Bivariate relationships

Week 5

13 February, 2017

Prof. Andrew Eggers

Financial Statements of the Oxford Colleges (2012-13)

The financial statements of the 36 colleges of Oxford University for the year ending 31 July 2013 are available as pdfs, together with an aggregated statement of financial activity (SOFA) and aggregated consolidated balance sheet.

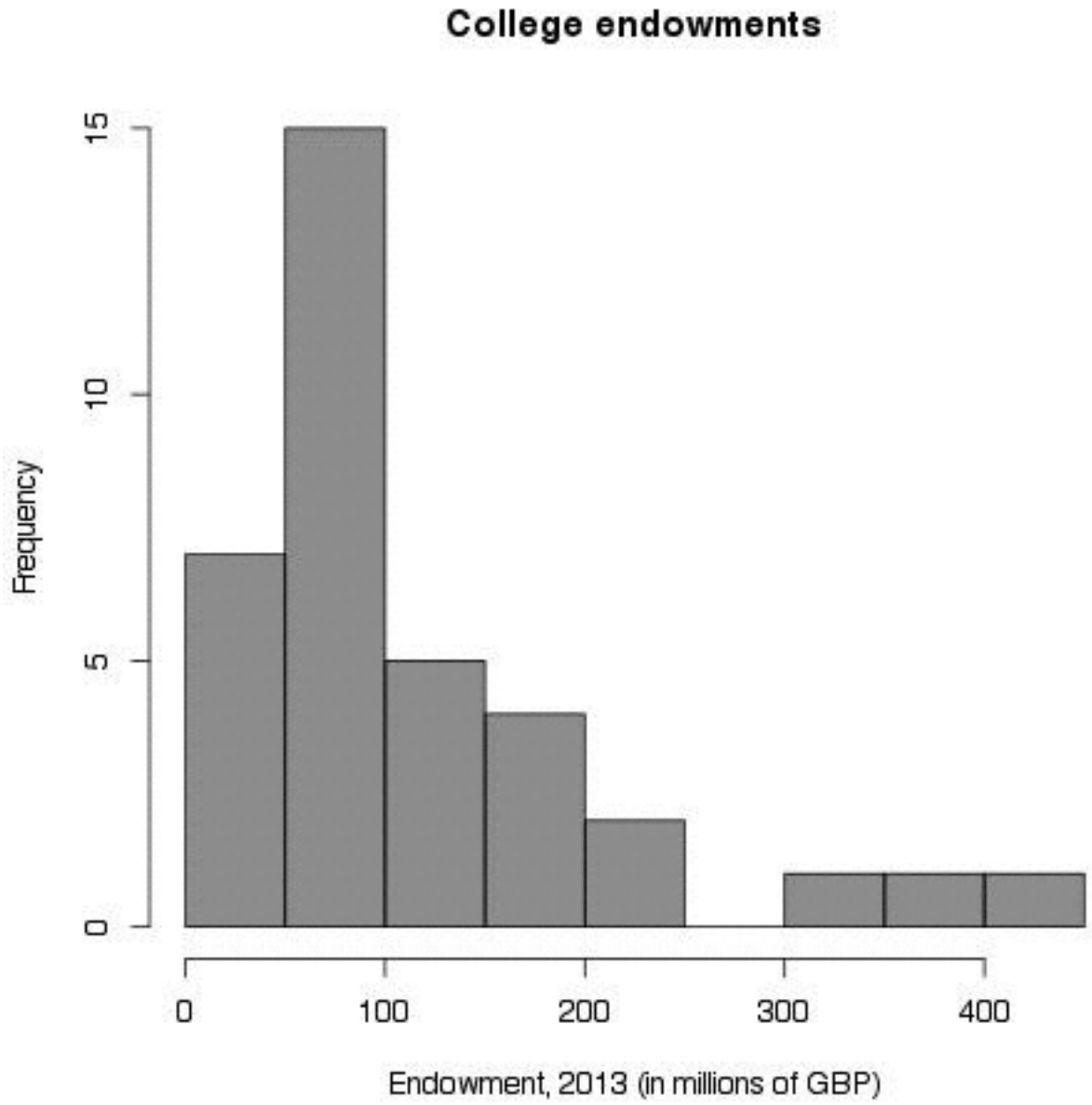
The colleges are independent, self-governing and financially autonomous and their accounts are published under the accounting convention developed by the Charity Commissions for use by charities in the UK (the Charity SORP).

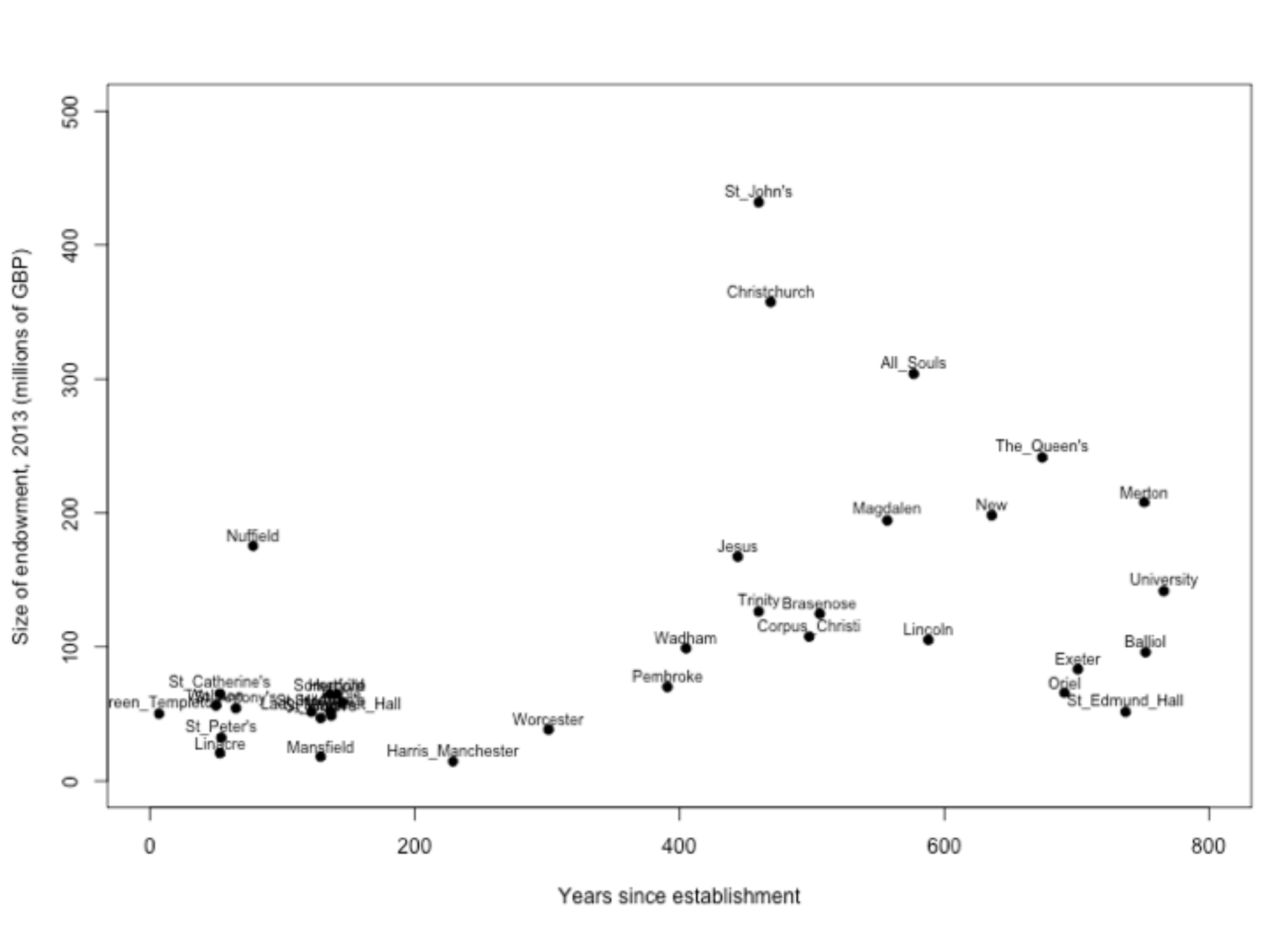
Kellogg College and St Cross College do not have Royal Charters and, for accounting purposes, are departments of the University. As such, their financial results are consolidated into the University's financial statements.

