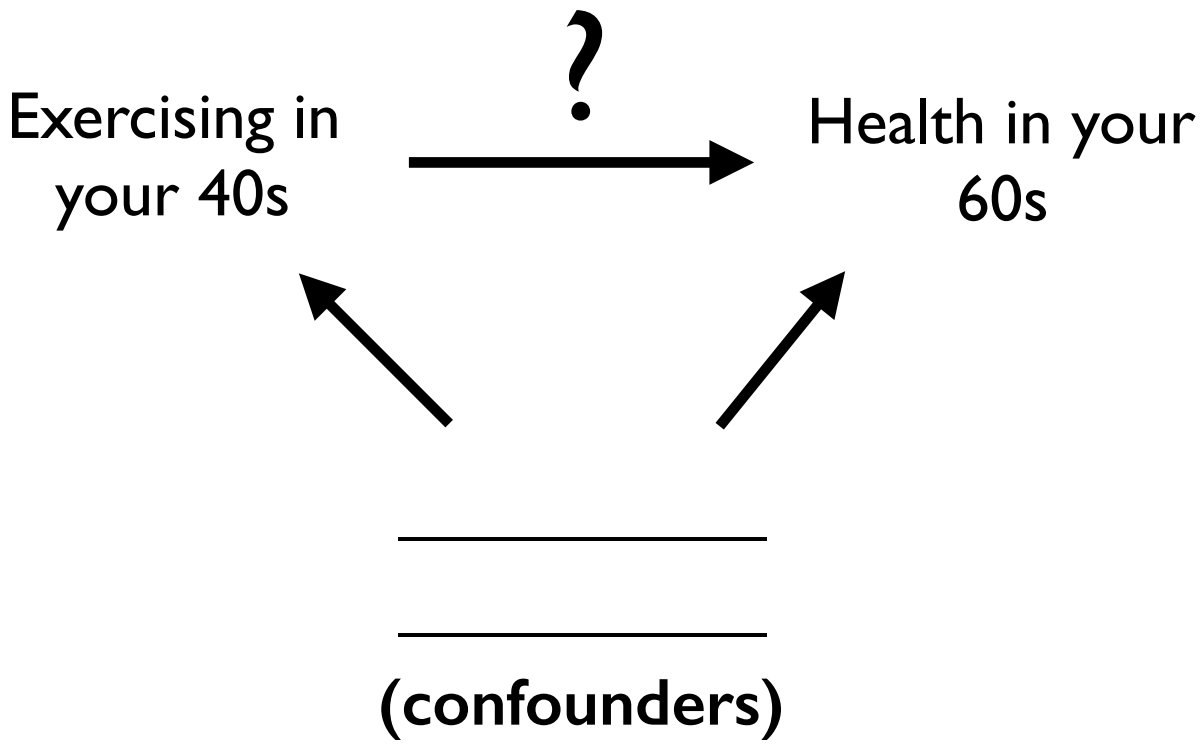

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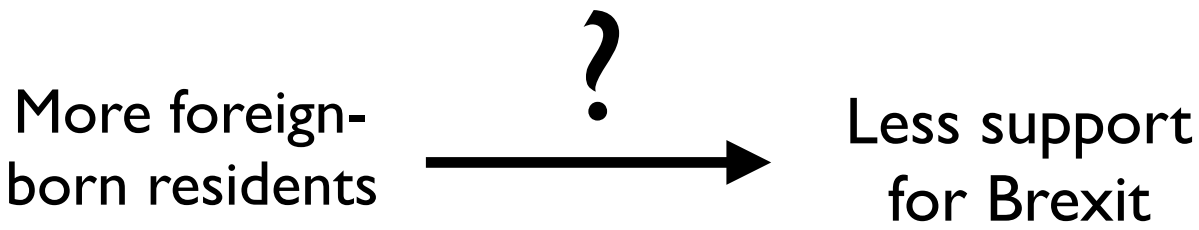

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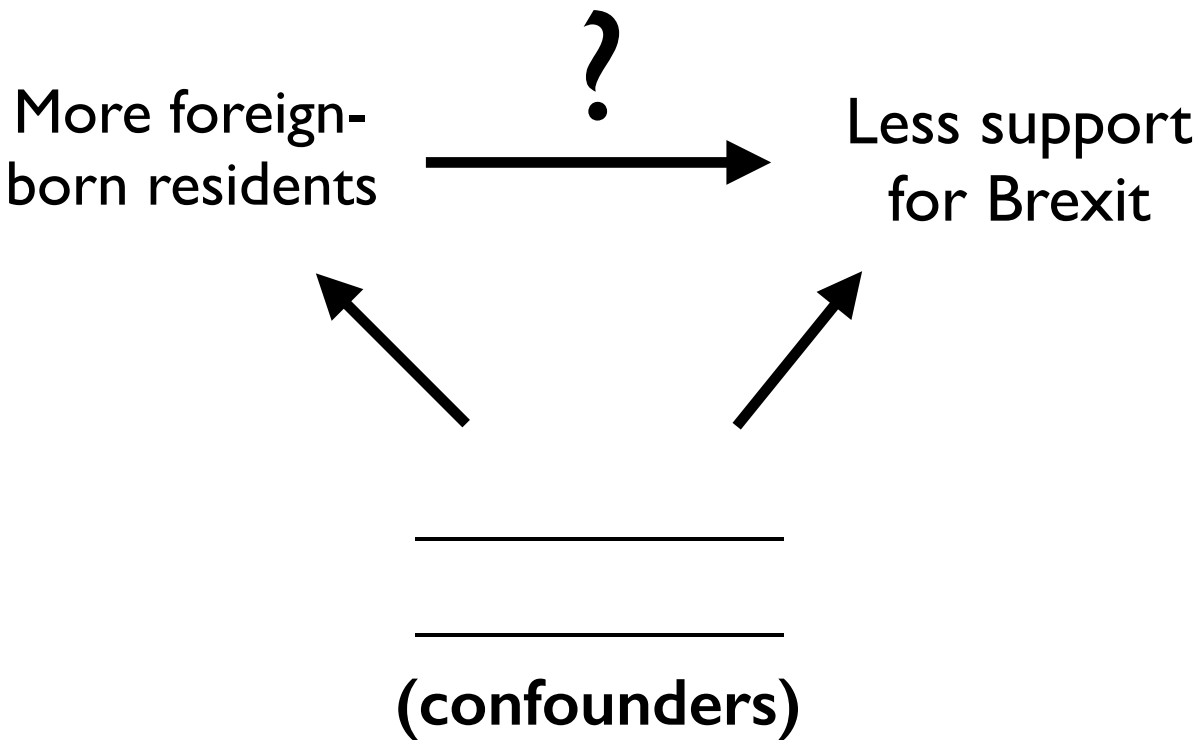

