Bivariate relationships: introduction to regression

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We want you to understand:

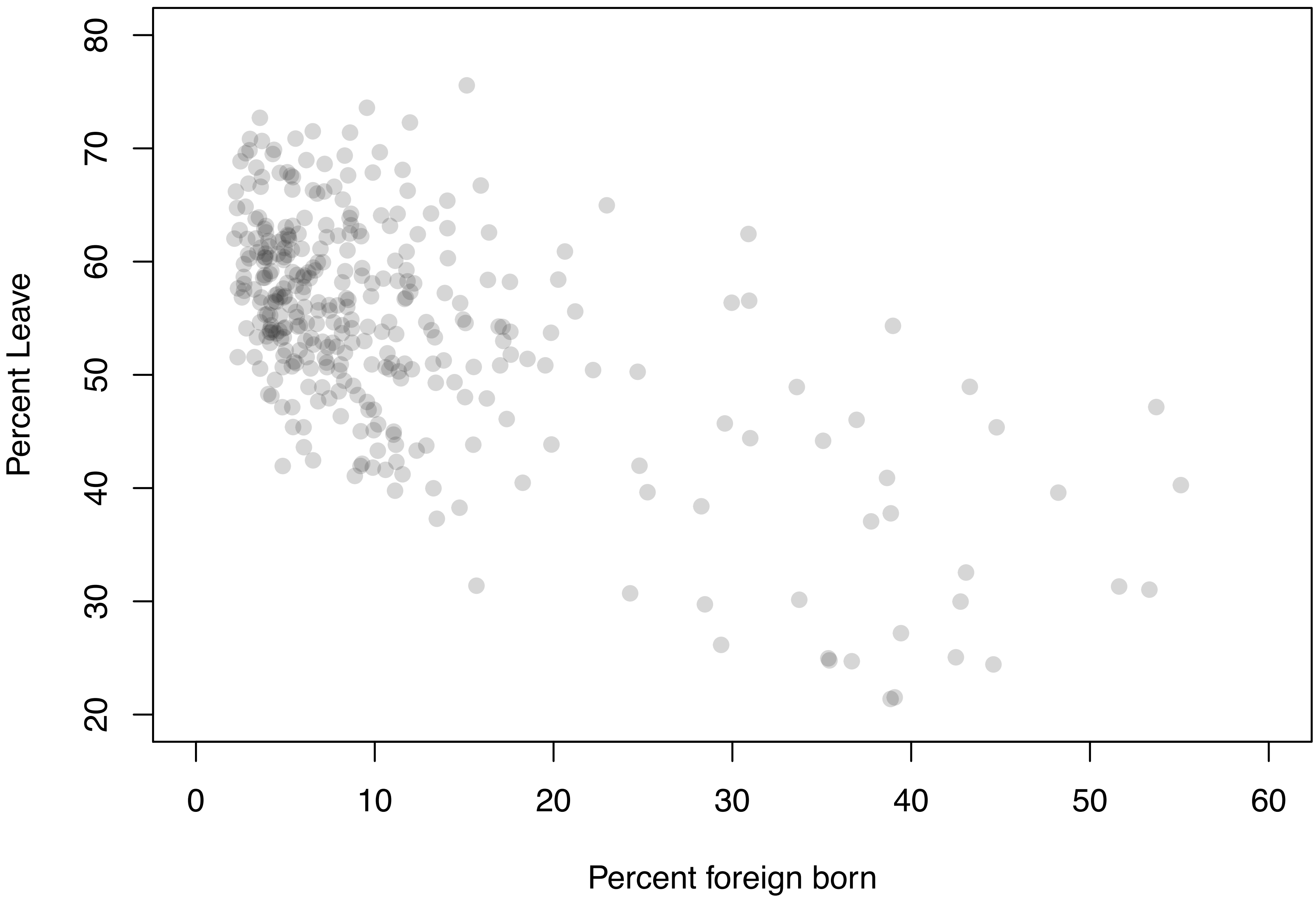
Dependent variable: Nobel Prizes awarded per capita (in log scale)

	(1)	(2)	(3)
Intercept	-1.629* (0.509)	-3.166* (0.511)	-2.982* (0.527)
Chocolate consumption per capita (log scale)	2.092* (0.298)	1.026* (0.326)	0.709 (0.415)
GDP/capita (thousands of USD)		0.105* (0.024)	0.106* (0.024)
NW Europe			0.549 (0.452)
R^2	0.70	0.85	0.86
N	34	34	34

Standard errors in parentheses. * Indicates $p<0.05$

- what a dependent variable is
- what an independent variable is
- what the coefficients mean (intercept, slopes)
- what the stars mean (i.e. what $p<0.05$ means)
- what the standard errors mean

Local authorities with more foreign-born residents were less supportive of Brexit



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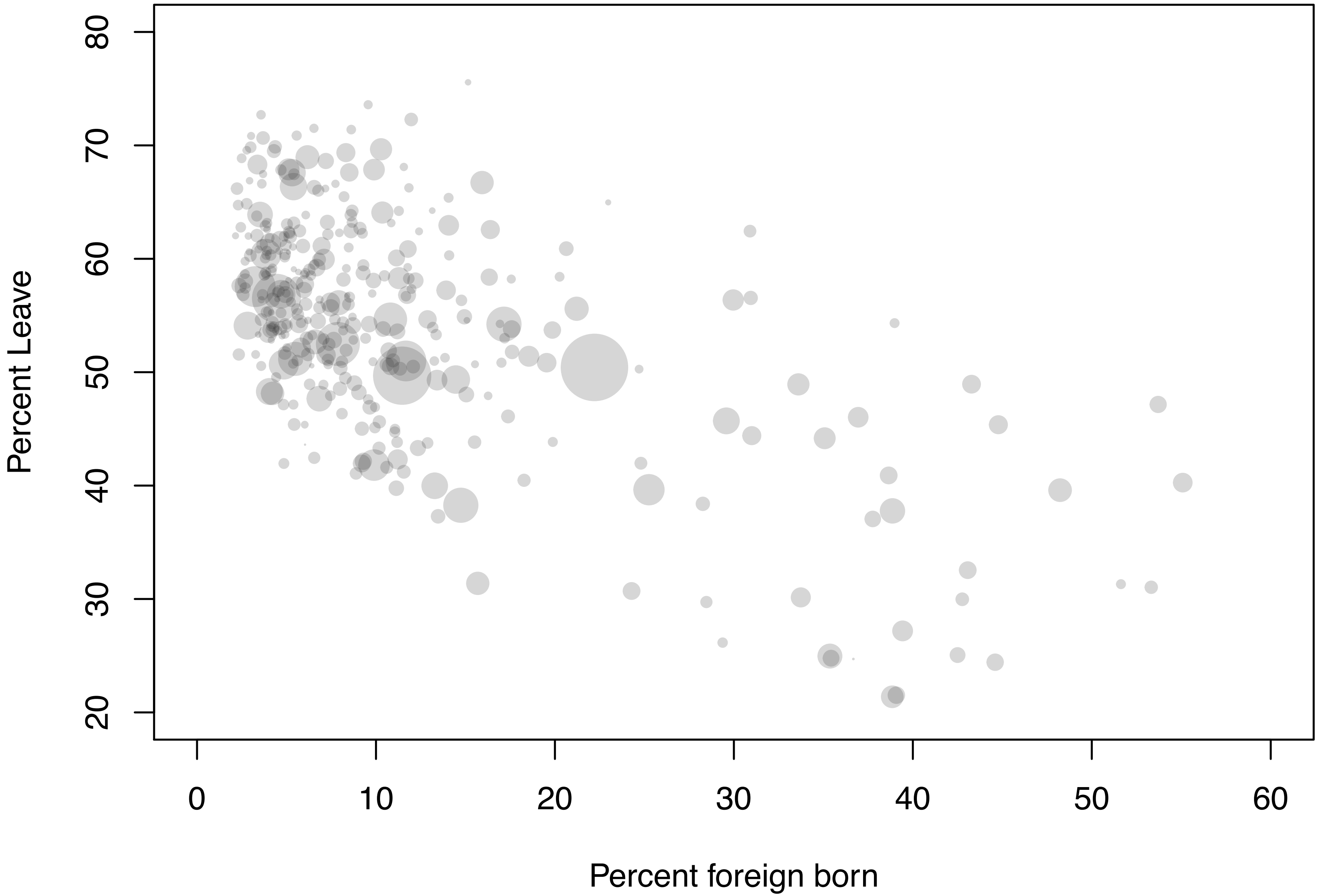

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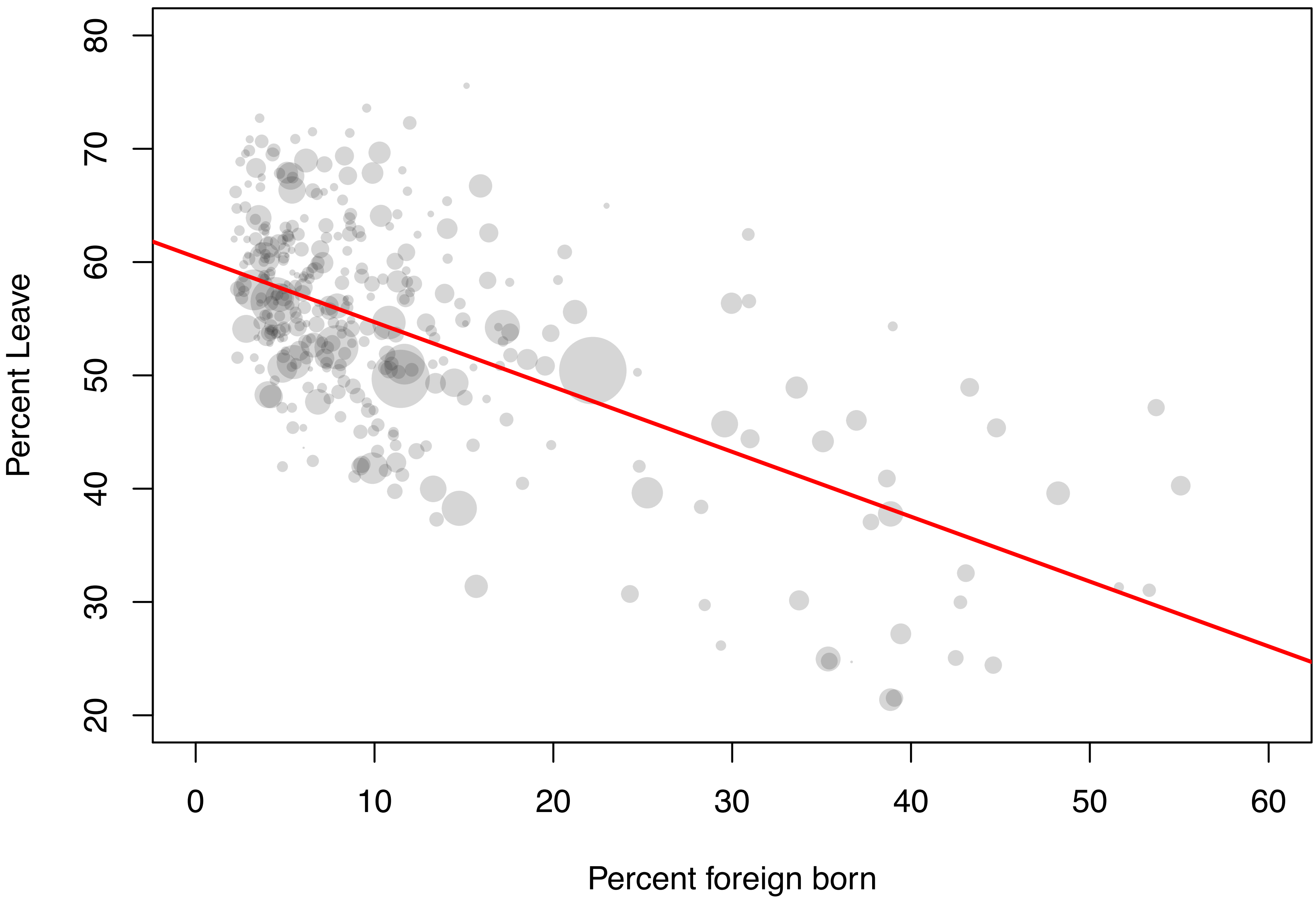

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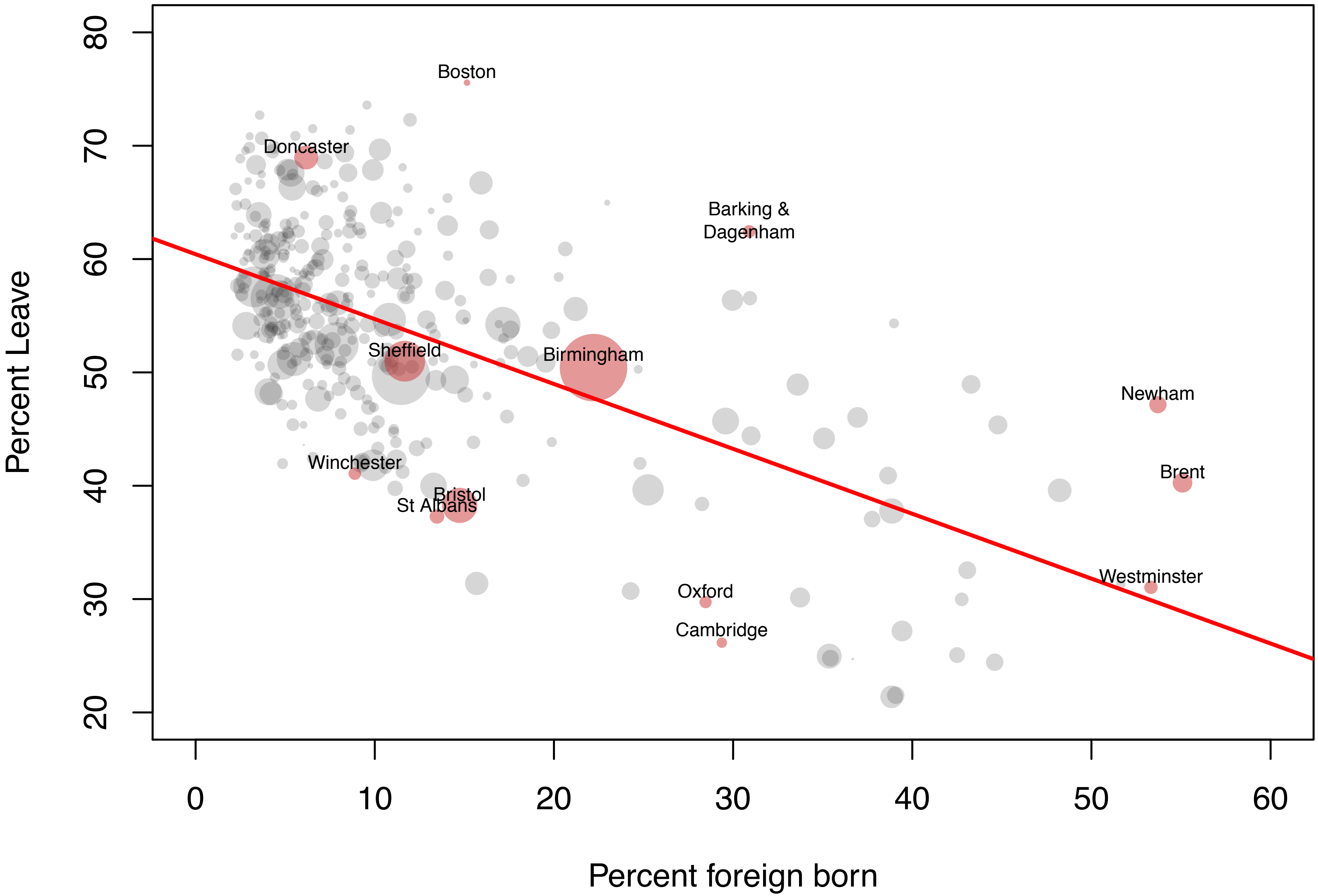

Contact hypothesis

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"Prejudice (unless deeply rooted in the character structure of the individual) may be reduced by equal status contact between majority and minority groups in the pursuit of common goals. The effect is greatly enhanced if this contact is sanctioned by institutional supports (i.e., by law, custom or local atmosphere), and provided it is of a sort that leads to the perception of common interests and common humanity between members of the two groups."