http://goo.gl/1pJA2r

Console ~/

```
> ## data analysis for lecture
> d = read.csv("http://andy.egge.rs/data/college_stats_edited.csv")
> d$rank = 37 - rank(d$endowments  )
>
> # just a table of the data
> d[order(d$endowments, decreasing = T), c("rank", "college", "endowments")]
   rank            college endowments
28    1          St_John's     432075
4     2       Christchurch     357667
1     3          All_Souls     303896
30    4        The_Queen's     241467
36    5             Merton     208054
17    6                New     198160
15    7           Magdalen     194344
18    8           Nuffield     175415
10    9              Jesus     167333
32   10         University     141872
31   11            Trinity     126350
3    12           Brasenose     124684
5    13    Corpus_Christi     107692
14   14            Lincoln     105245
33   15             Wadham      98935
2    16            Balliol      96044
6    17             Exeter      83383
20   18           Pembroke      70195
```

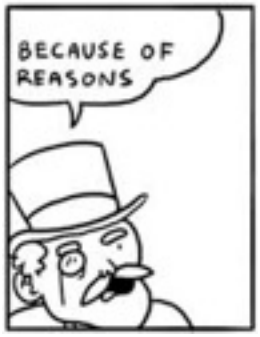

College endowments
Frequency
0
5
10
15
0
100
200
300
400
Endowment, 2013 (in millions of GBP)


Size of endowment, 2013 (millions of GBP)
0
100
200
300
400
500
0
200
400
600
800
Years since establishment
St_John's
Christchurch
All_Souls
The_Queen's
Merton
New
Magdalen
Nuffield
Jesus
University
Trinity
Brasenose
Corpus_Christi
Lincoln
Wadham
Balliol
Exeter
Pembroke
Oriel
St_Edmund_Hall
St_Catherine's
Worcester
St_Peter's
Linacre
Mansfield
Harris_Manchester

Research questions you might have about Oxford colleges' endowments and age

Descriptive/predictive questions:

- Do older colleges have more money?
- What is the average endowment of a college that is X years old?

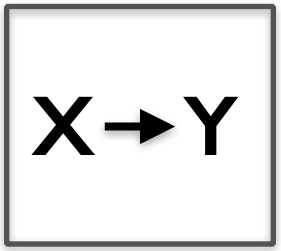


Explanatory questions **(reverse causal)**:

- Why do some colleges have more money than others? (Maybe age is the/an answer.)

Forward causal questions:

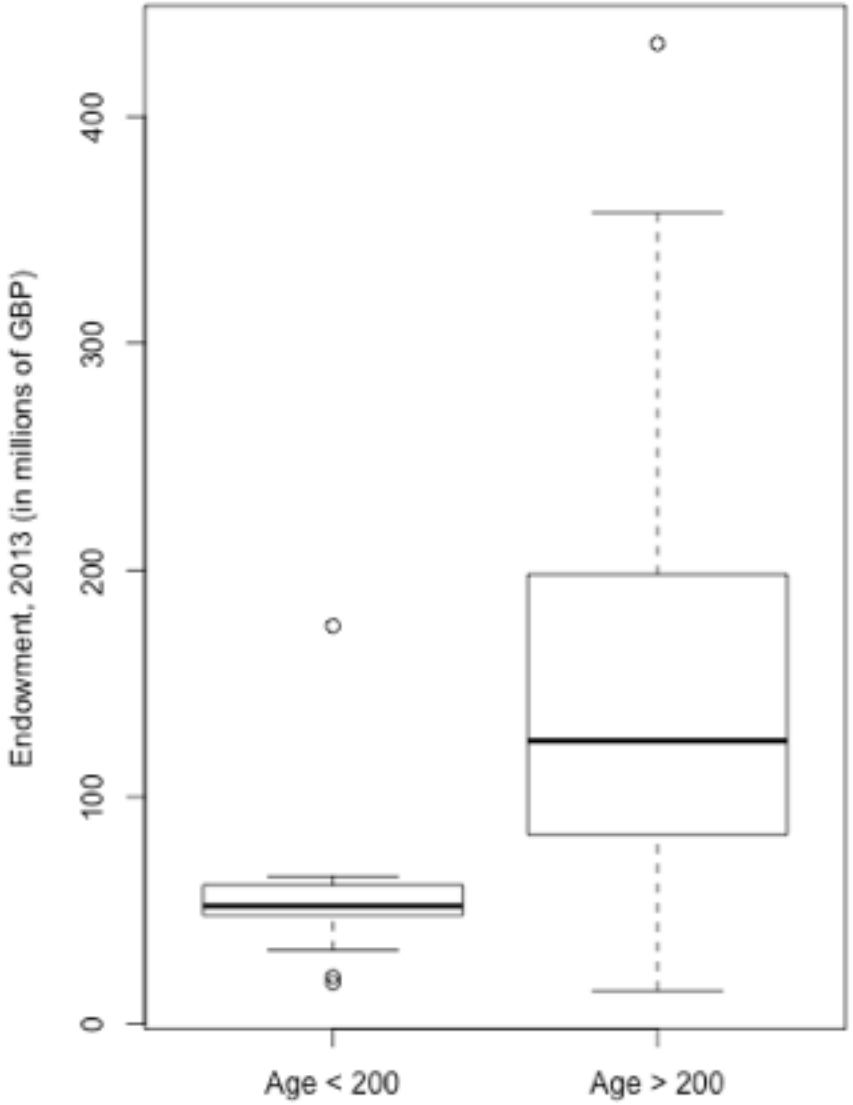
- (What is the effect of greater age on a college's endowment?)