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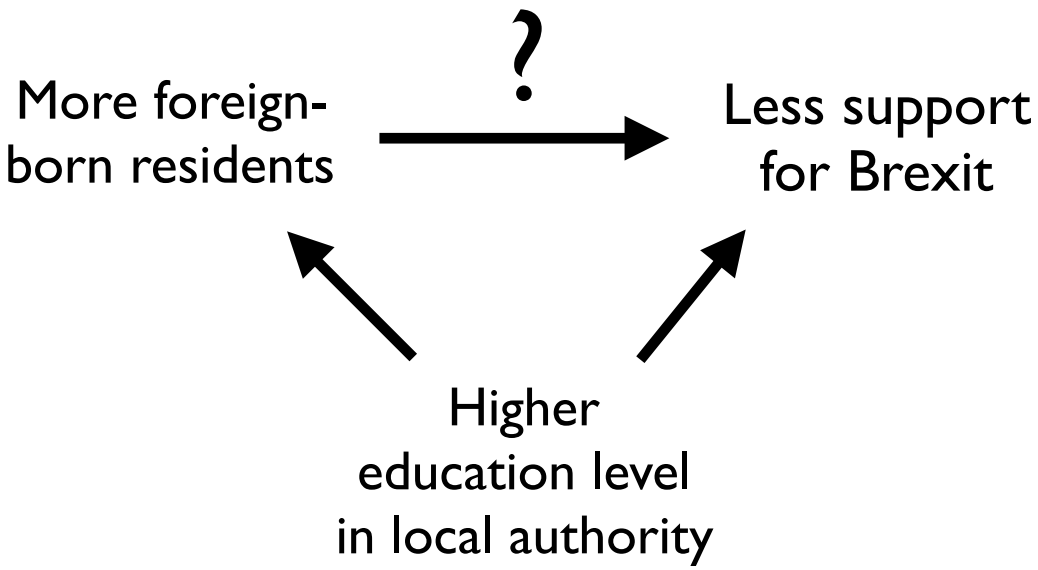
- Are people who exercise less likely to develop dementia, controlling for diet and age?
- Are countries with more inclusive political systems less likely to experience violence, controlling for economic development and the number of ethnic groups?
- Are local authorities with more foreign-born residents less likely to support Brexit, controlling for _____?

How do we control for confounders?

Let's focus on education as a confounder in our Brexit example:



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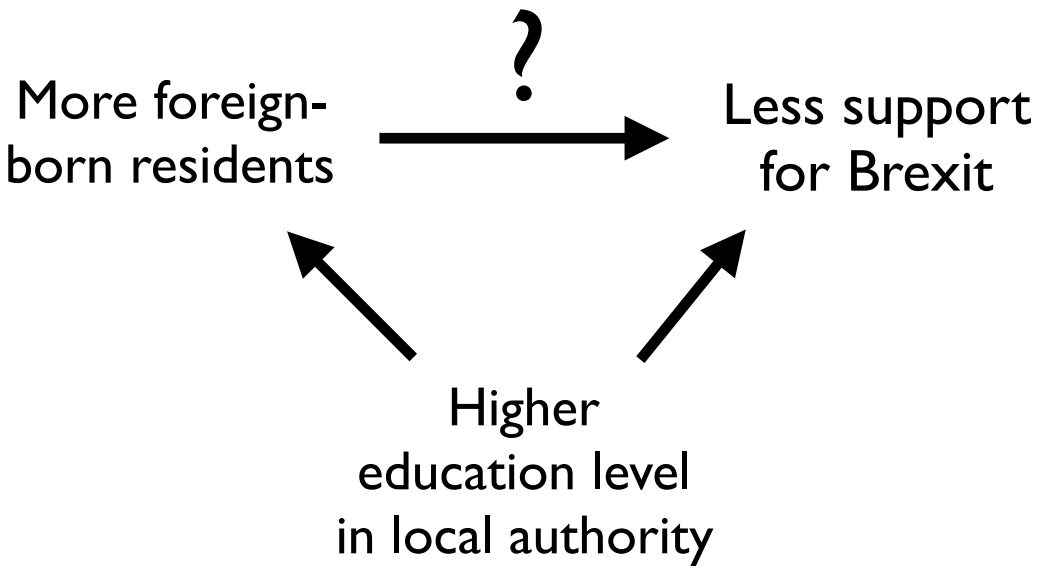
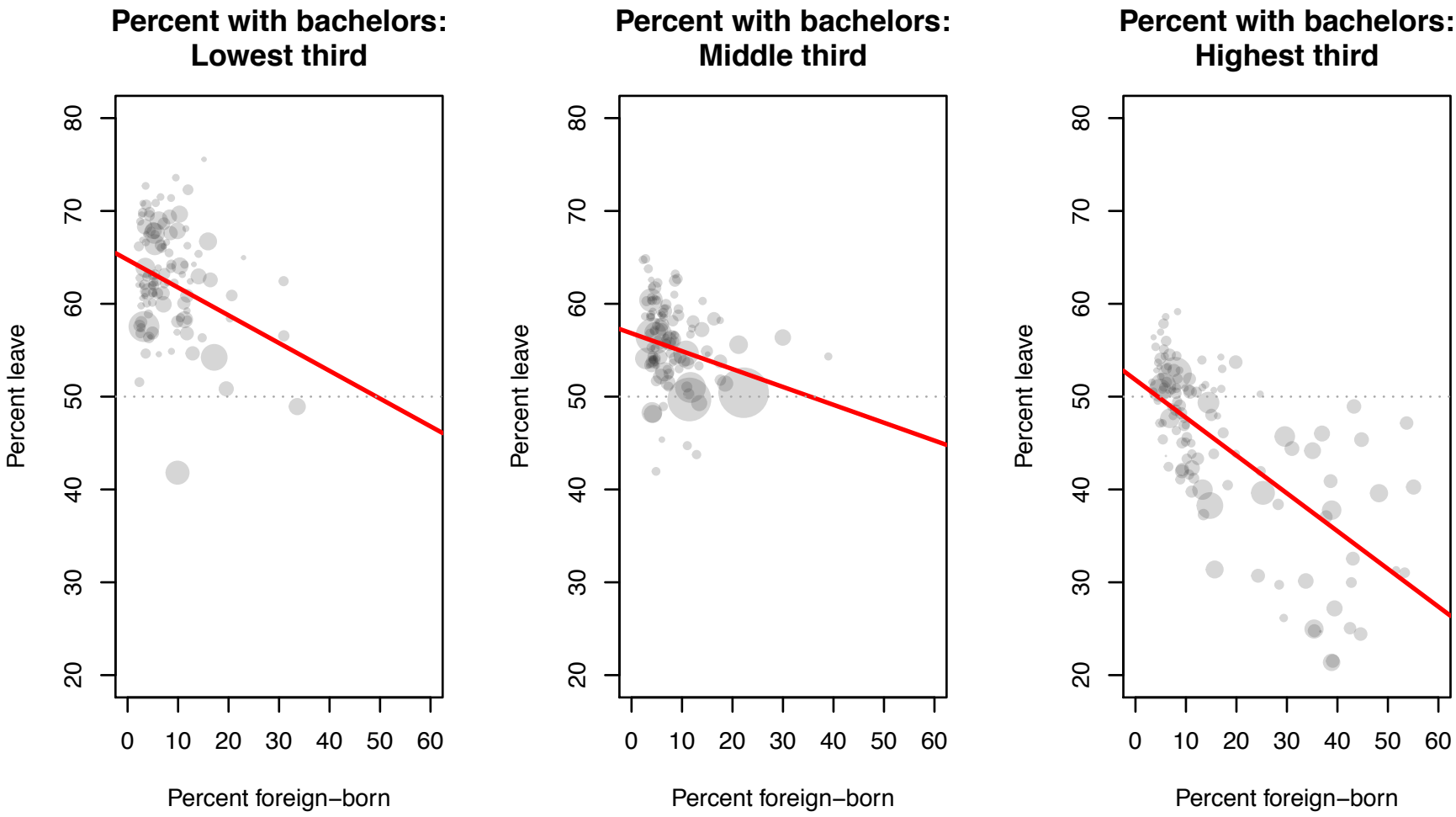


How can we measure the relationship between a local authority's proportion of foreign-born residents and its support for Brexit, controlling for its education level?

One idea: measure relationship between %foreign-born and %leave after stratifying by education level



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Basic idea: measure relationship between

- "support for Leave" and
- the part of "% foreign-born" that is not explained by "% bachelors degree"

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$$\text{PercentLeave} = \beta_0 + \beta_1 \text{PercentForeignBorn}$$

β_1 was the **derivative** of predicted PercentLeave w.r.t. PercentForeignBorn, i.e. the slope.