— Gordon Allport (1954) *The Nature of Prejudice*

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- How could this pattern be explained by the contact hypothesis? (easy)
- How could this pattern be explained by other factors? (harder)

My lectures in weeks 5, 6, and 7

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Running question: Why is there such a strong relationship between % foreign born and opposition to Brexit?

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- How do we summarize our uncertainty about our conclusions?
  - standard errors, p-values, confidence intervals

Summarizing bivariate relationships: options other than OLS regression

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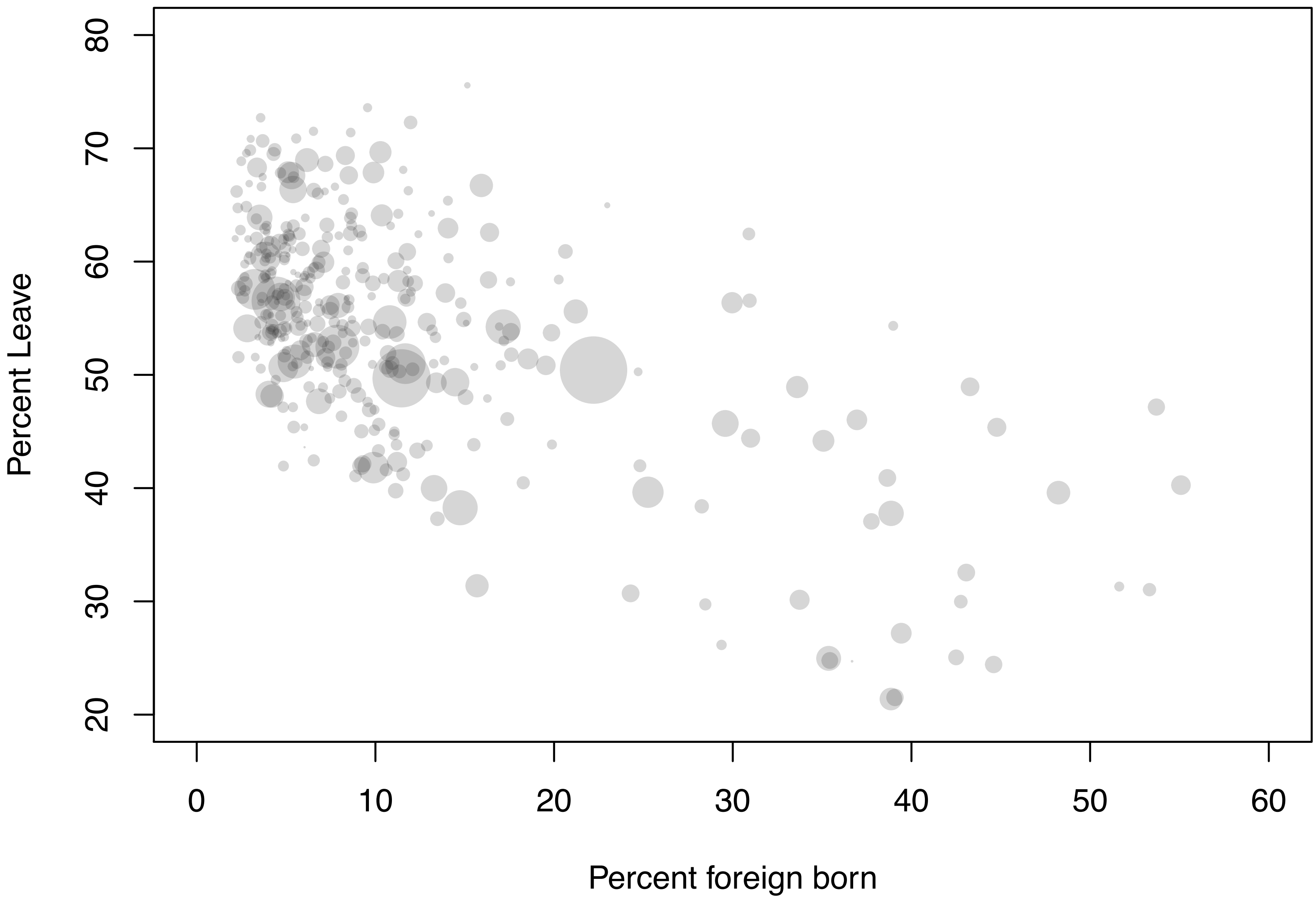
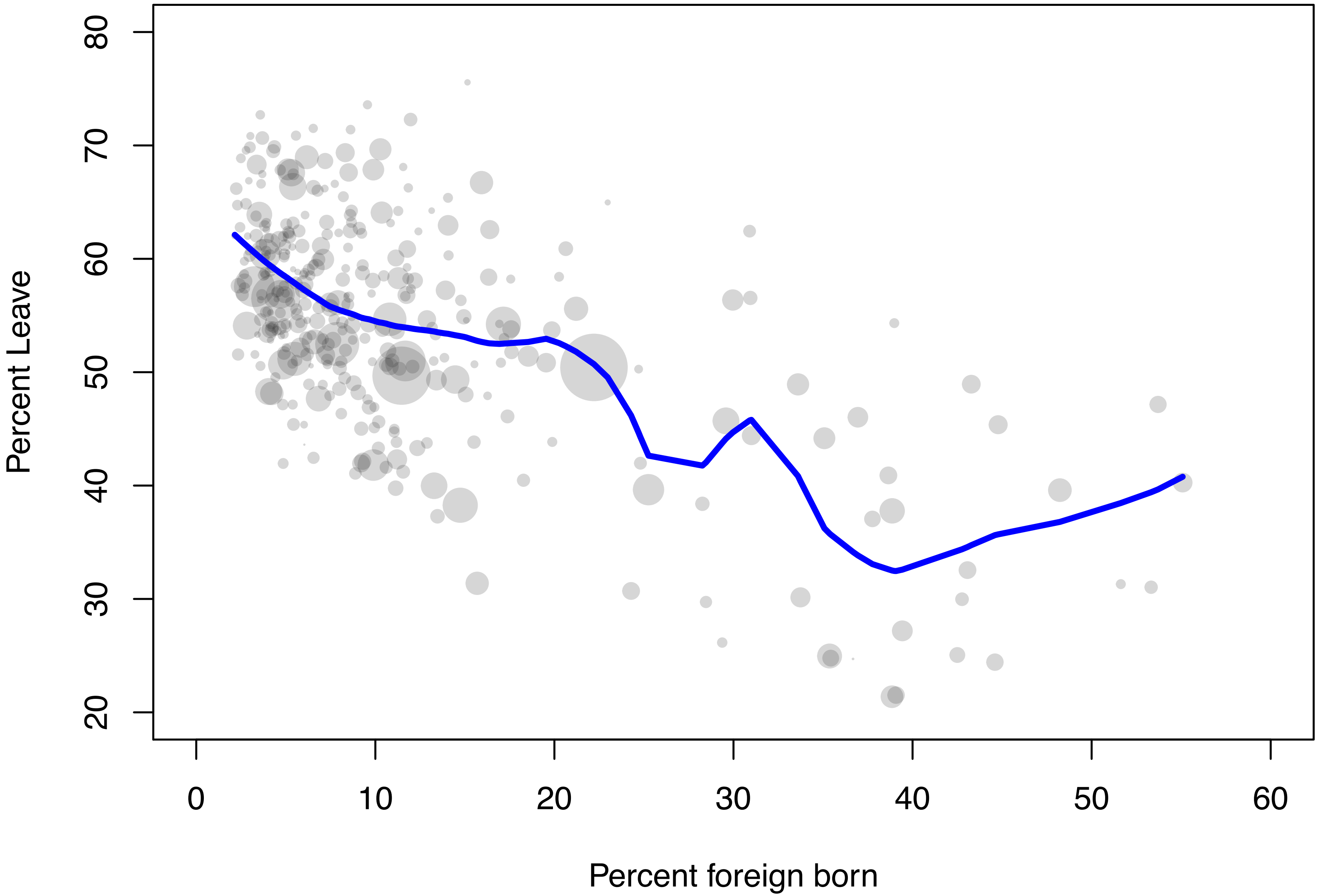


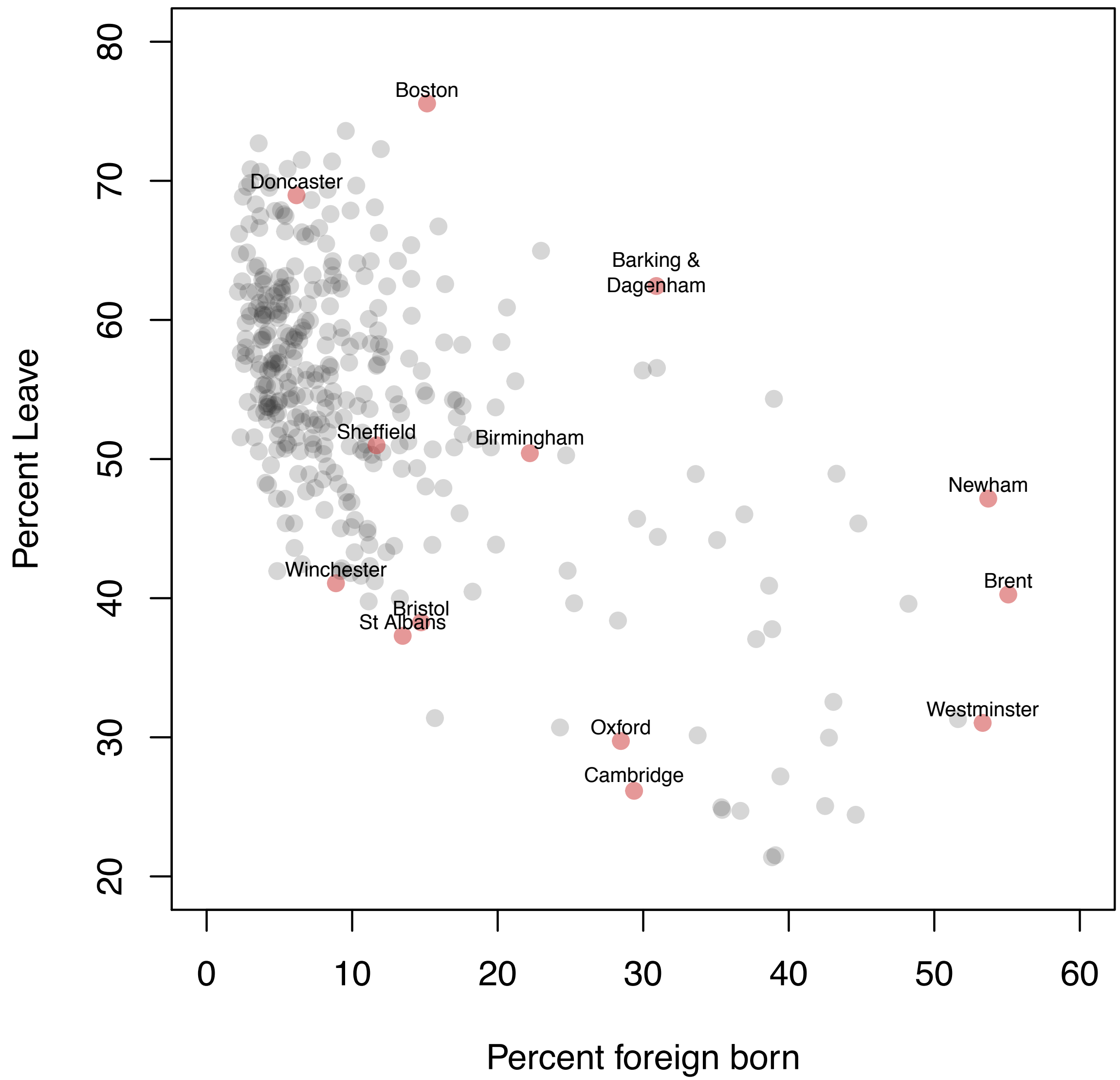
Kernel smoother (lokern function in R)



Single-number summaries: covariance

How do x and y tend to move together, i.e. how do they **covary**?

When x is above its mean, is y also above its mean? By how much?