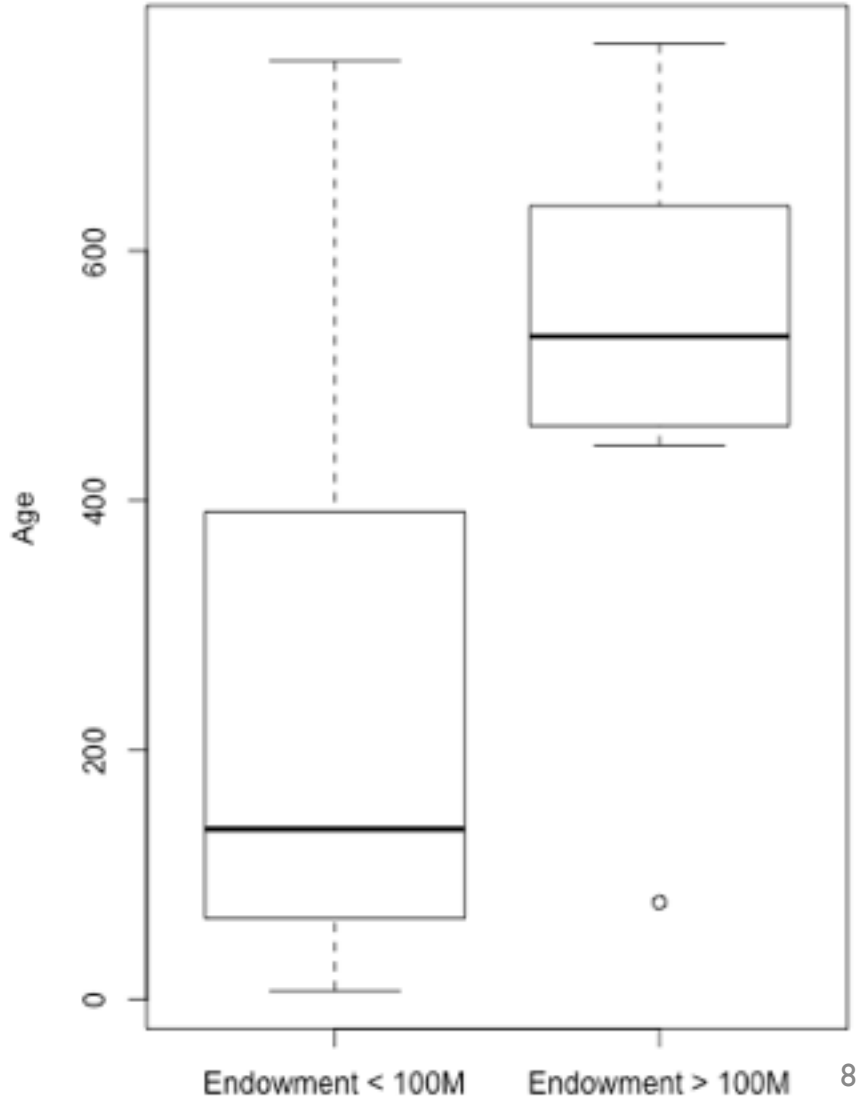


Various ways to summarize a relationship between two variables

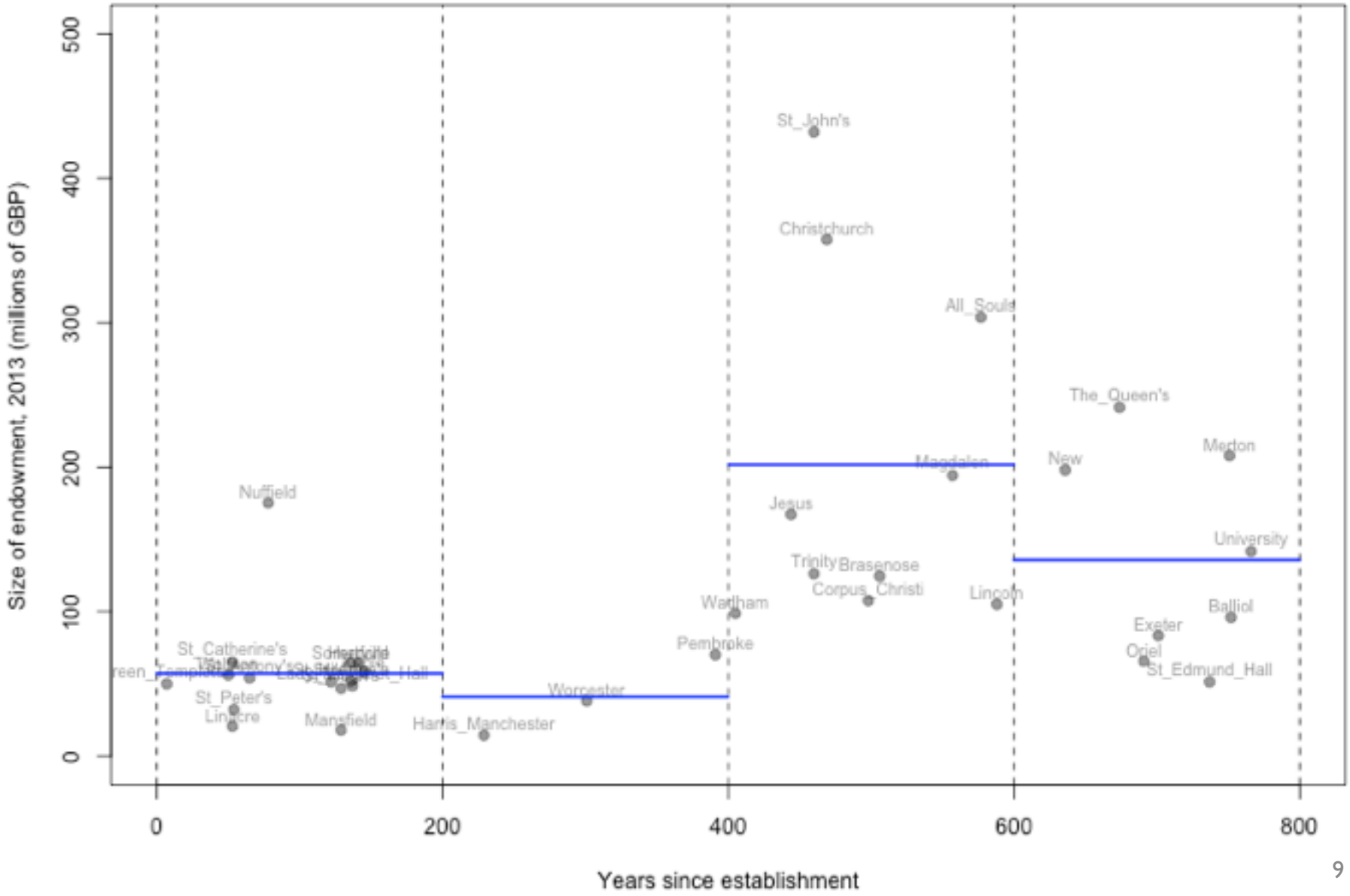
- Show the scatterplot!
- Compare boxplots of one variable across categories of the other
- Show/report mean of one variable across categories of the other (binned means or kernel average smoother)
- Report covariance/correlation
- Report regression coefficient(s)
- Report predicted values of one variable based on the other from regression

Boxplots: which do you prefer?