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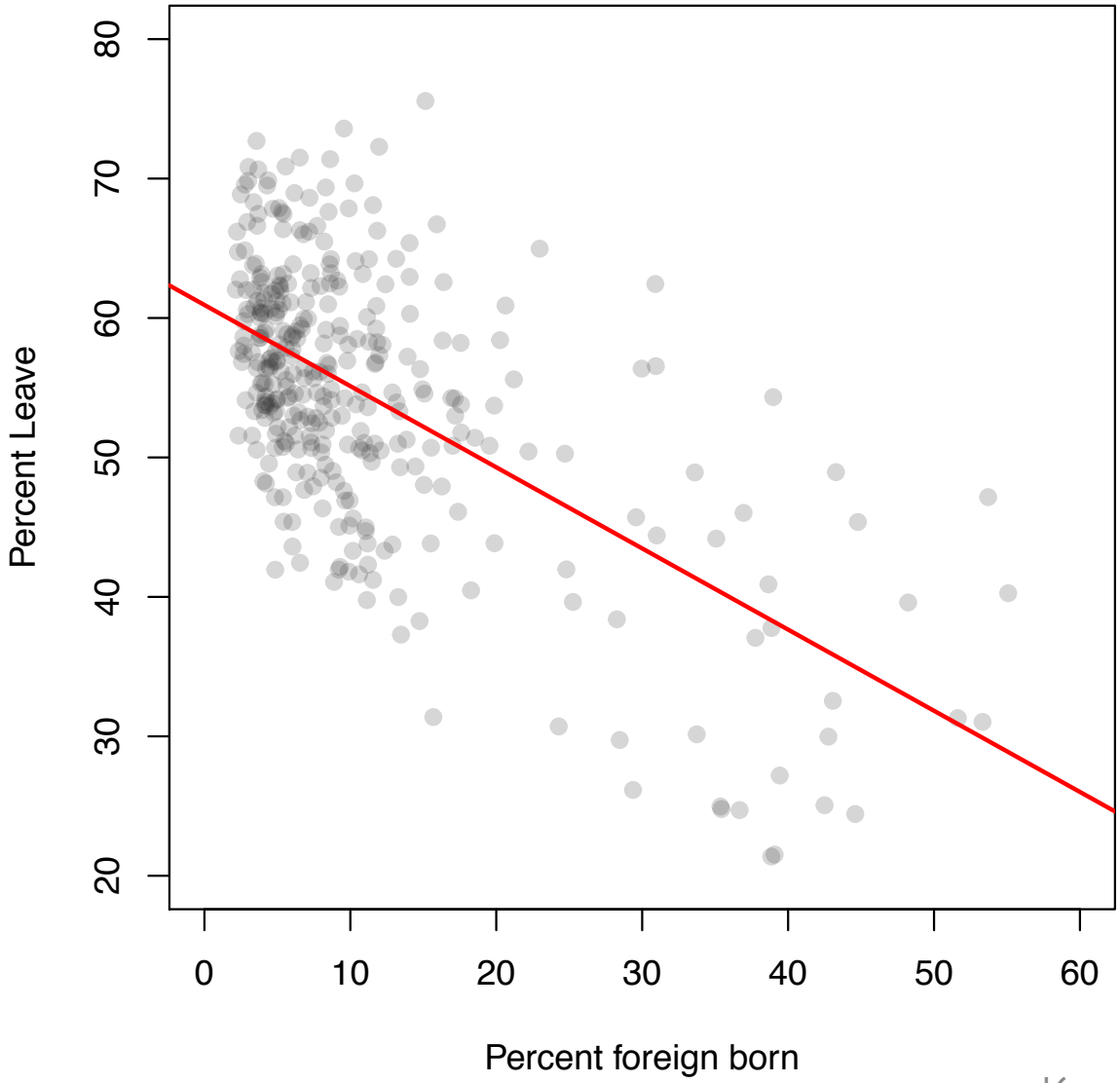
$$\text{PercentLeave} = \beta_0 + \beta_1 \text{PercentForeignBorn} + \beta_2 \text{Education}$$

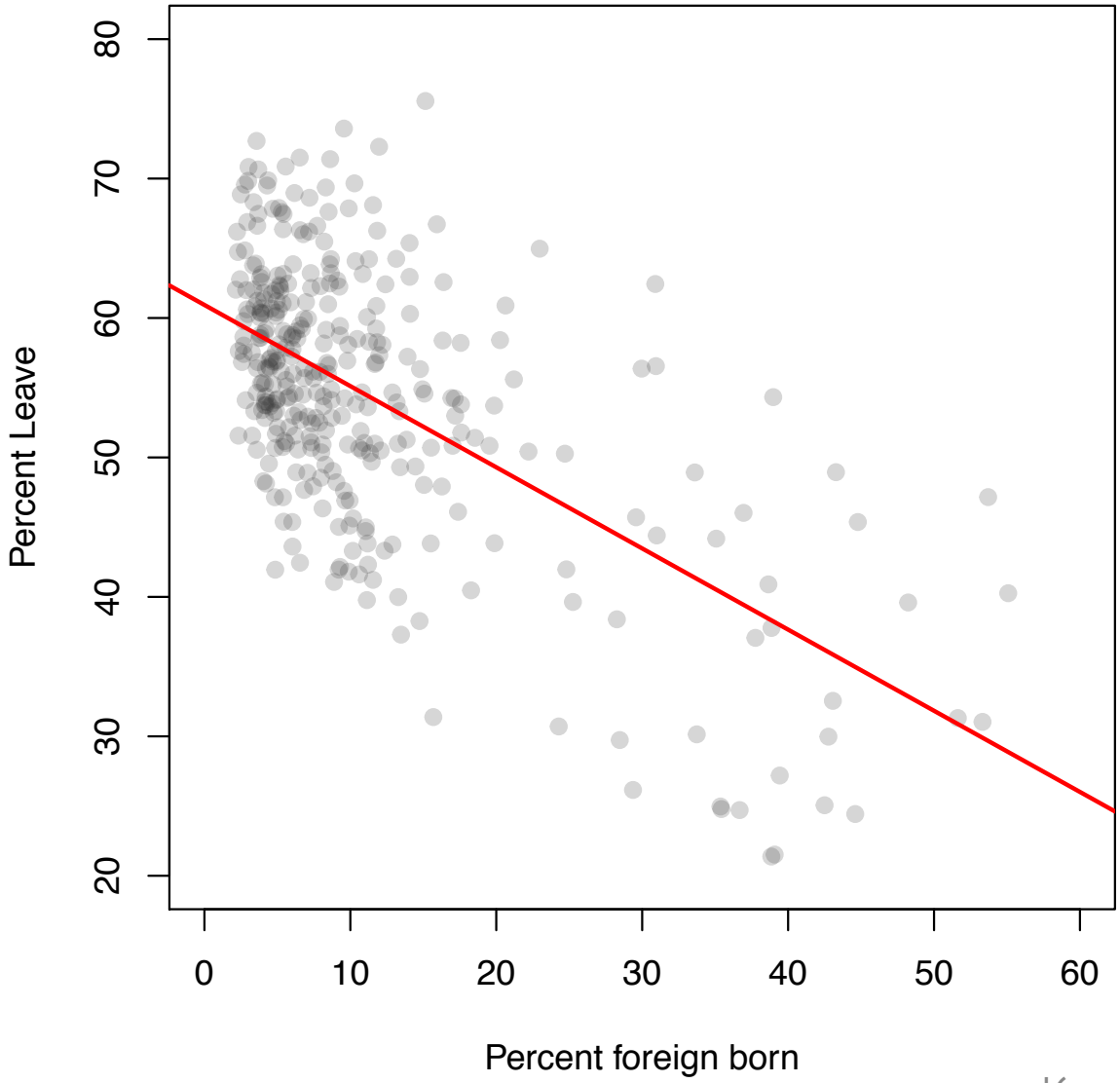
β_1 is now the **partial derivative** of predicted PercentLeave w.r.t. PercentForeignBorn, i.e. the slope holding constant Education.