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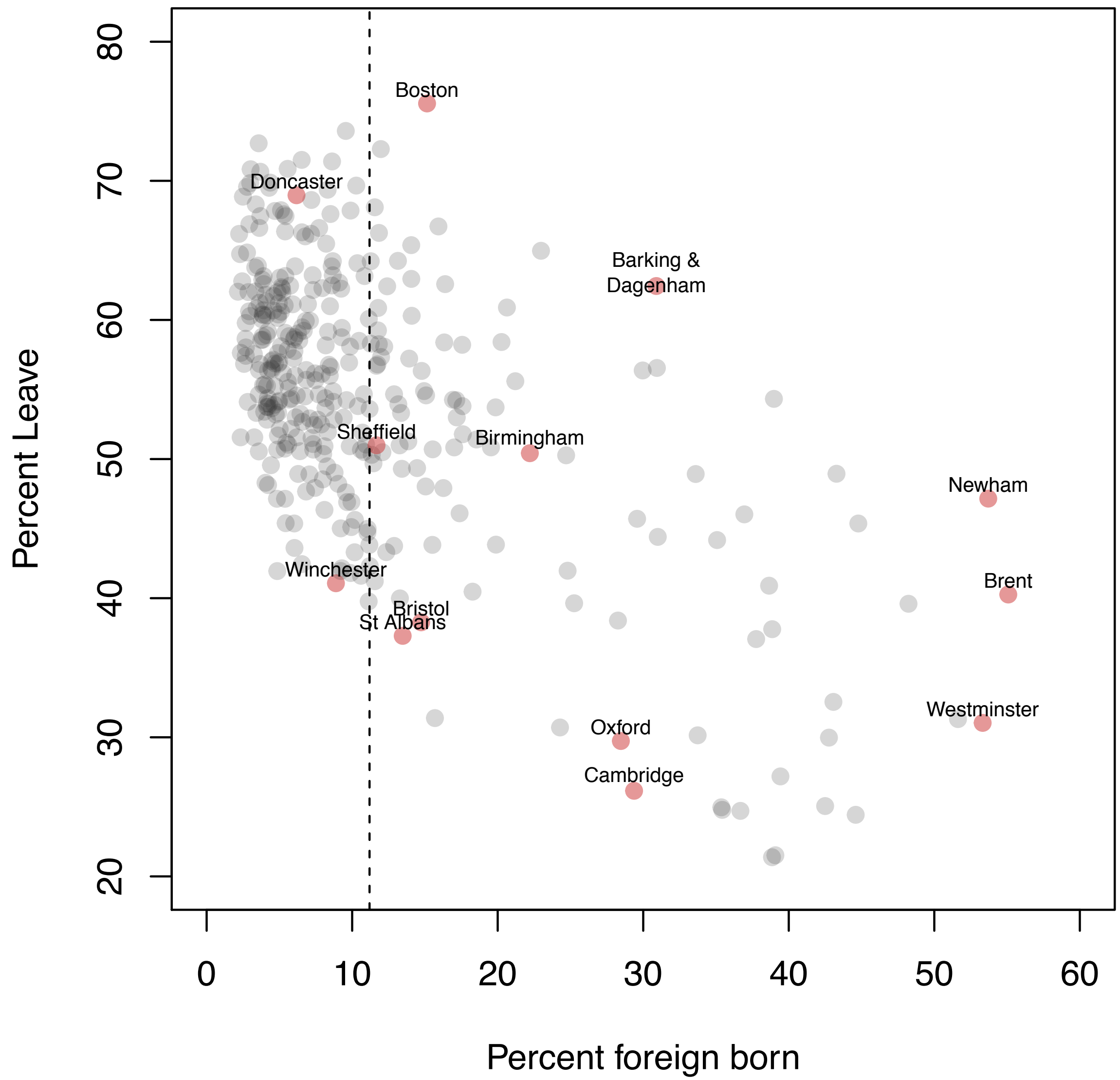

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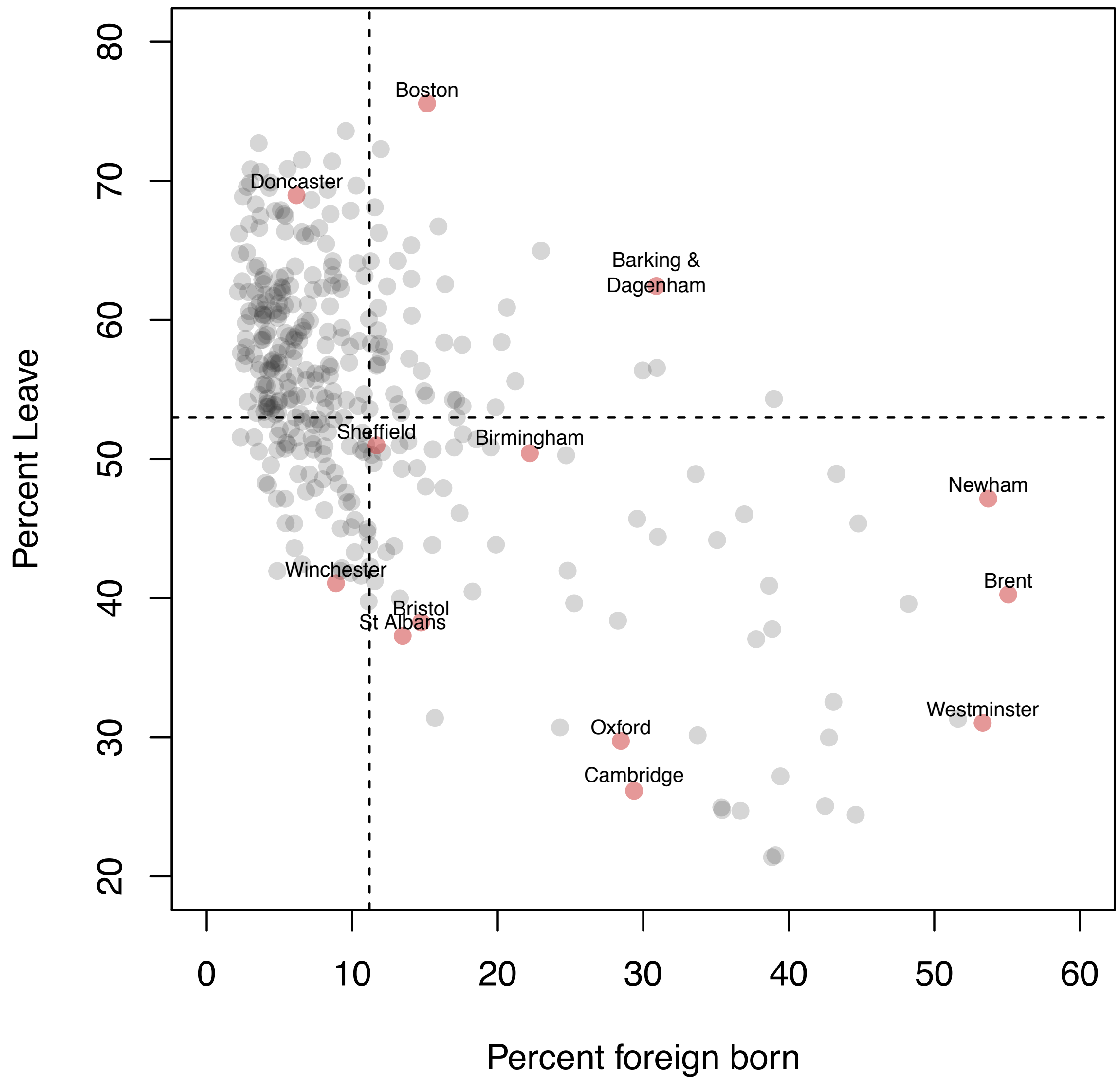

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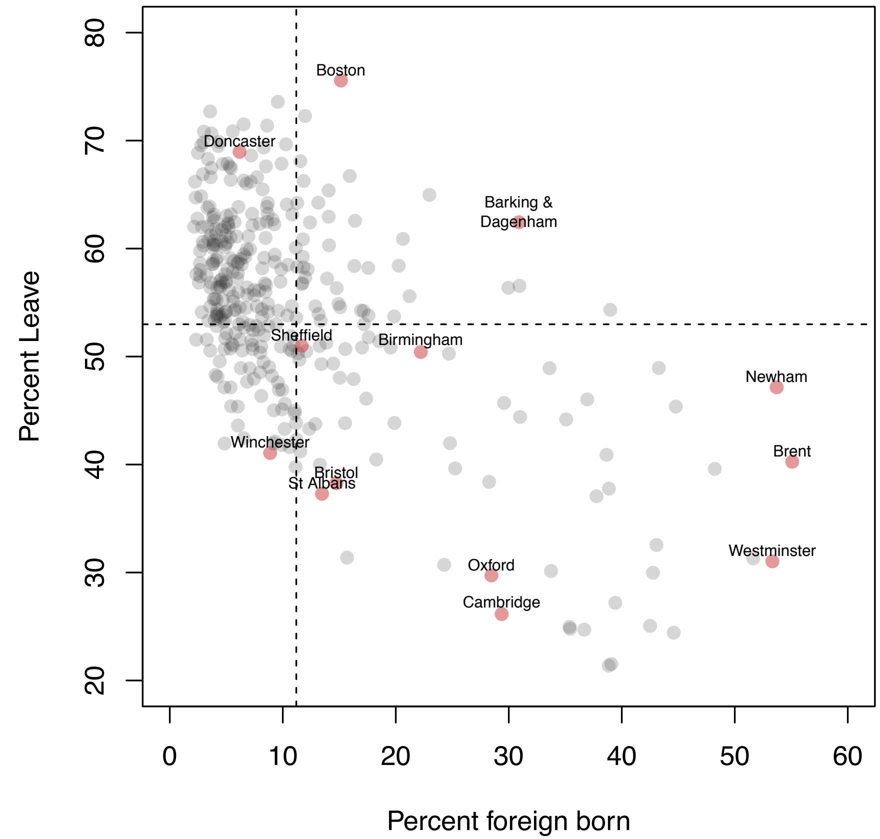

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```
> cov(d$Percent_foreign_born, d$Percent_Leave, use = "complete")
[1] -62.17755
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> cor(d$Percent_foreign_born, d$Percent_Leave, use = "complete")
[1] -0.6125353
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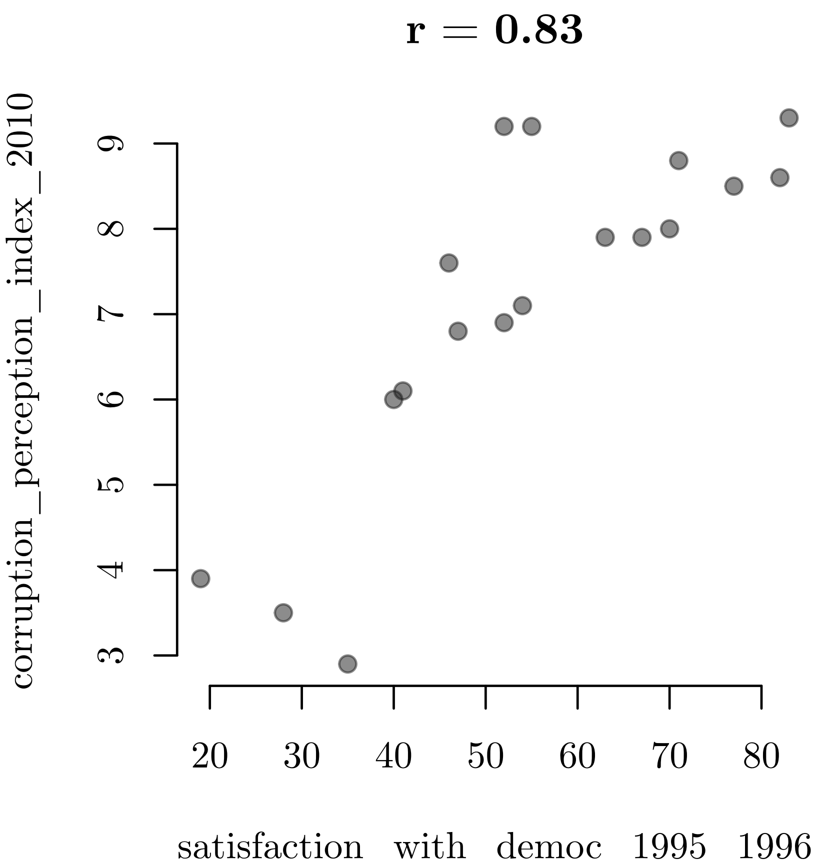