Mean endowment within 200-year intervals



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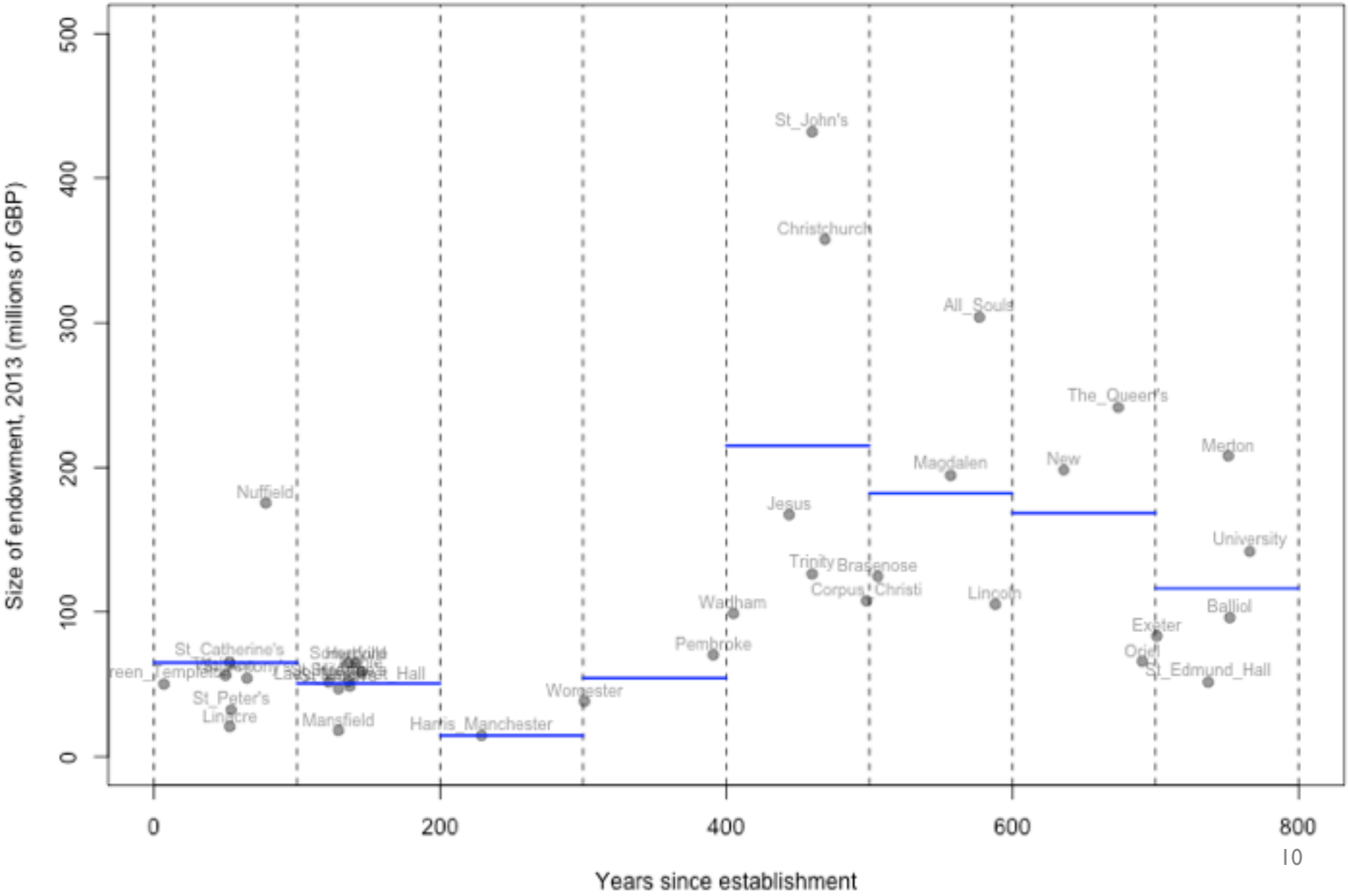
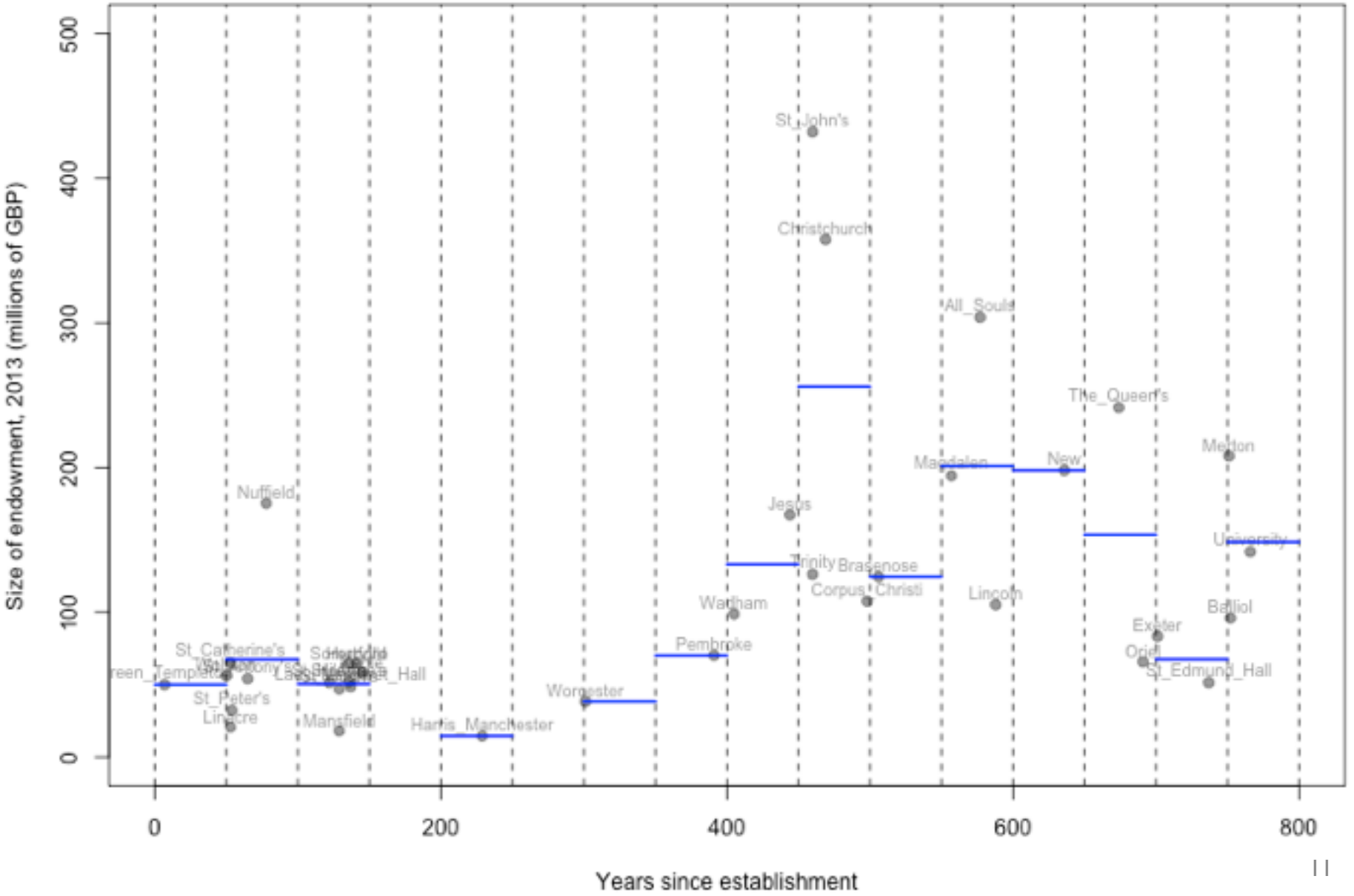
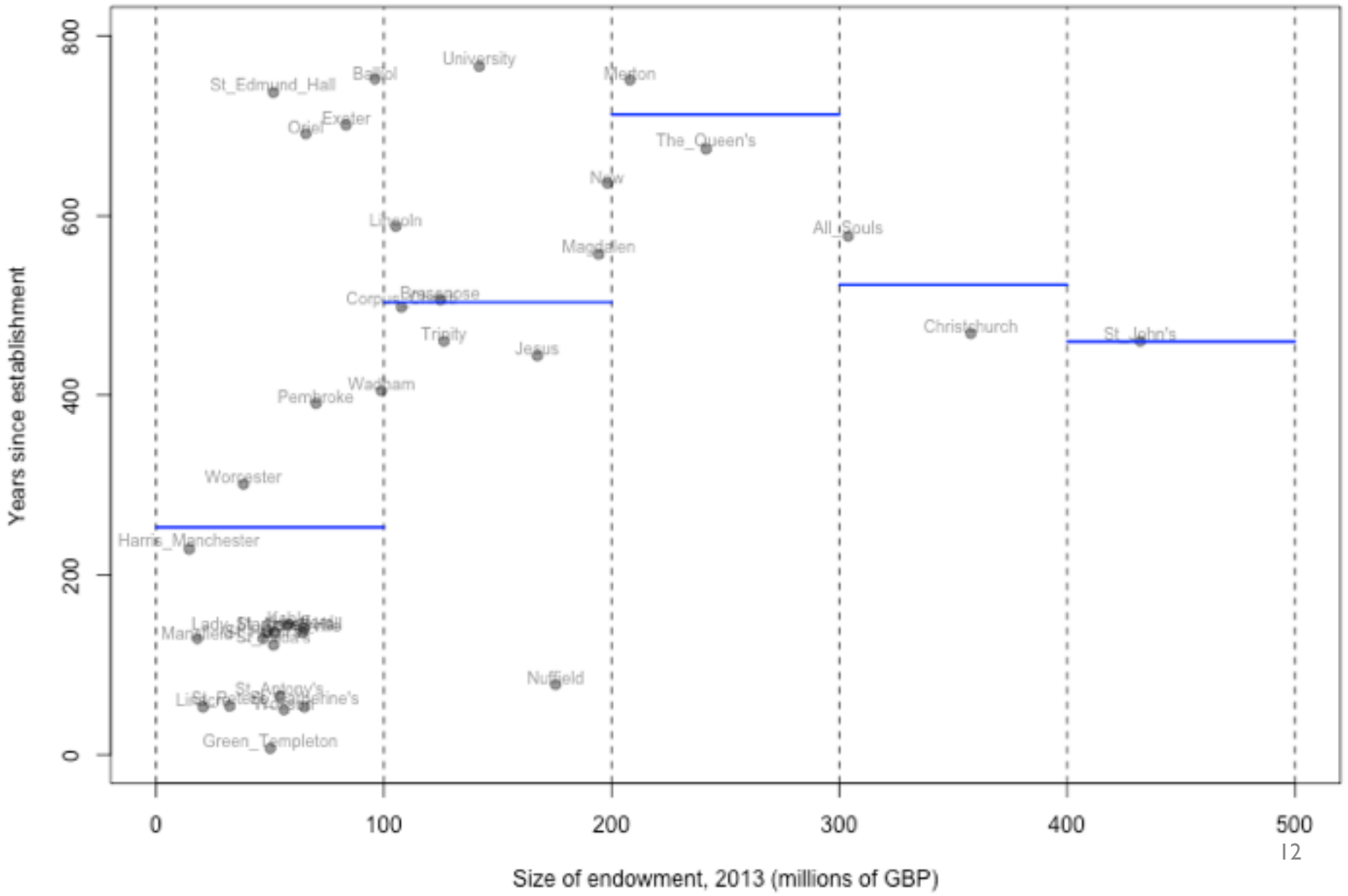