Visualizing bivariate regression

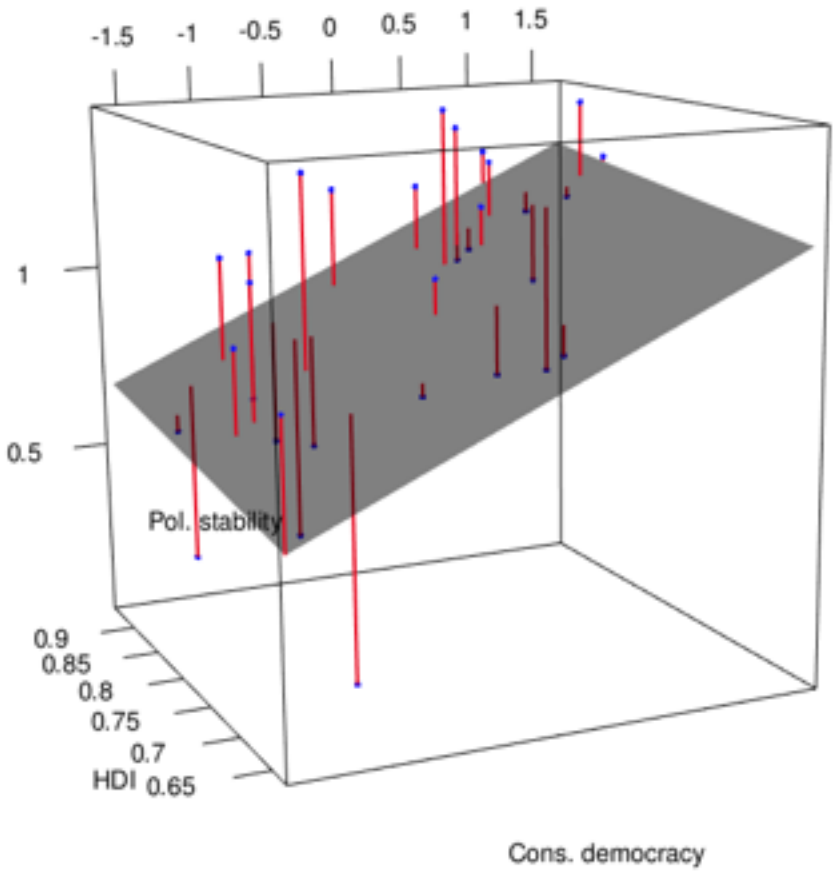


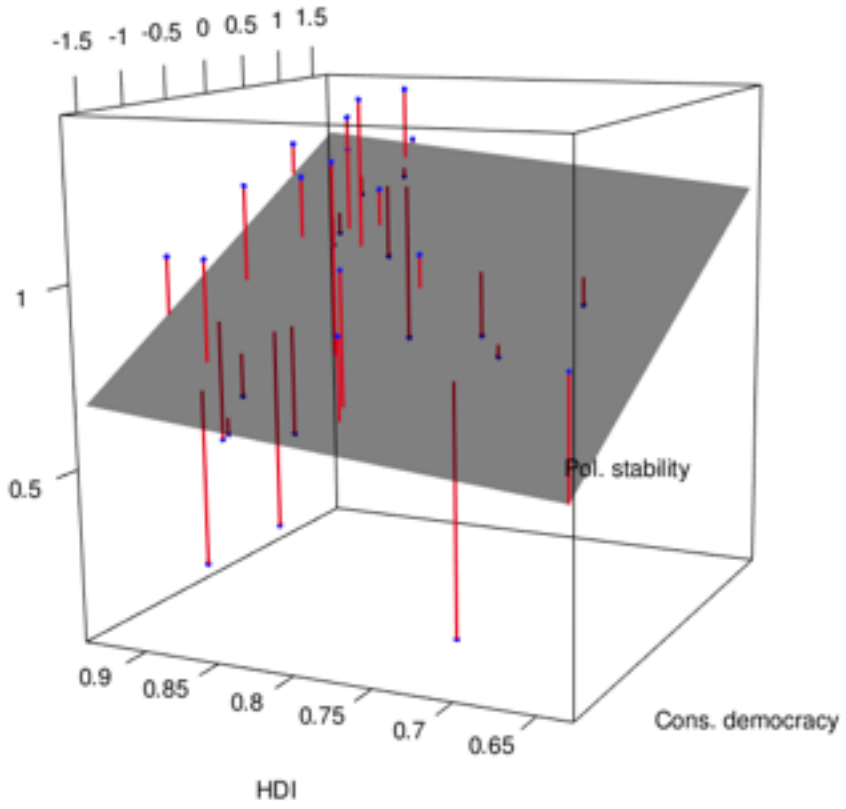
Visualizing bivariate regression

With just two variables (bivariate regression), the regression equation is a **line**.