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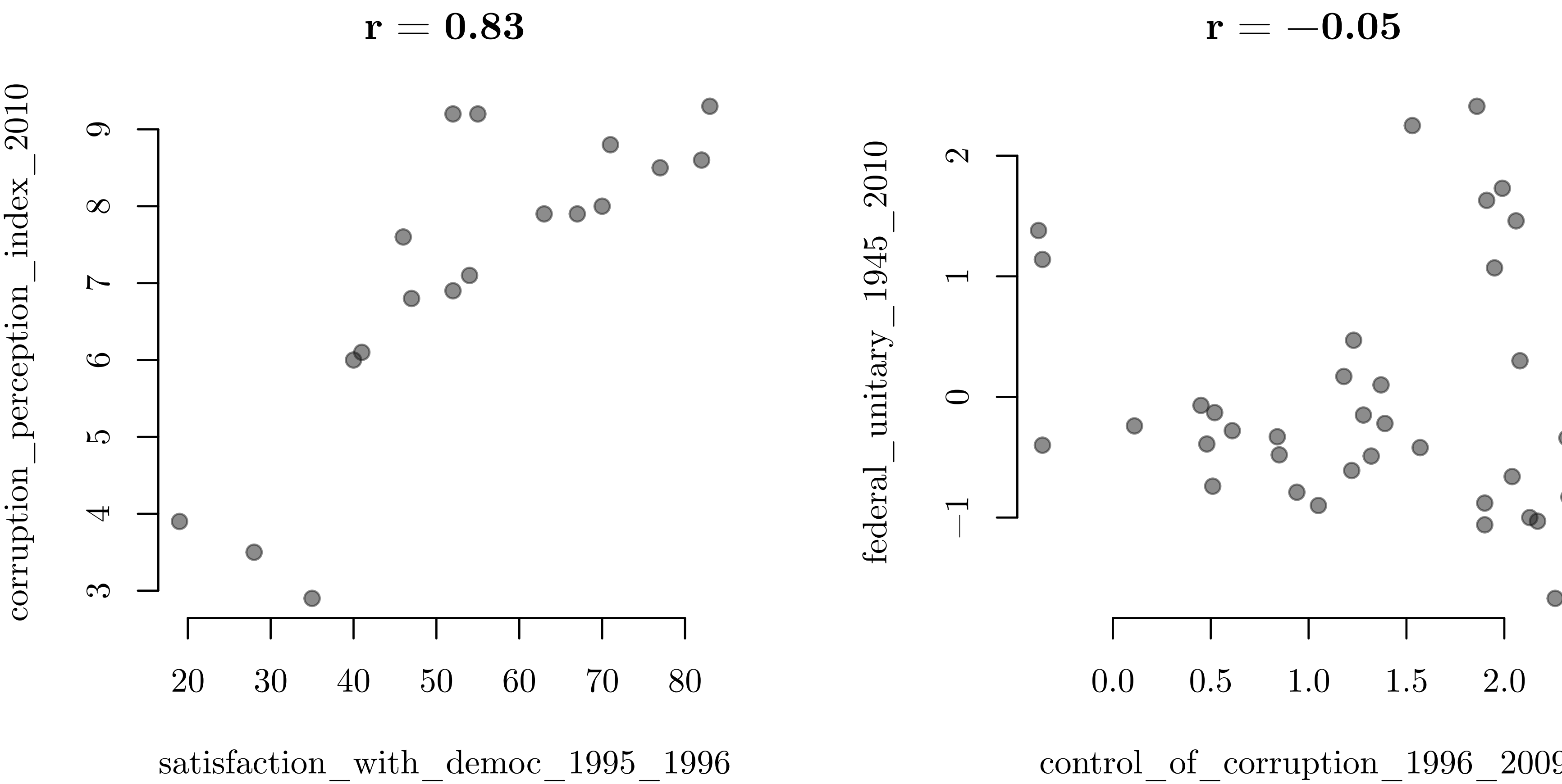
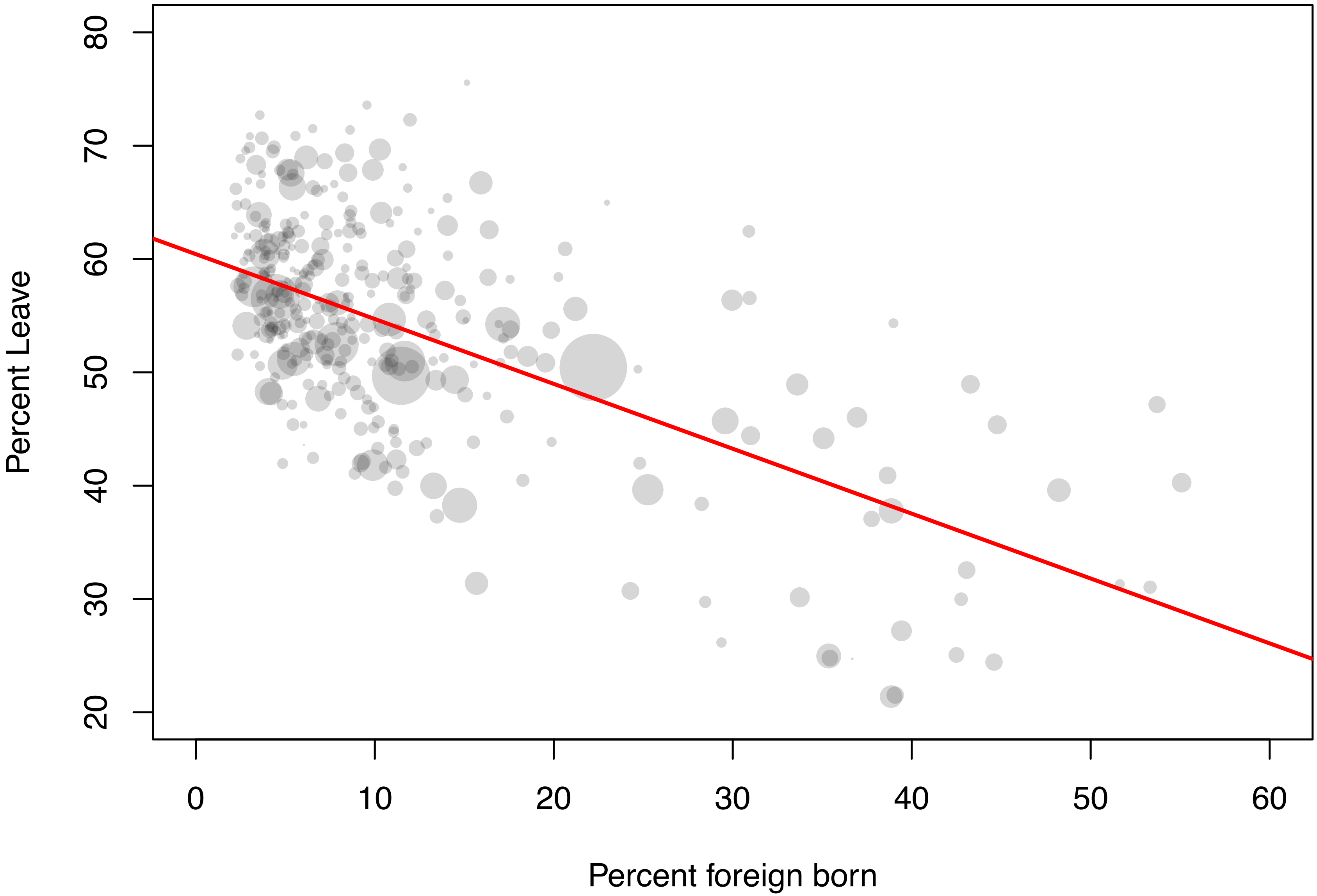
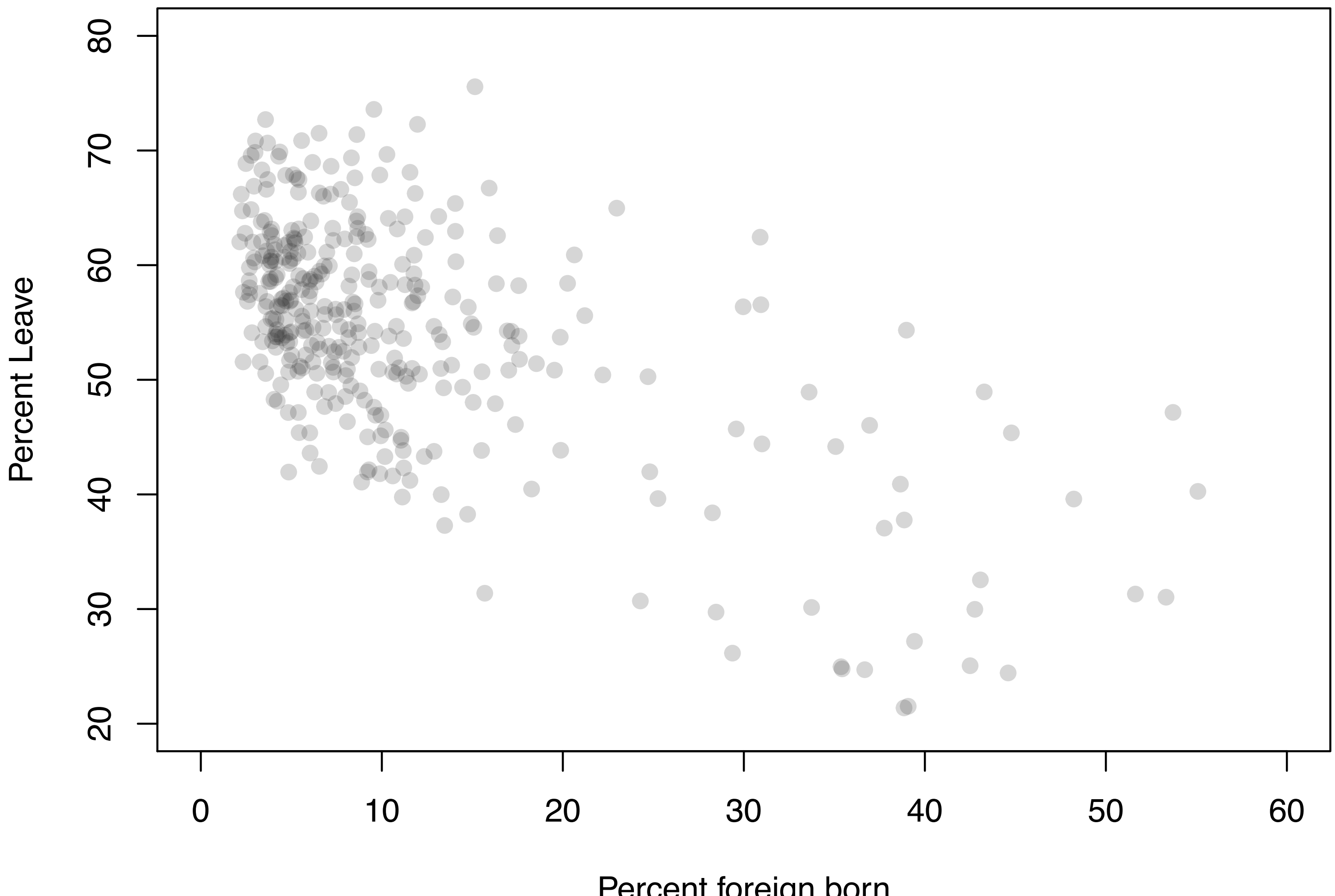


The most important summary: OLS regression



Step 1 for understanding OLS: residuals

A **residual** is the difference between the *actual* y-value and the *predicted* y-value.



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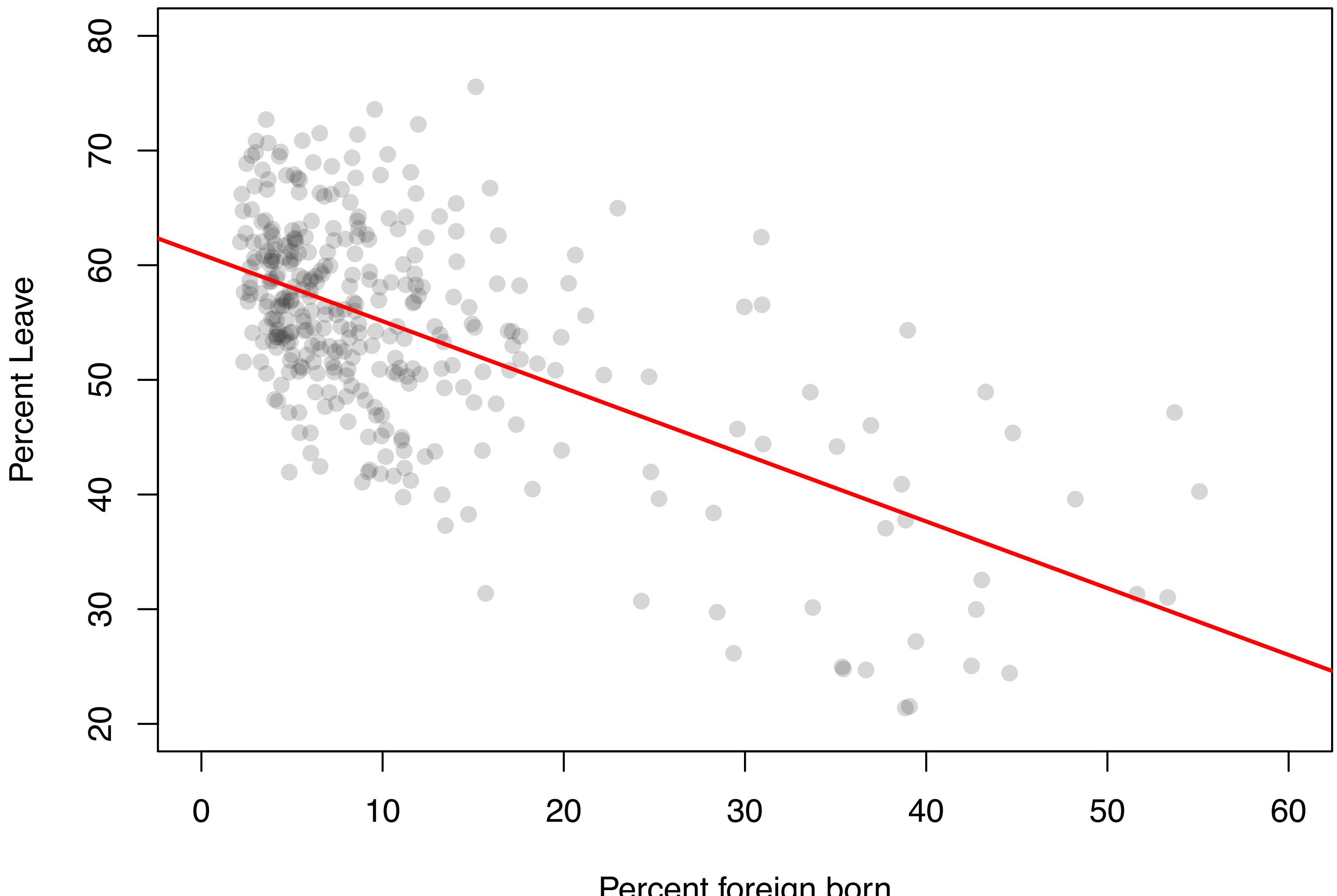

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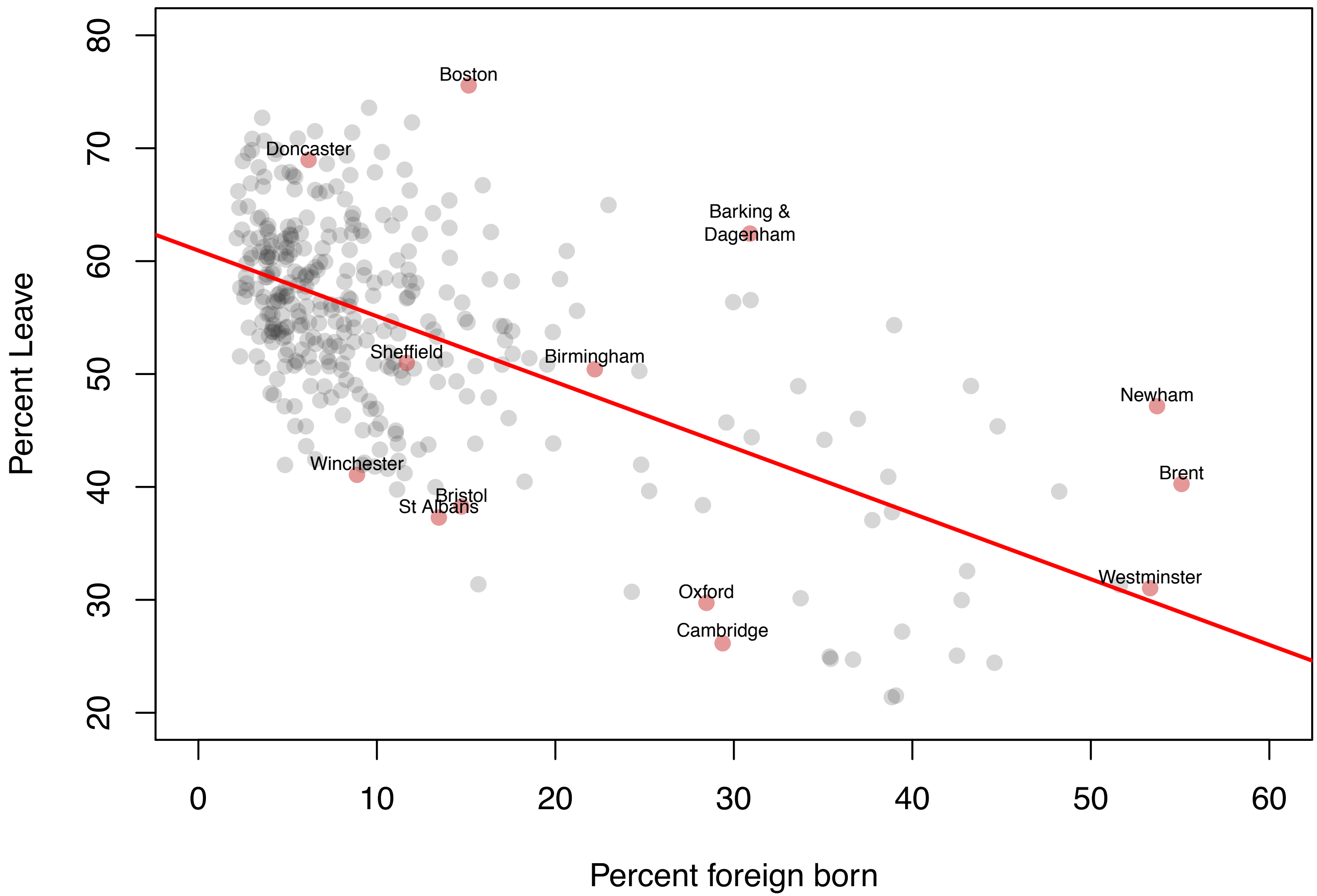