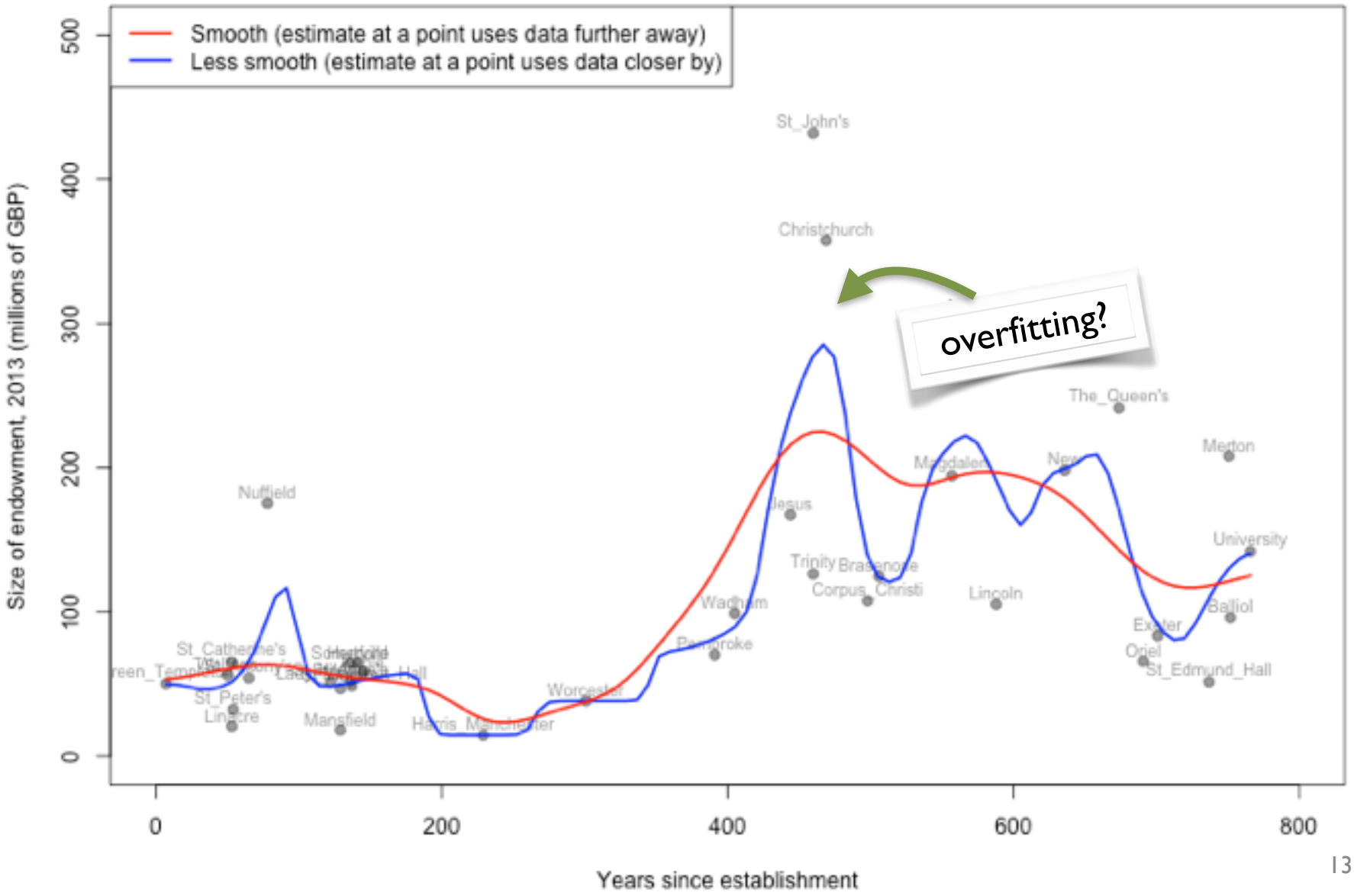
Mean endowment within 50-year intervals



Mean age within intervals of endowment



Kernel smoother: estimate at a point is (weighted) average of nearby points



Covariance, correlation, and regression

Covariance: a measure of linear association

How do x and y tend to move together, i.e. how do they **covary**?

When x is above its mean, is y also above its mean? By how much?

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"mean of y"

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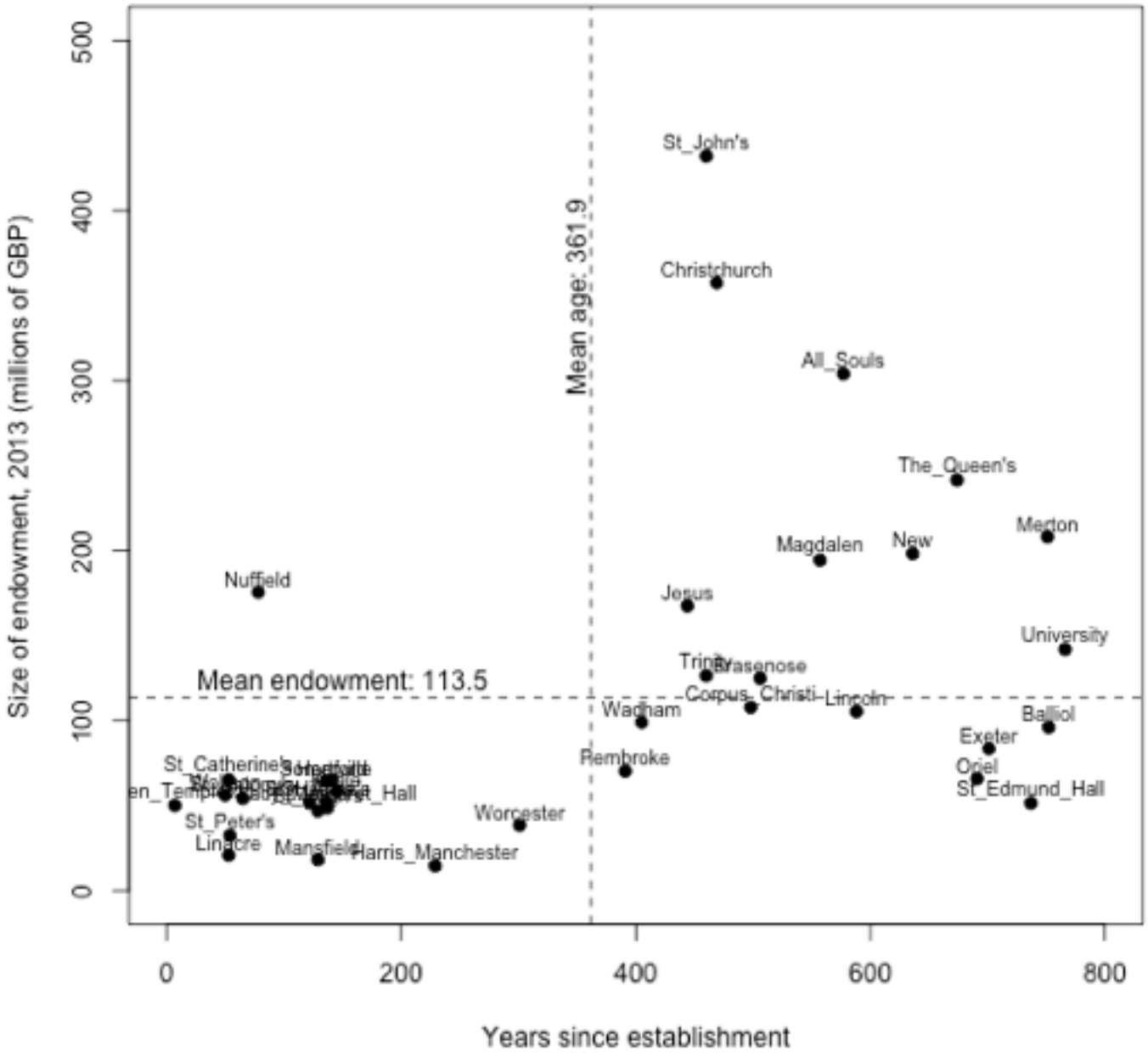

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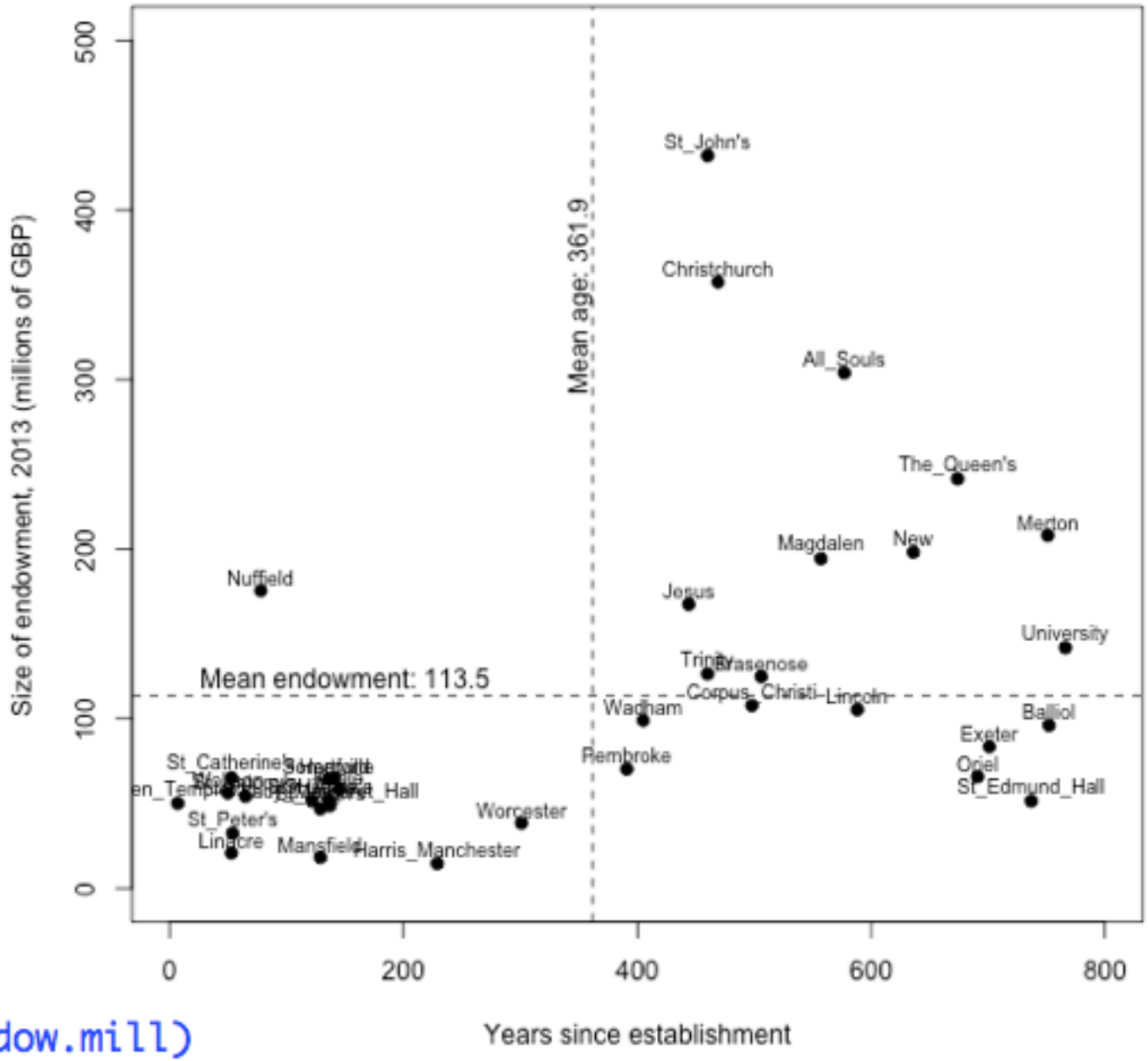


```
> cov(d$age, d$endow.mill)
[1] 11828.5
```