Visualizing multivariate regression





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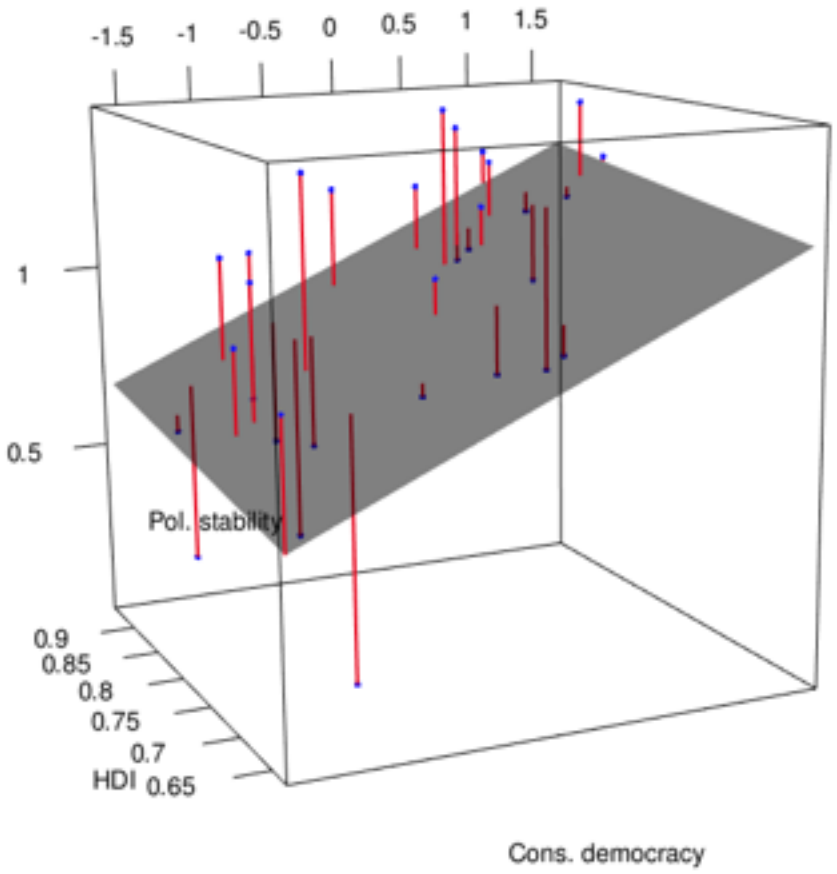

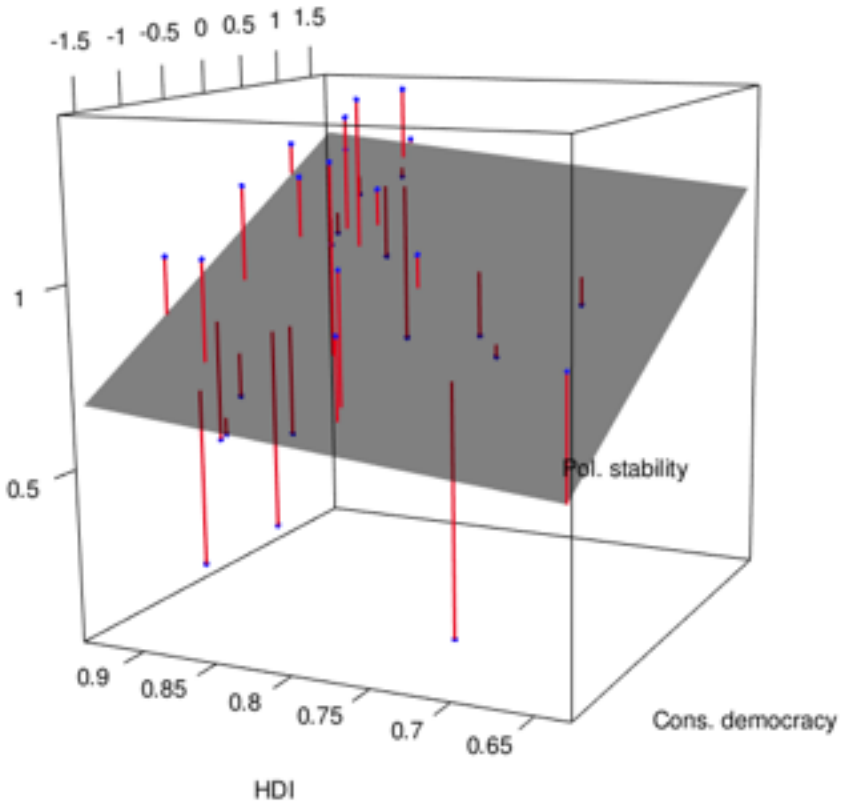

With three variables (multivariate regression), the regression equation is a **plane**.