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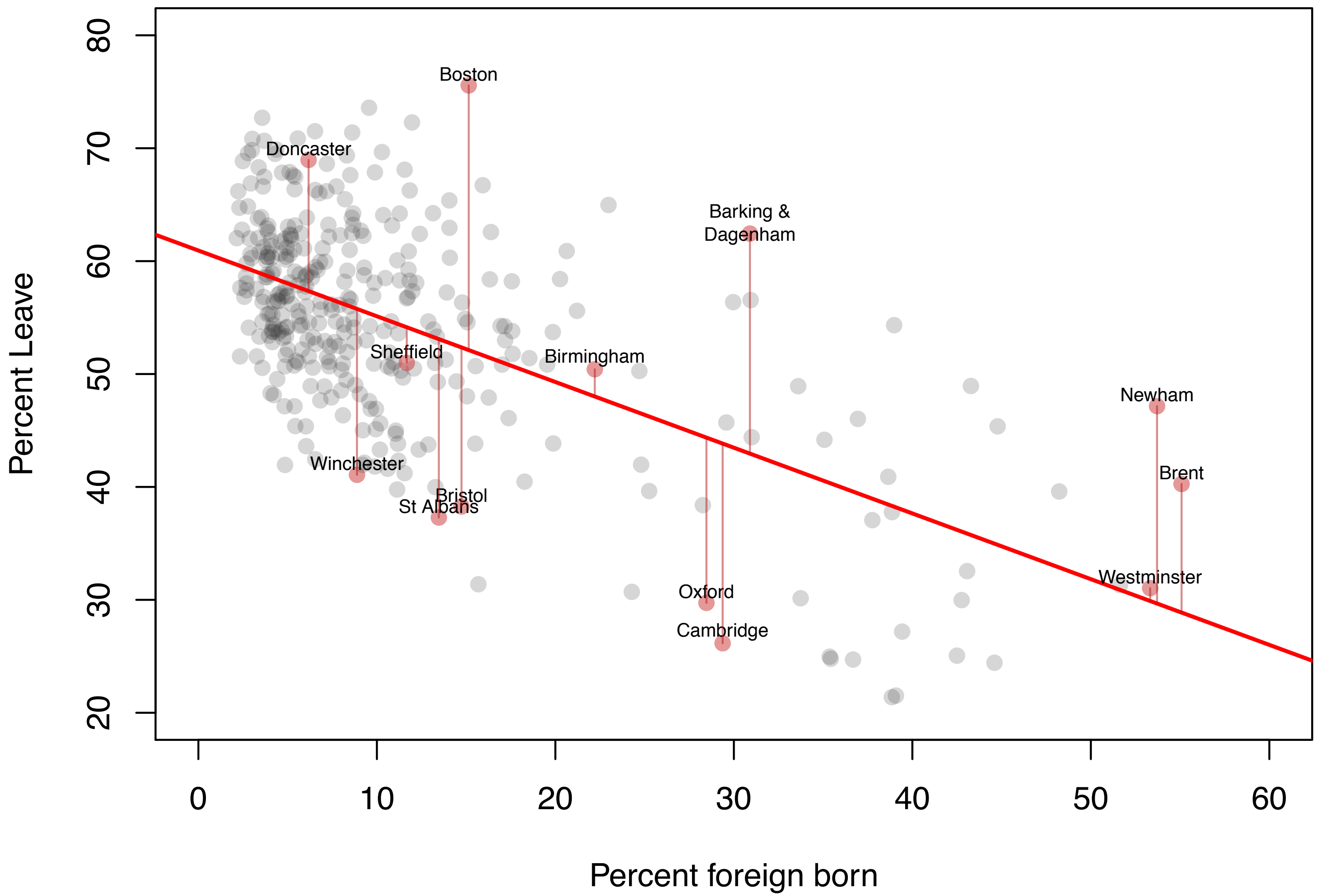

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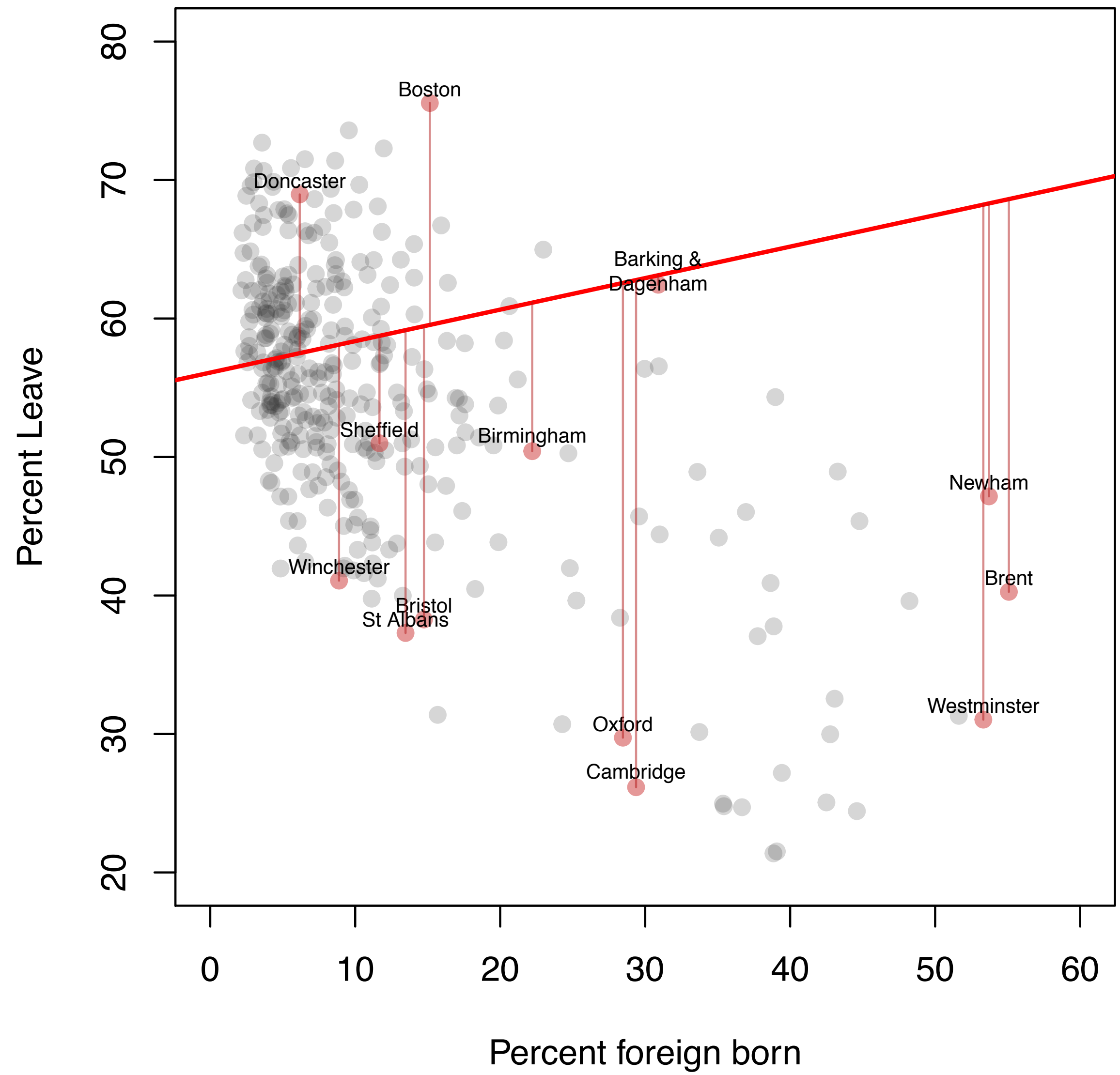

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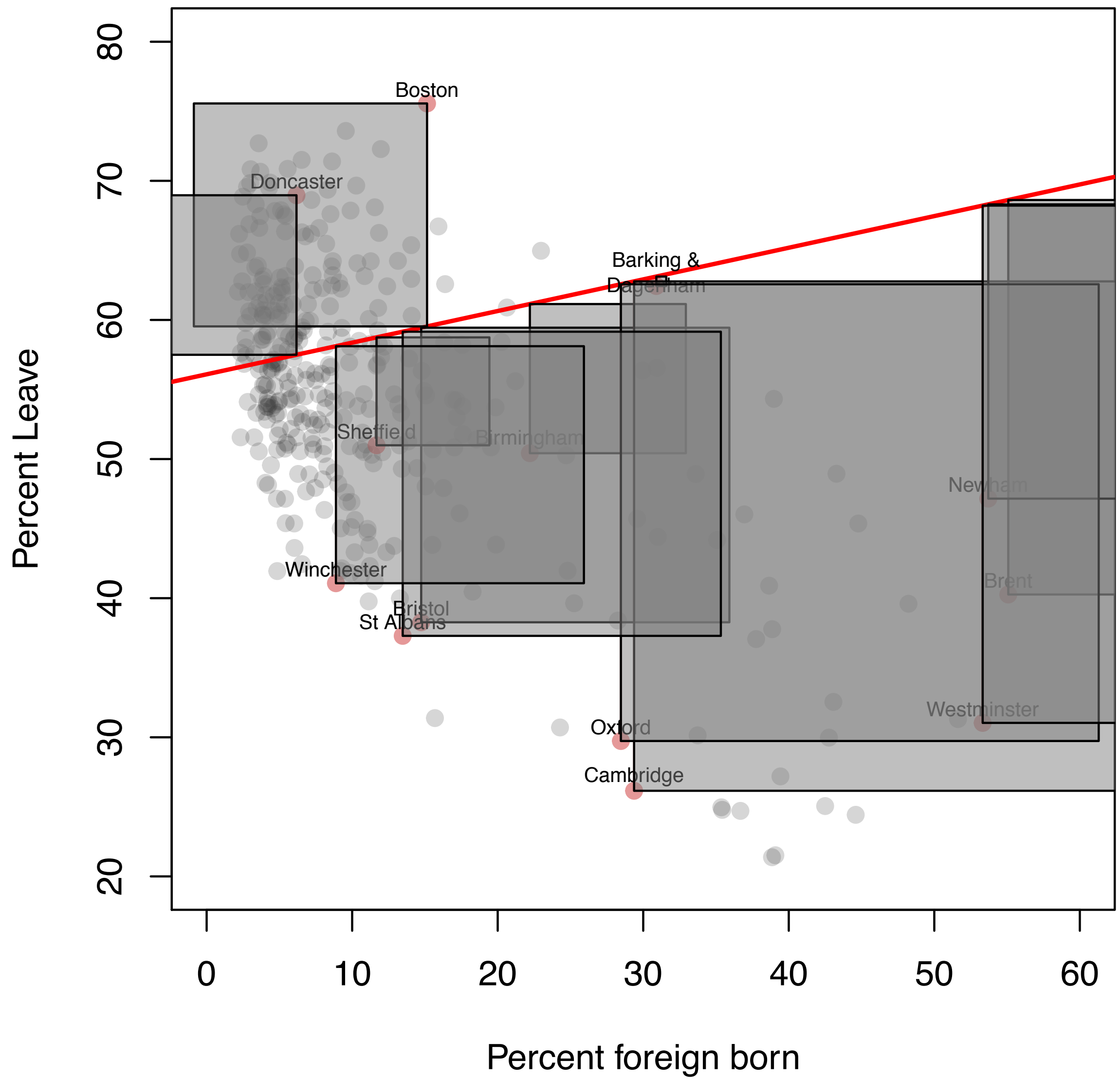

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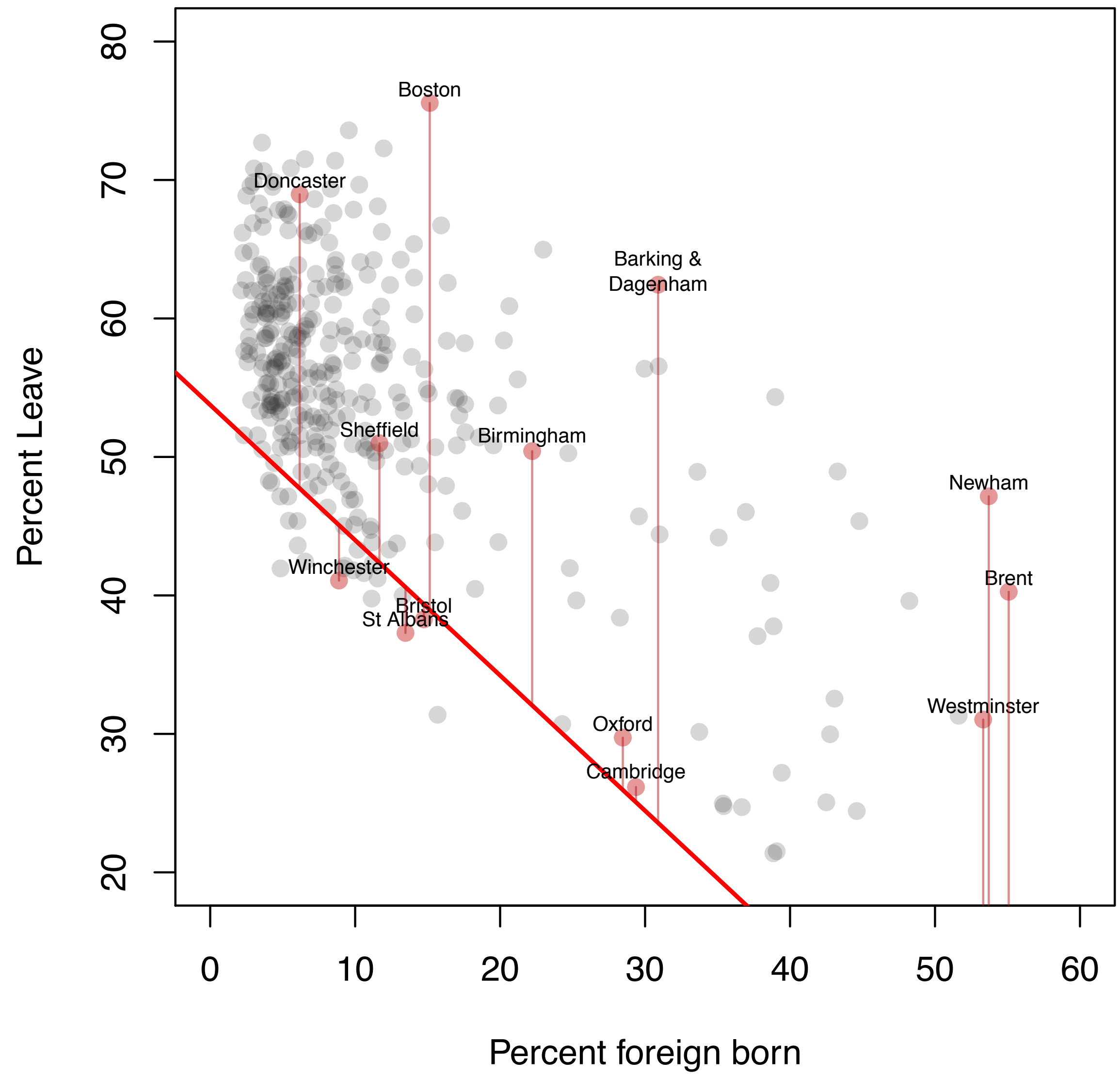