Correlation: a "scale-invariant" measure of linear association

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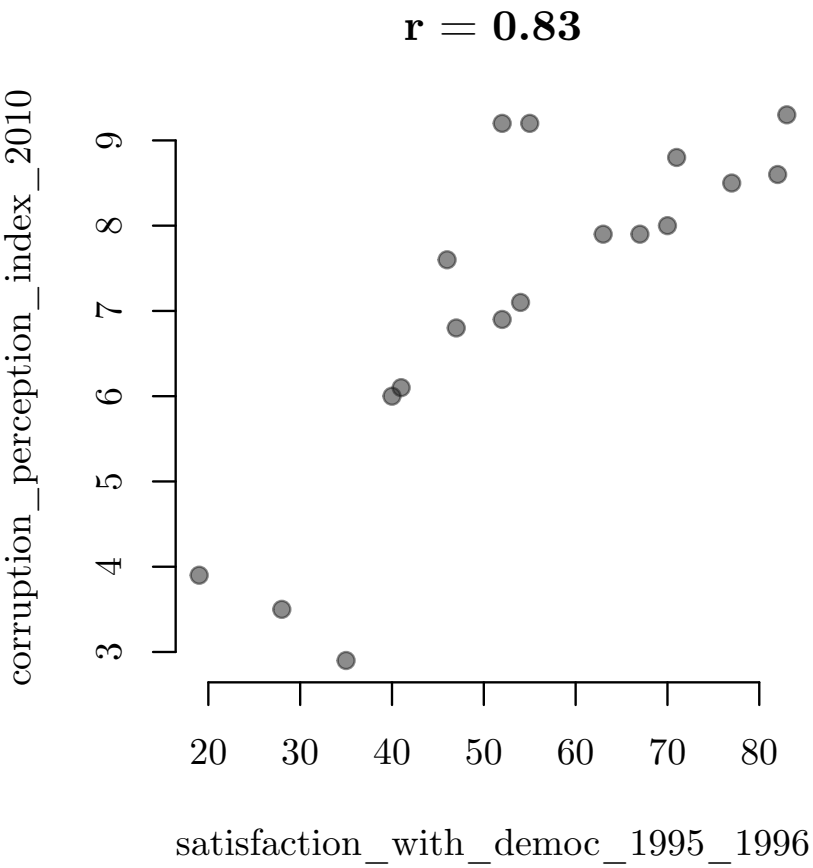

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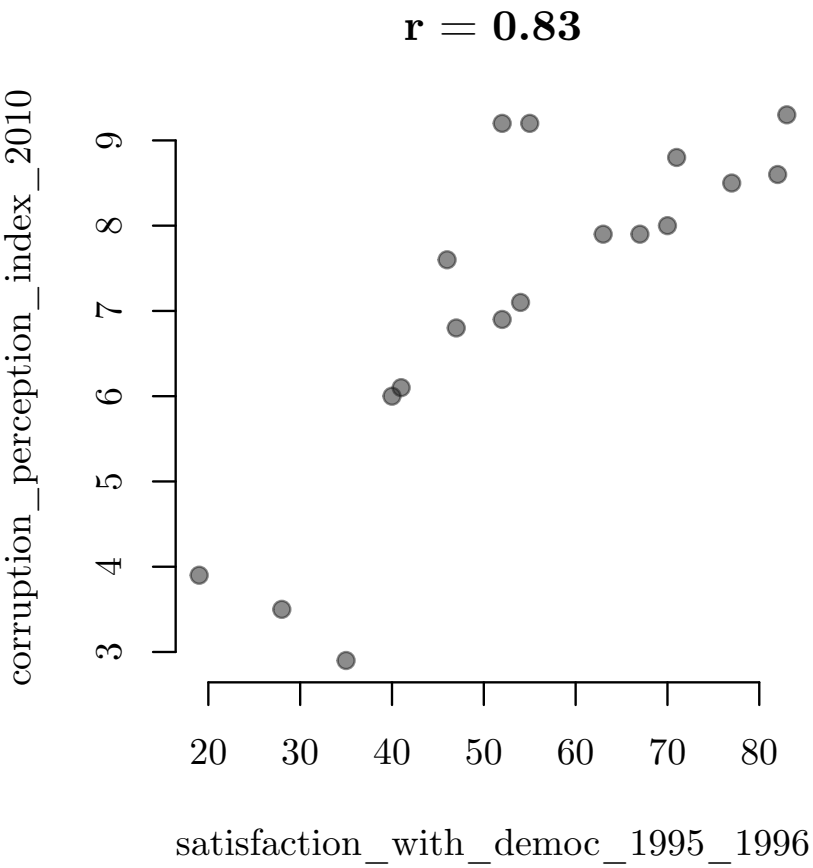