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Step 3: regress outcome (% leave) on residuals from step 2

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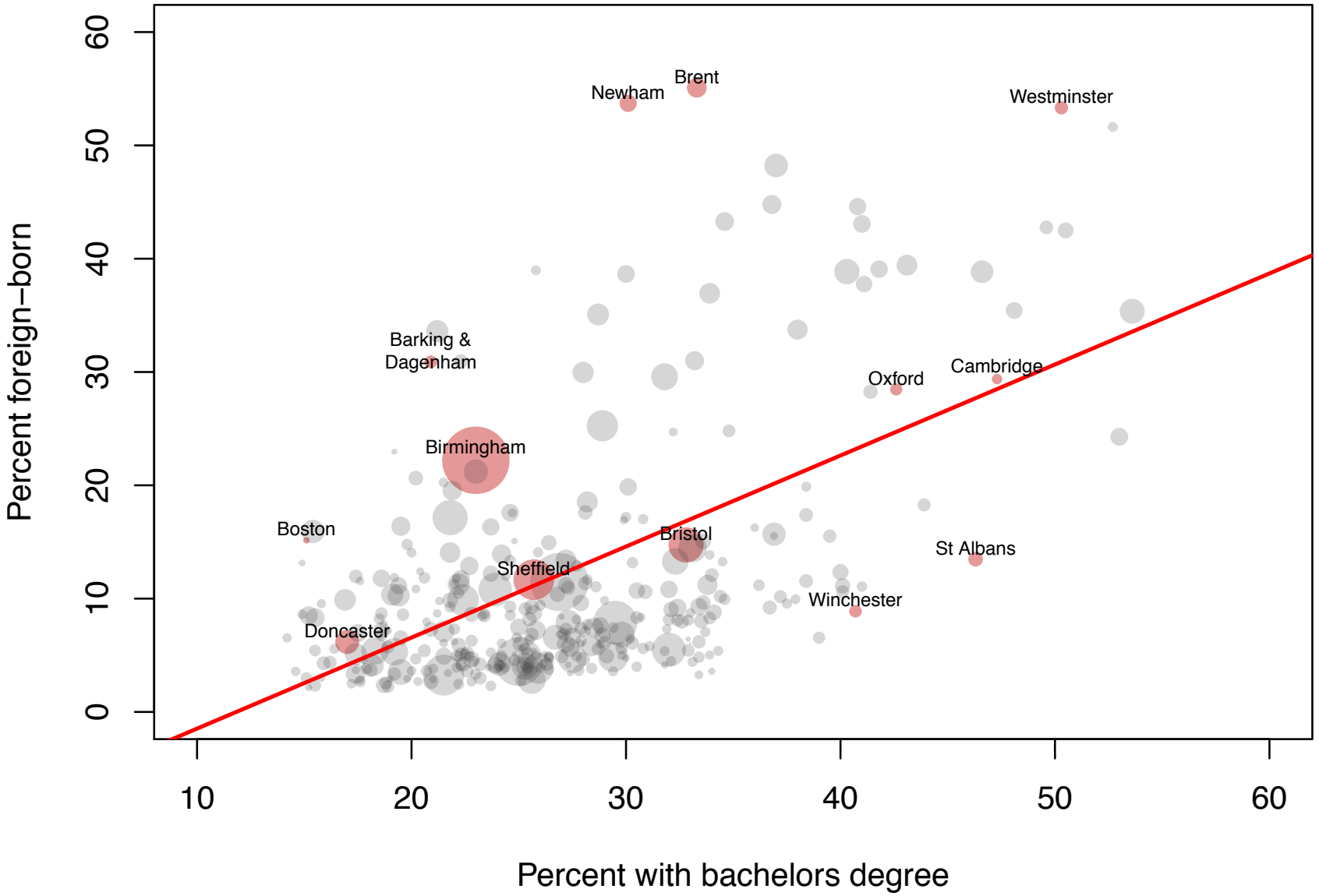
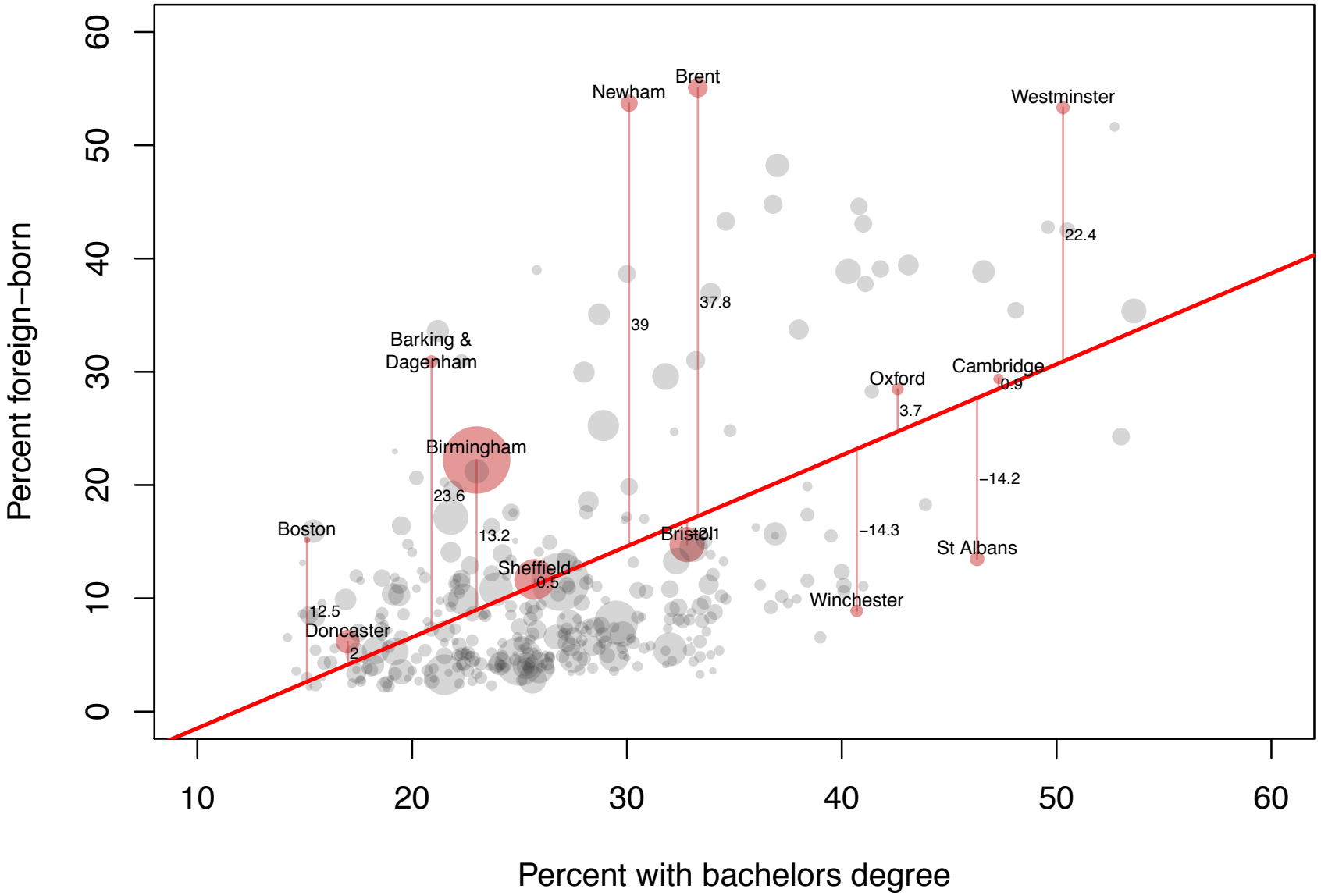