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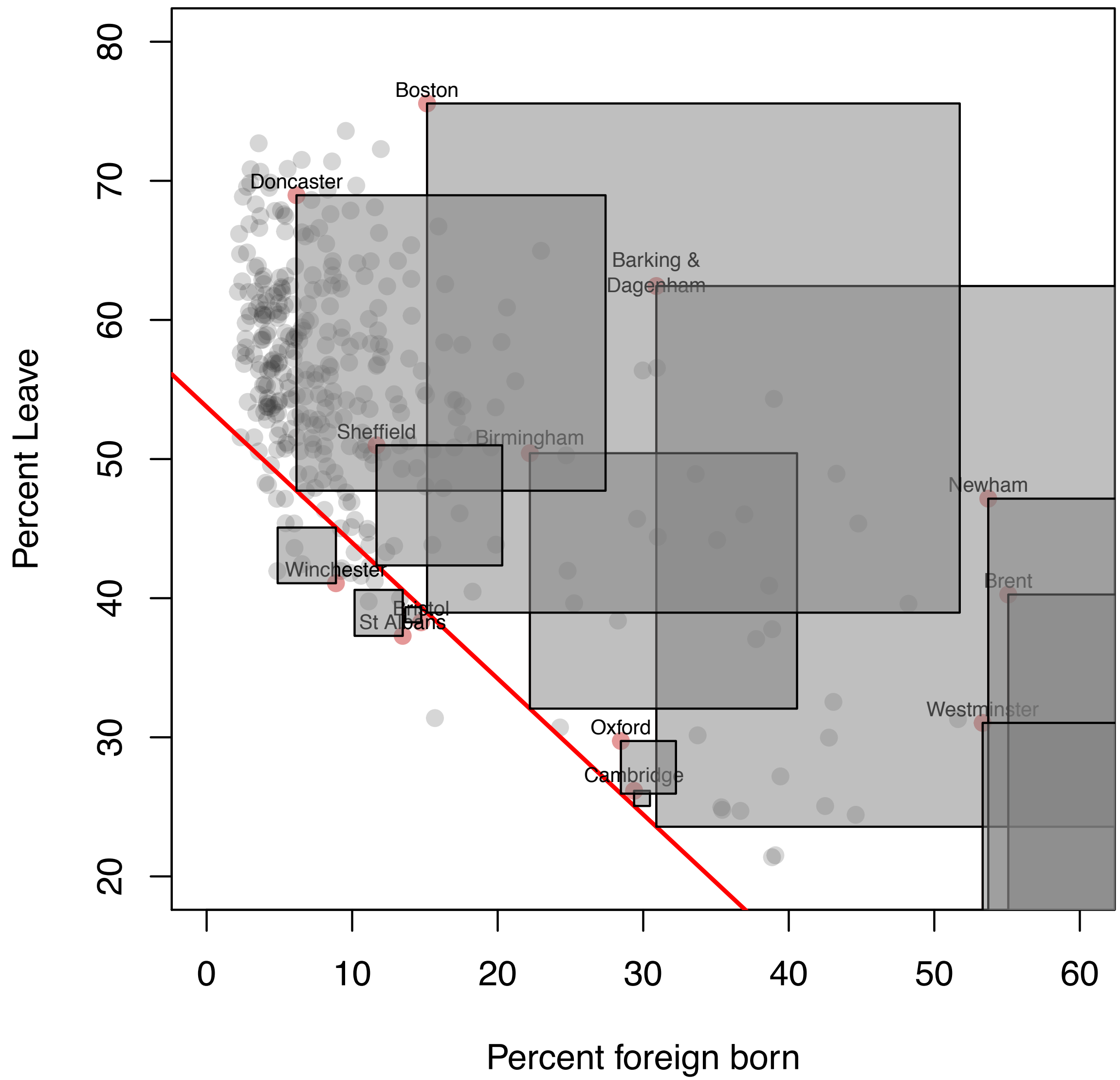

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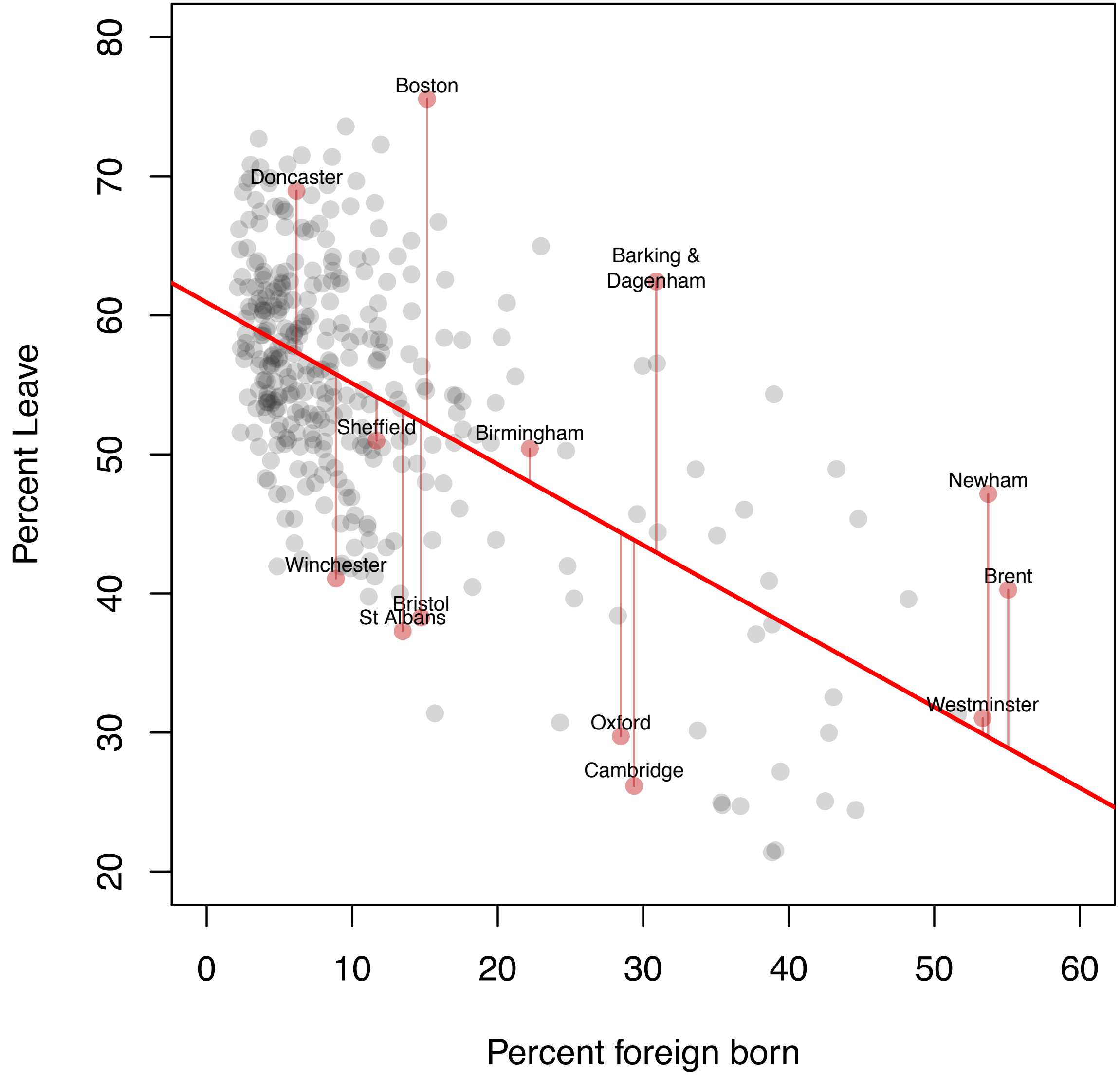

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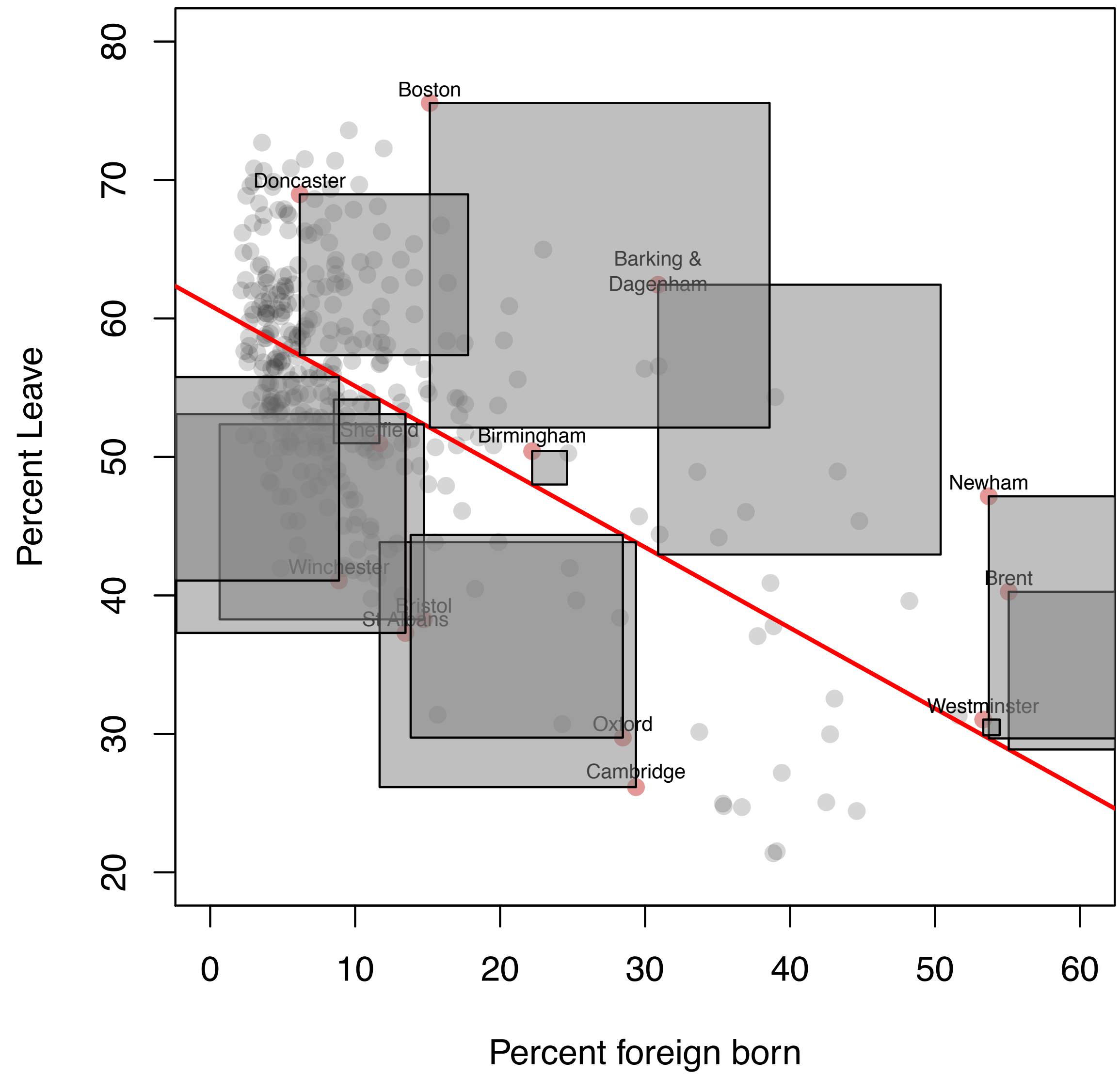
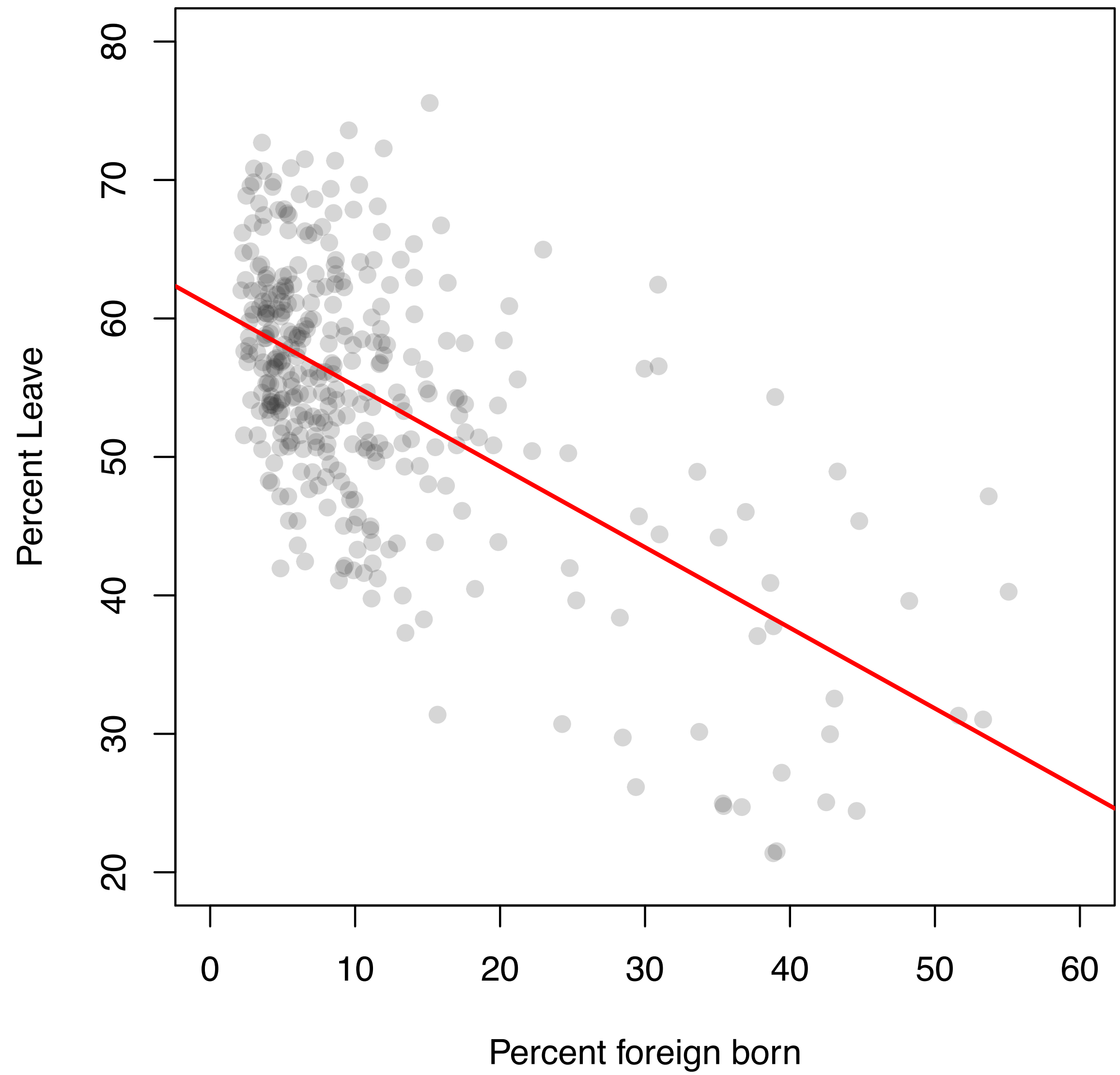