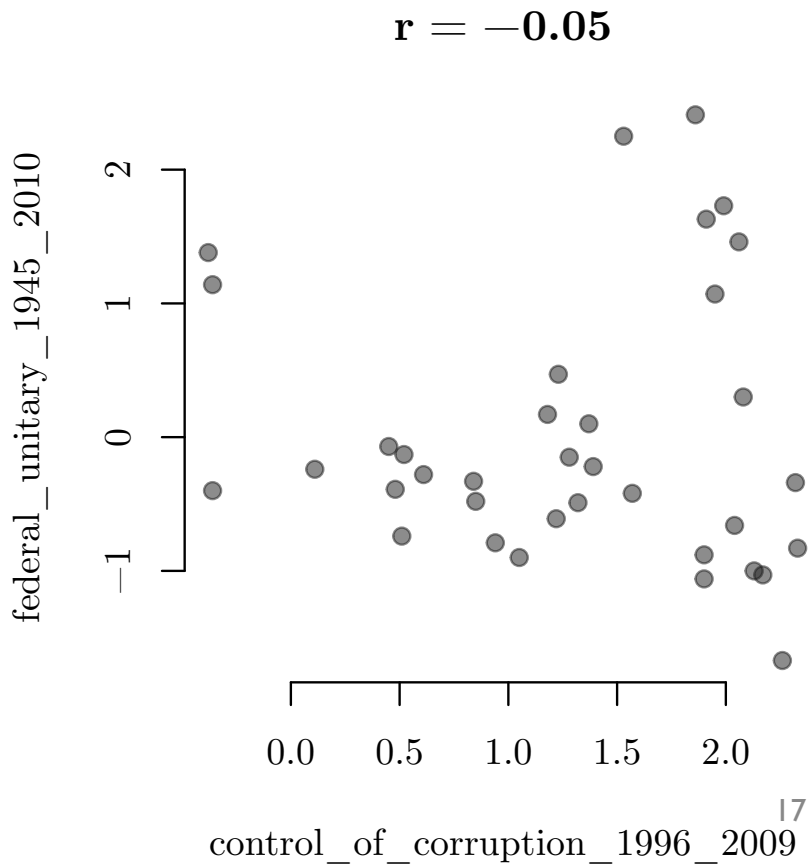

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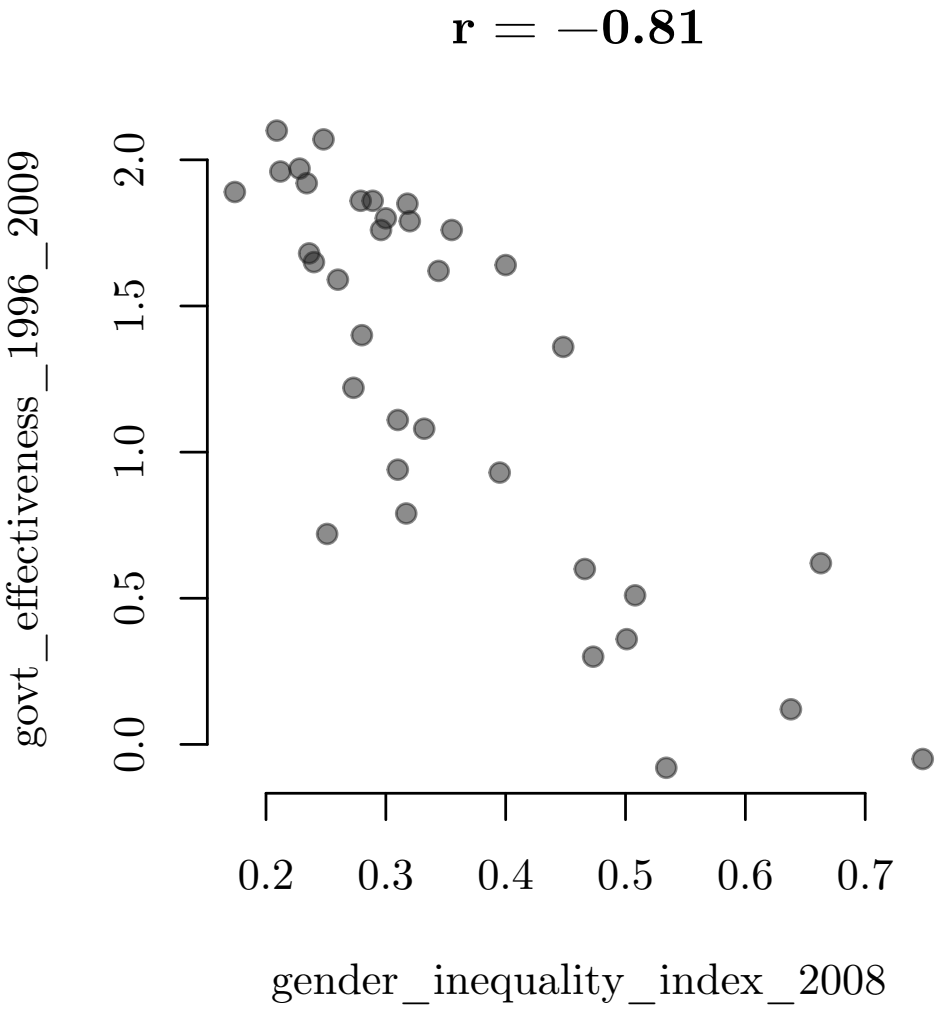

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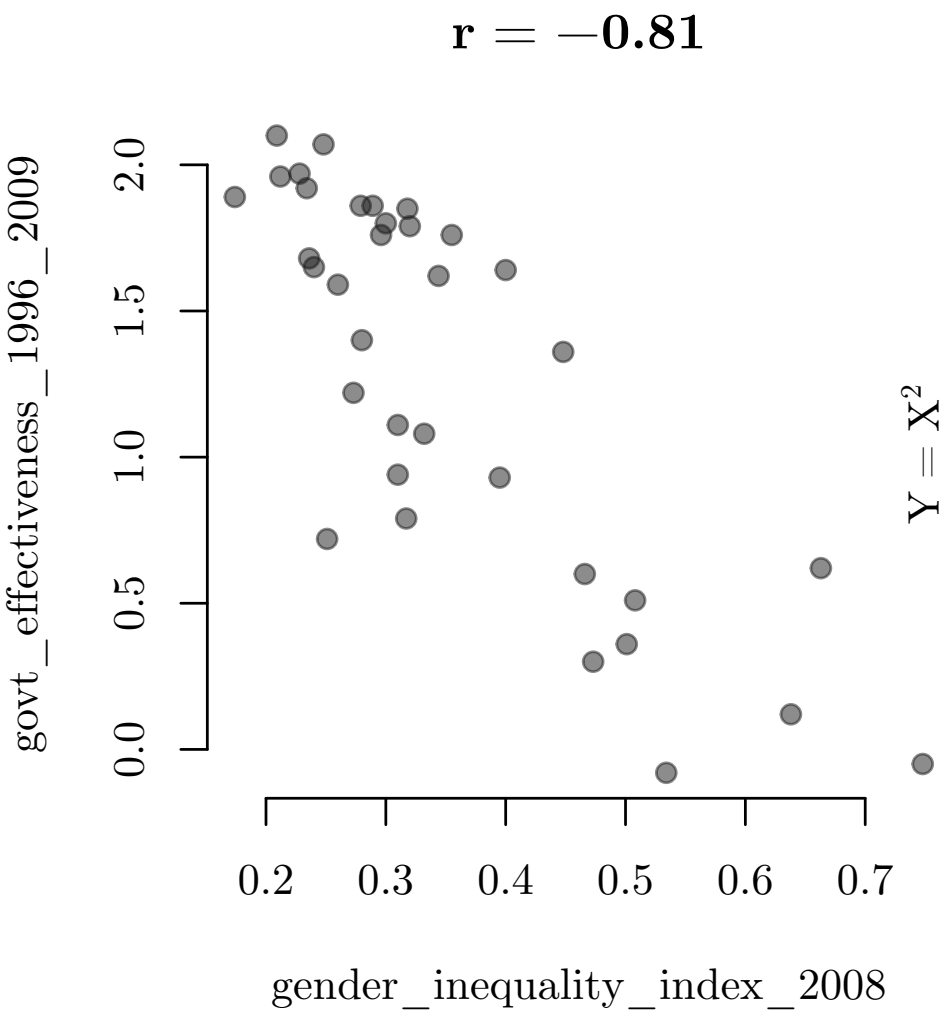

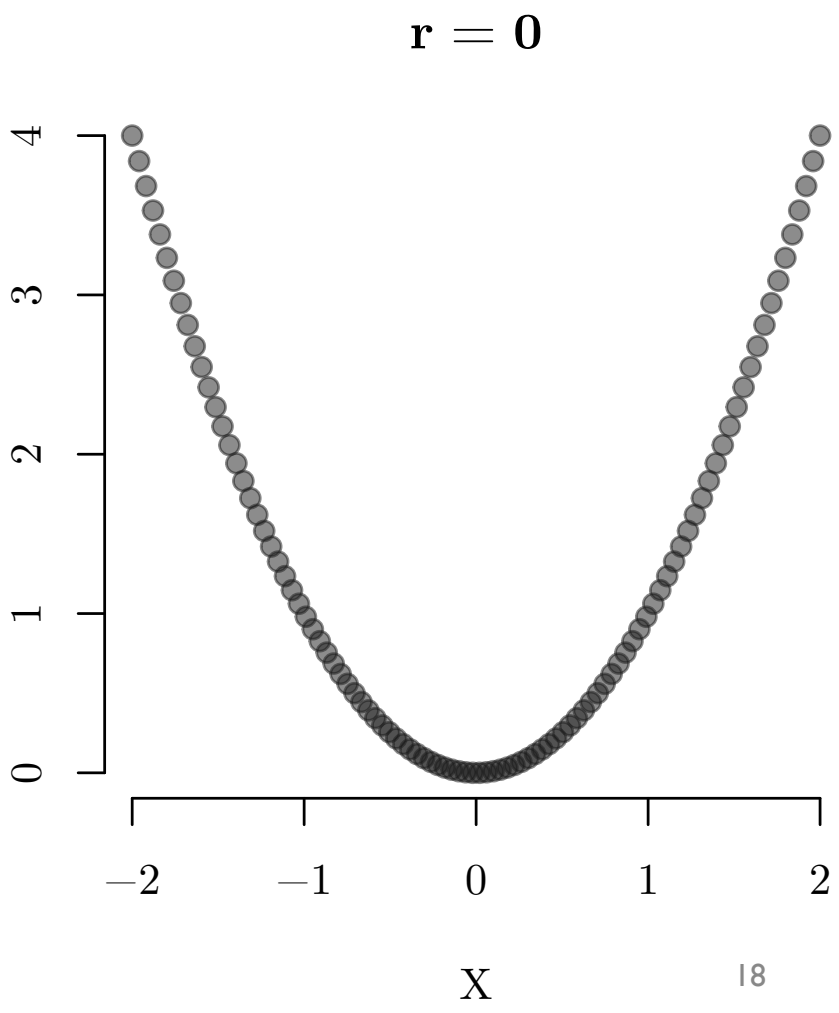

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"standard deviation of y"