Step 2: calculate residuals, i.e. the part of %foreign-born not "explained" by education



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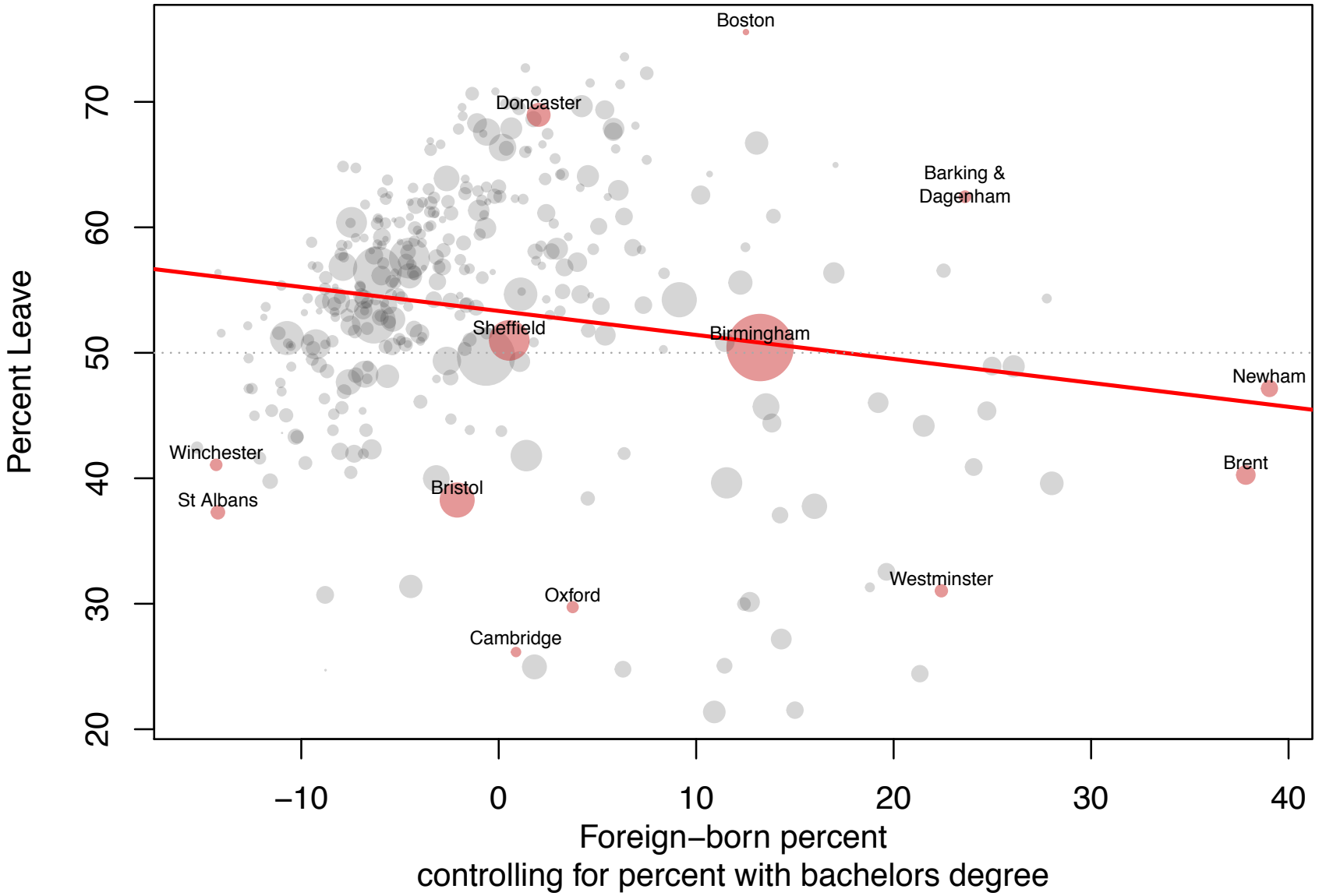

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2. Use calculus to find the slope and intercept that minimize the sum of squared residuals.
3. Use lm() function in R:

```
> lm(d$Percent_Leave ~ d$Percent_foreign_born + d$Bachelors_deg_percent)

Call:
lm(formula = d$Percent_Leave ~ d$Percent_foreign_born + d$Bachelors_deg_percent)

Coefficients:
            (Intercept)   d$Percent_foreign_born  d$Bachelors_deg_percent
                83.1386                 -0.1742                 -0.9875
```