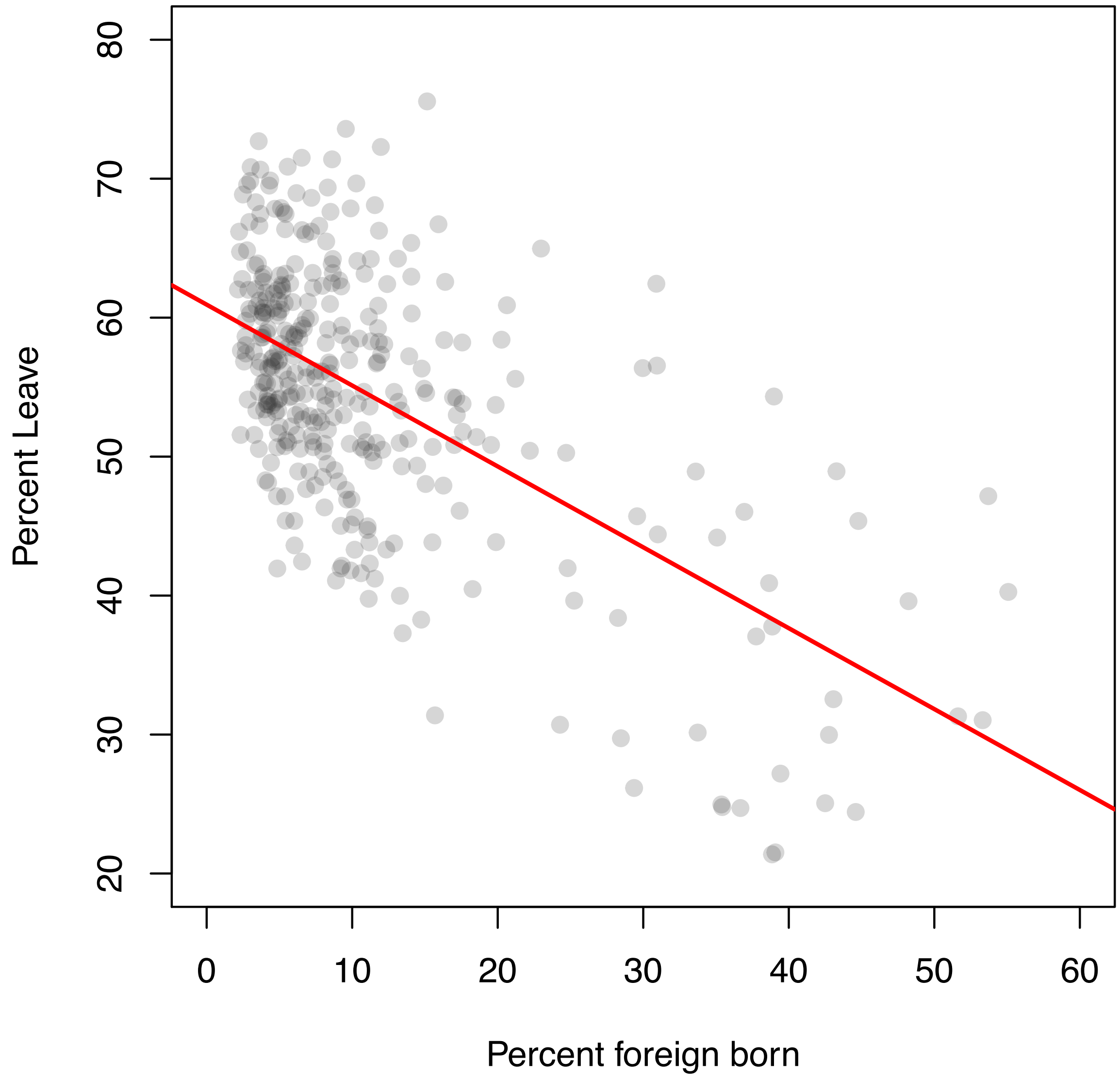
Step 3 for understanding OLS: minimizing the sum of squared residuals

The OLS regression line minimizes the sum of squared residuals (SSR).

Hence ordinary **least squares**.



Coefficients: the two variables in a bivariate regression



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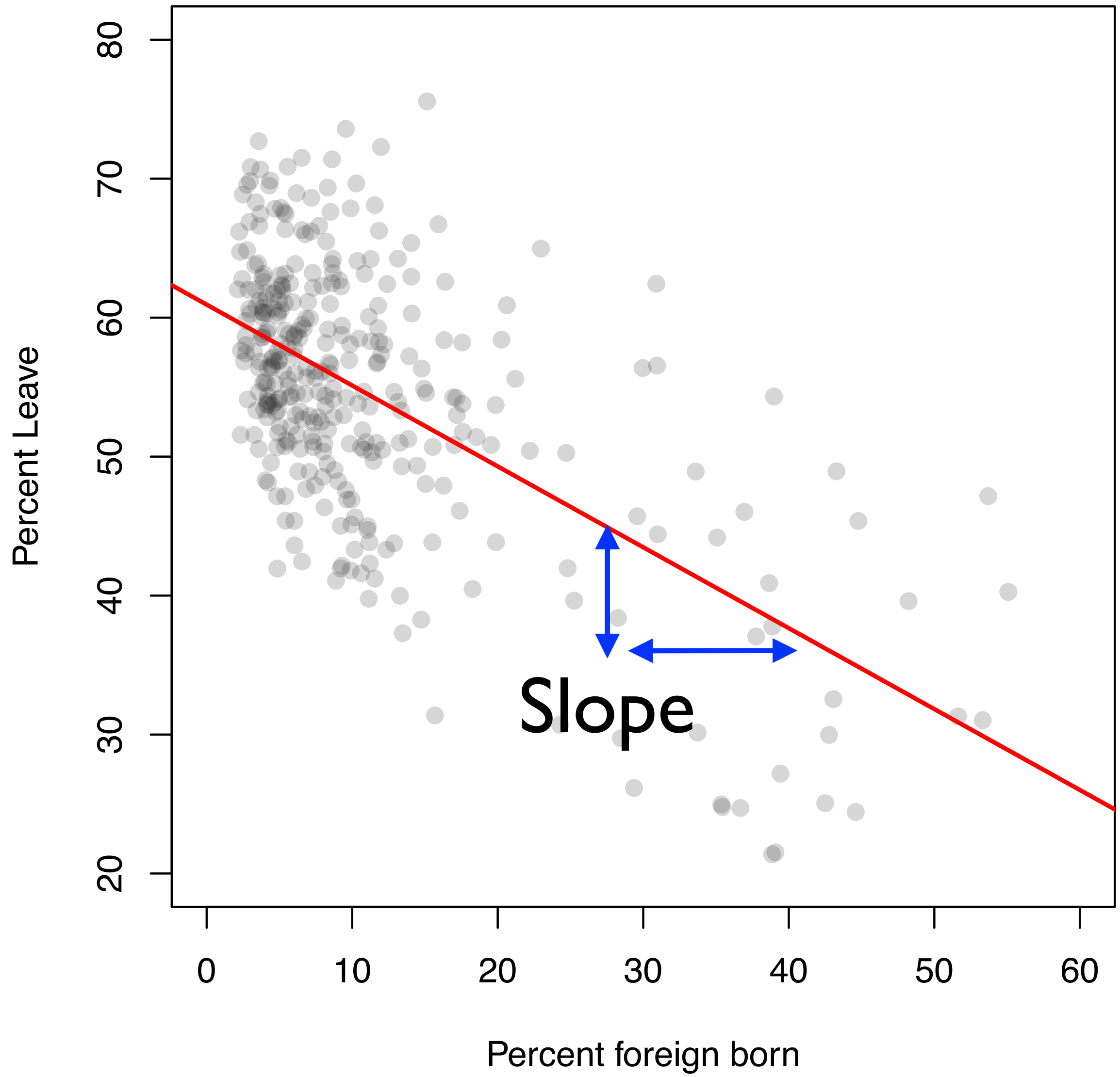

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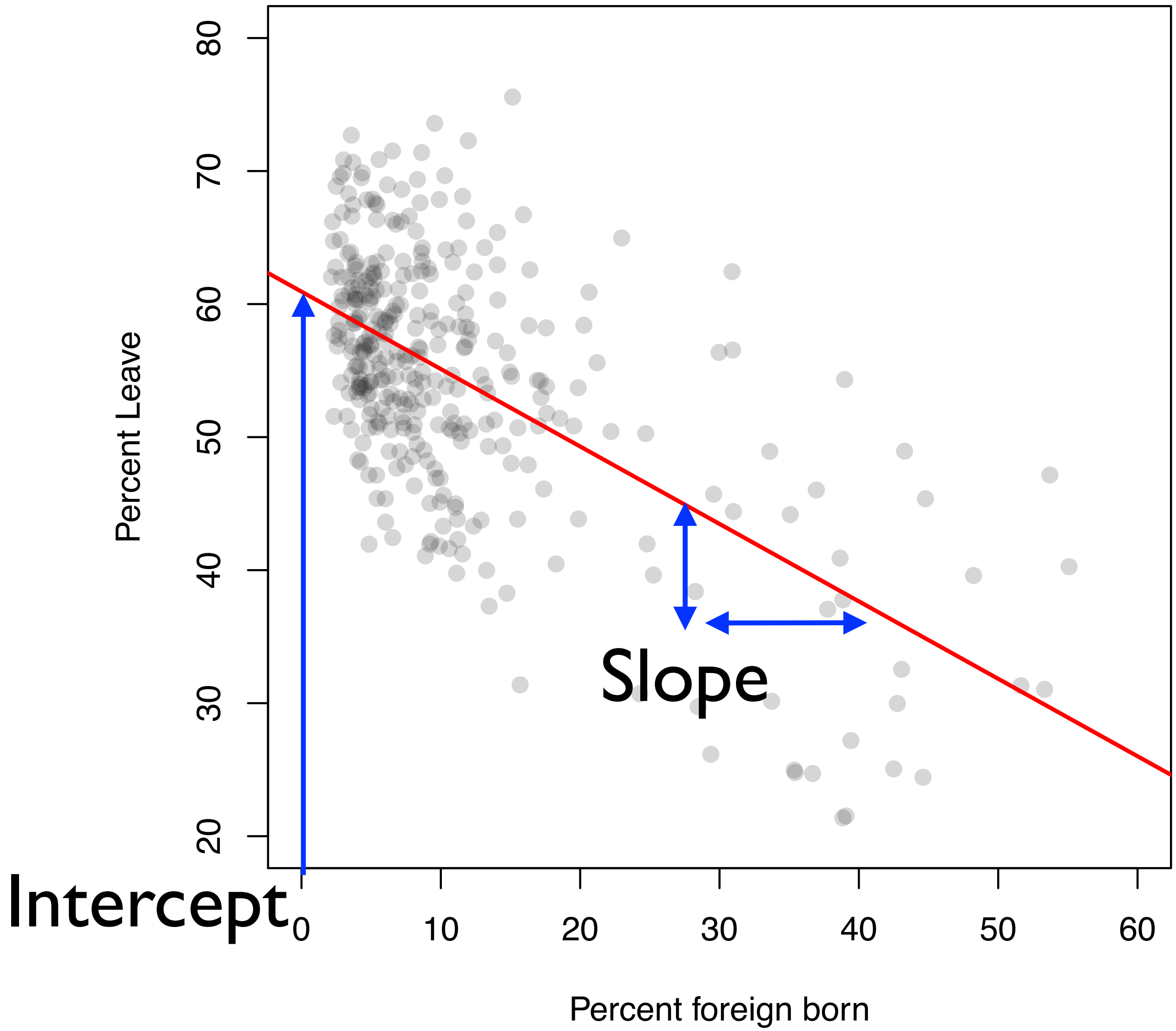

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3. Use lm() function in R:

```
> lm(d$Percent_Leave ~ d$Percent_foreign_born)

Call:
lm(formula = d$Percent_Leave ~ d$Percent_foreign_born)

Coefficients:
           (Intercept)  d$Percent_foreign_born
               60.9373                 -0.5821
```

A (surprising?) fact about the slope coefficient

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Slope from OLS regression of y on x:

$$\hat{\beta} = \frac{\mathrm{Cov}(x,y)}{\mathrm{Var}(x)}$$

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```
> cov(d$Percent_Leave, d$Percent_foreign_born, use = "complete")/var(d
$Percent_foreign_born, na.rm = T)
[1] -0.582101
> lm(d$Percent_Leave ~ d$Percent_foreign_born)

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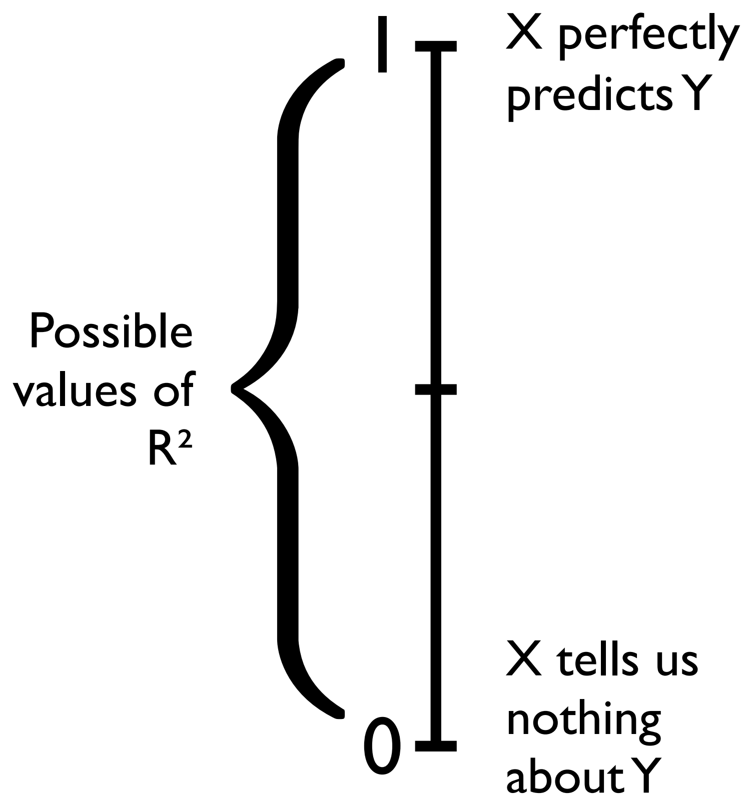
How well does our regression line predict the outcome? R^2

```
> summary(lm(d$Percent_Leave ~ d$Percent_foreign_born))

Call:
lm(formula = d$Percent_Leave ~ d$Percent_foreign_born)

Residuals:
     Min       1Q   Median       3Q      Max 
-20.4253  -4.7247  -0.0025   4.4336  23.4417 

Coefficients:
                       Estimate Std. Error t value Pr(>|t|)    
(Intercept)            60.93732    0.61845   98.53   <2e-16 ***
d$Percent_foreign_born -0.58210    0.04062  -14.33   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 7.775 on 342 degrees of freedom
  (38 observations deleted due to missingness)
Multiple R-squared:  0.3752,Adjusted R-squared:  0.3734 
F-statistic: 205.4 on 1 and 342 DF,  p-value: < 2.2e-16
```

R^2: intuition

How much better are the predictions from our OLS regression line than the predictions from a flat line (i.e. not using X at all)?



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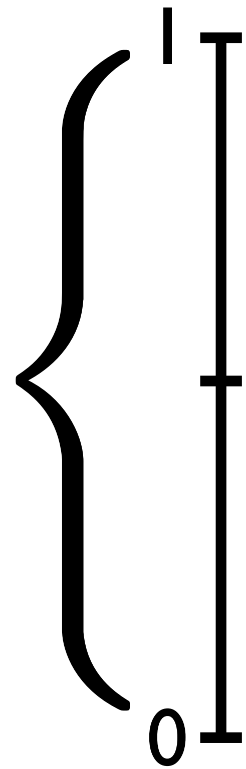
How much of the variation in Y is "explained" by the variation in X?



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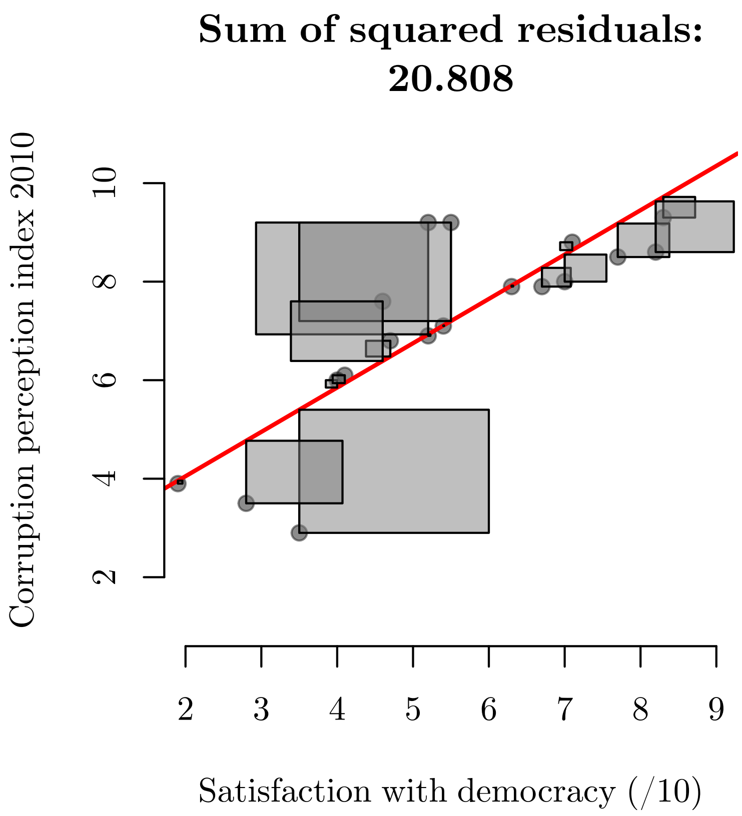
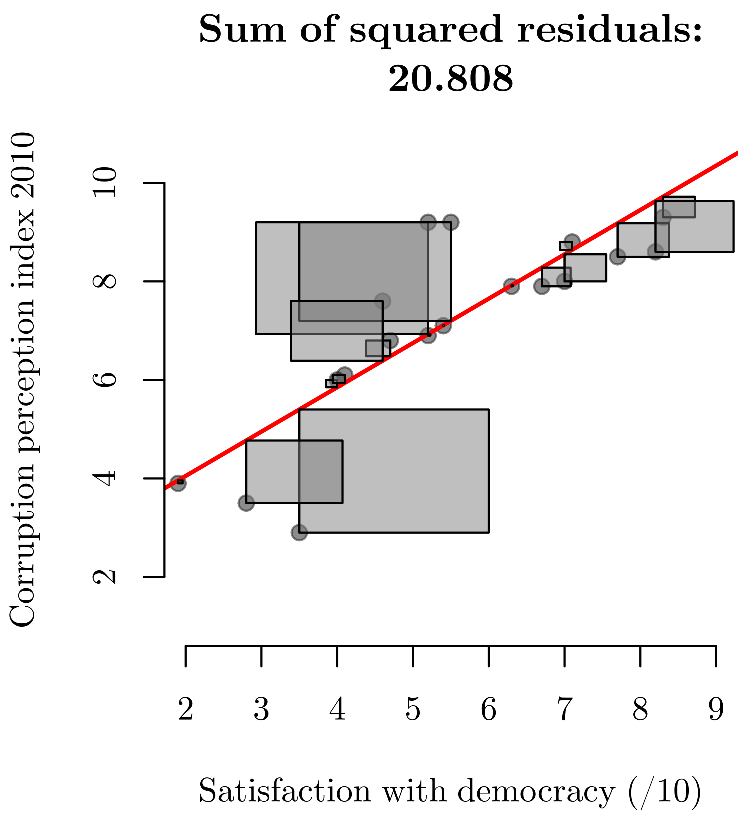
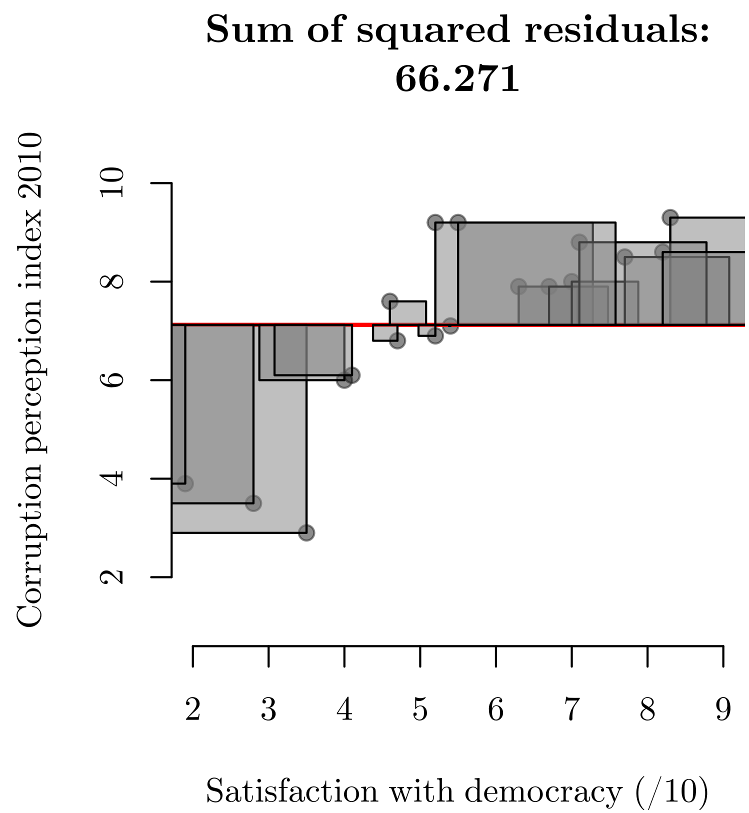


R^2: calculation

“Total sum of squares”





R^2: calculation

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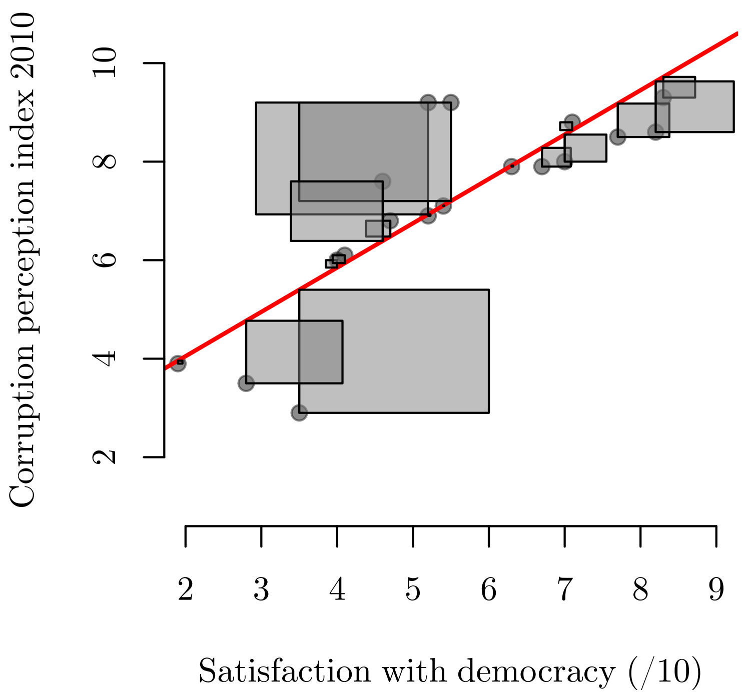
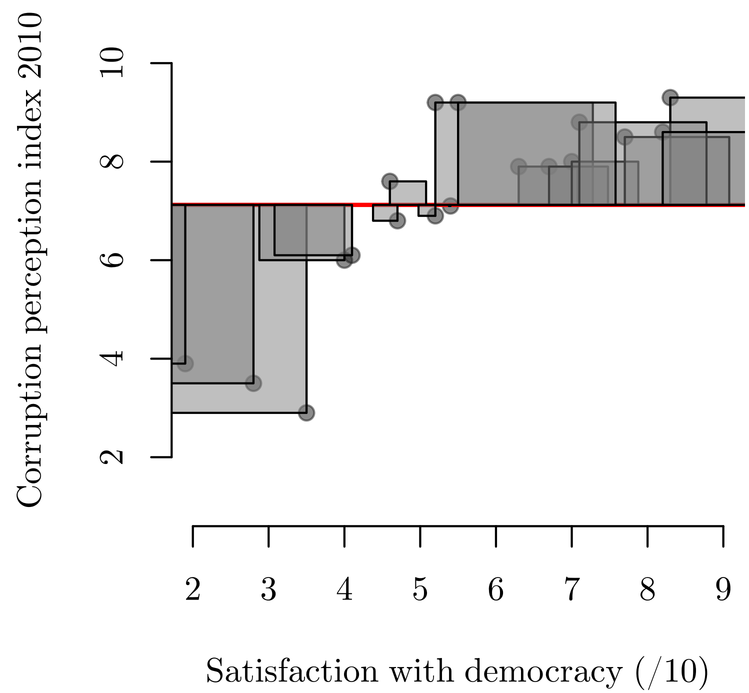
Sum of squared residuals: 20.808



Sum of squared residuals: 66.271



$$1 - \frac{20.808}{66.271} =$$

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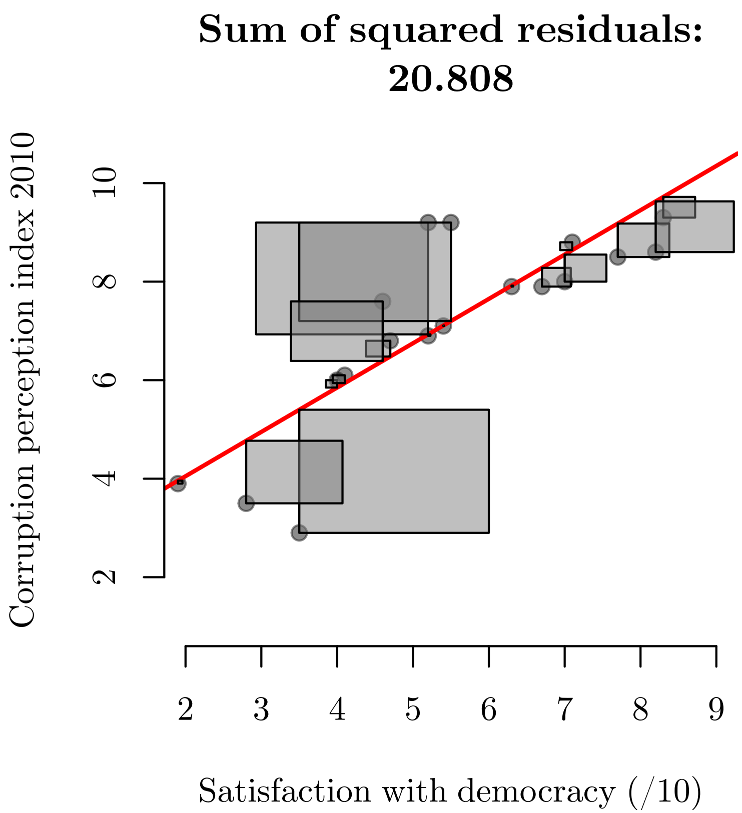

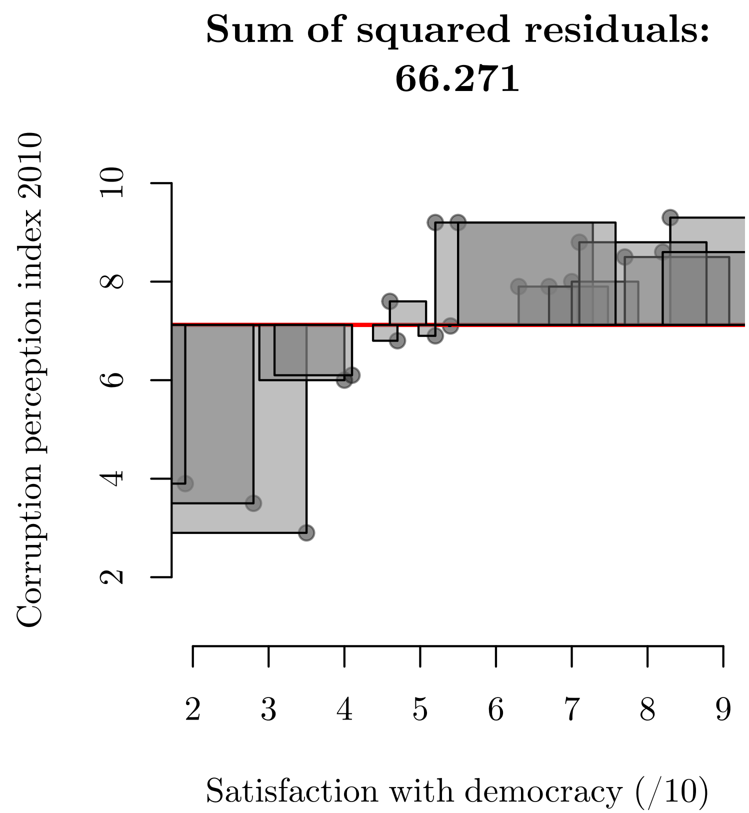

$$1 - \frac{20.808}{66.271} = 0.6864$$

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Key measures:

- covariance
- correlation
- OLS regression output:
  - intercept
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Regression slope (but not covariance or correlation) depends on which is Y and which is X

Covariance and regression slope (but not correlation) depend on the units

Why are we minimizing squared residuals?

There are other ways to draw a predictive line.

But OLS (minimizing squared residuals)

- produces nice analytical solutions
- recovers the mean
- among unbiased estimators, minimizes variance