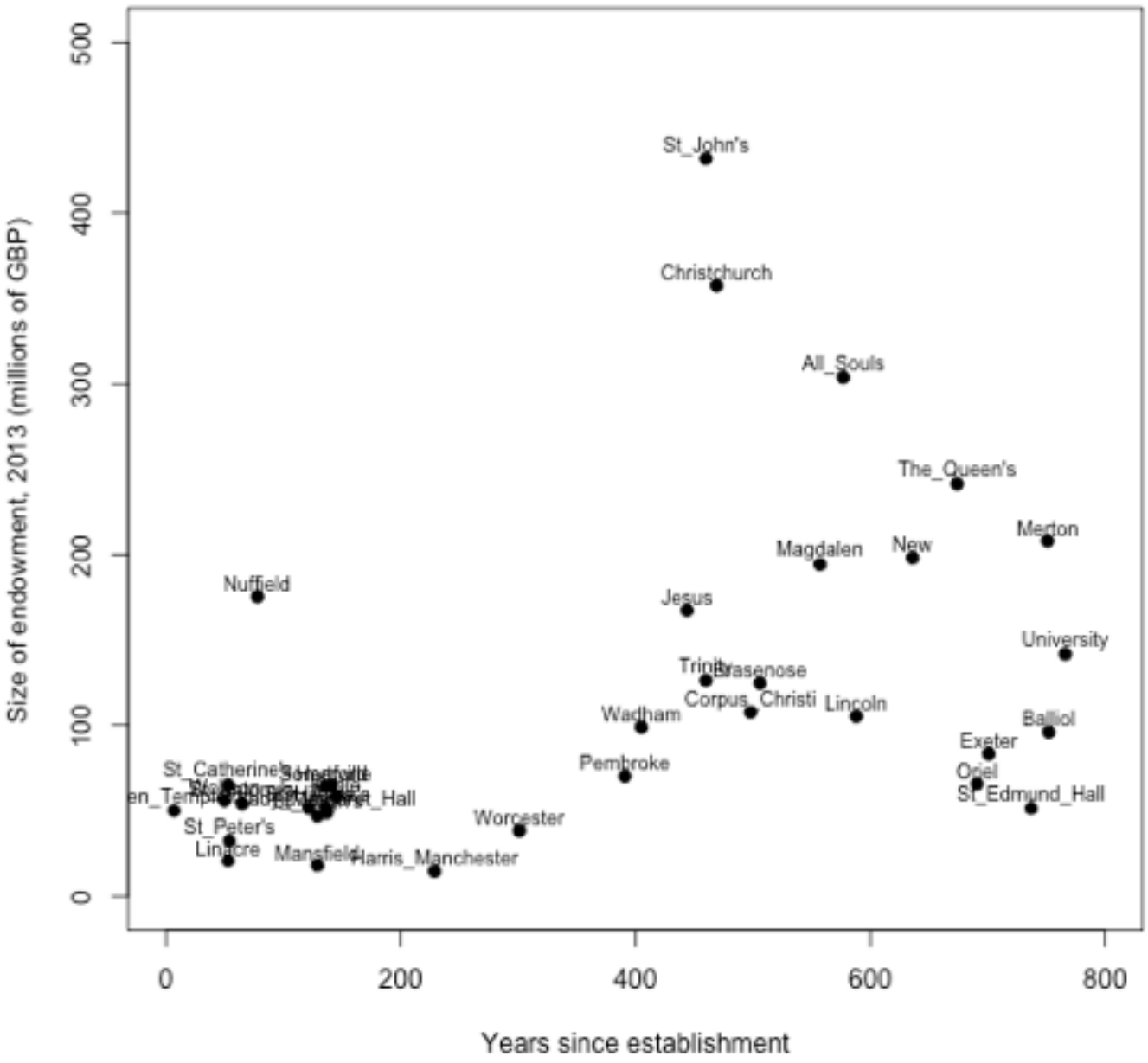
Two things to remember about correlation

- It is always between -1 and 1.
- Correlation ≠ causation.

Regression: linear prediction

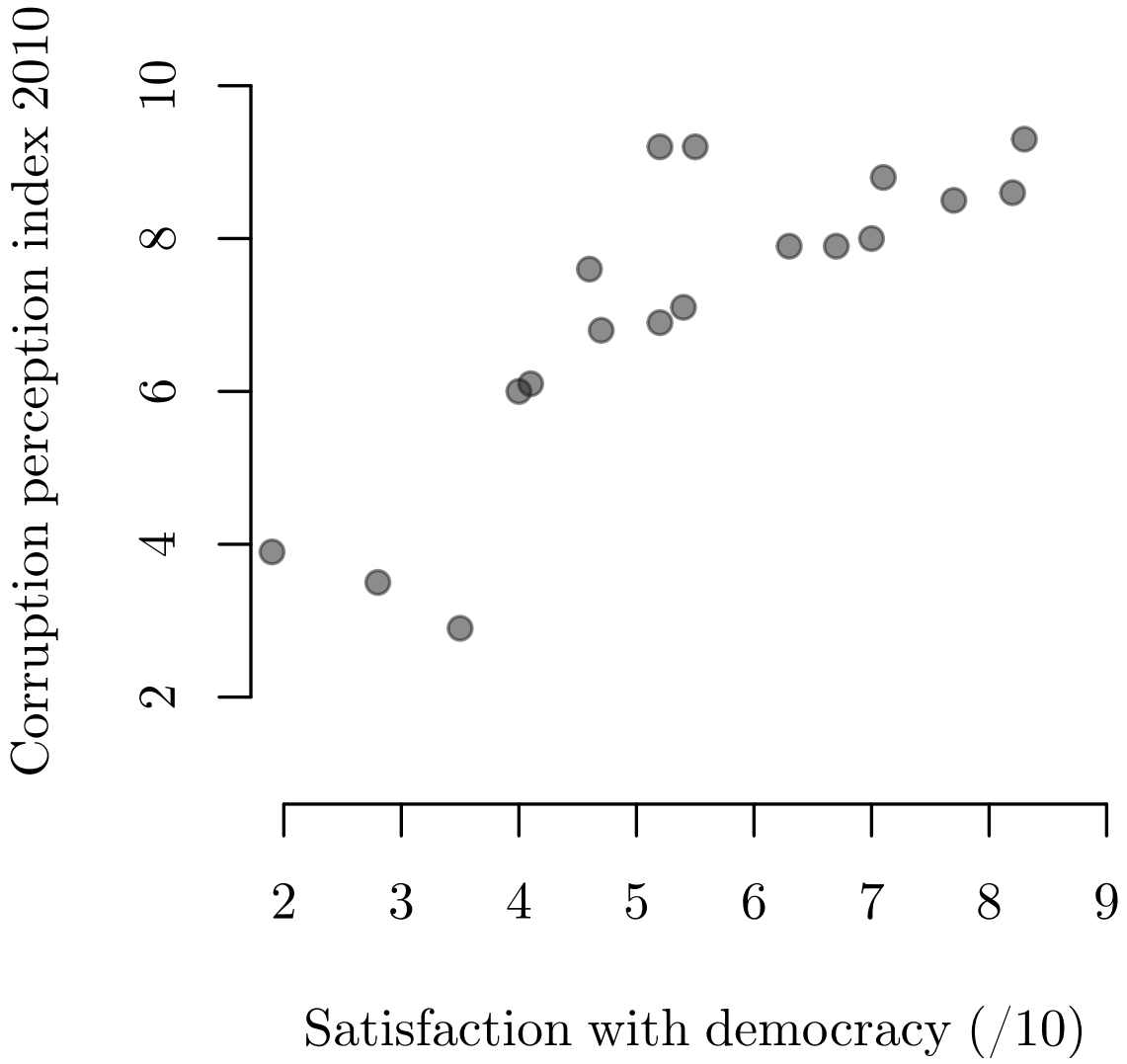
If you knew how old a college was, what would be your **best prediction** for the size of its endowment?

What about your best **linear** prediction?



Regression: linear prediction

If you knew the average satisfaction with democracy in a country in 1995-1996, what would be your **best linear prediction** for its corruption perception index in 2010?



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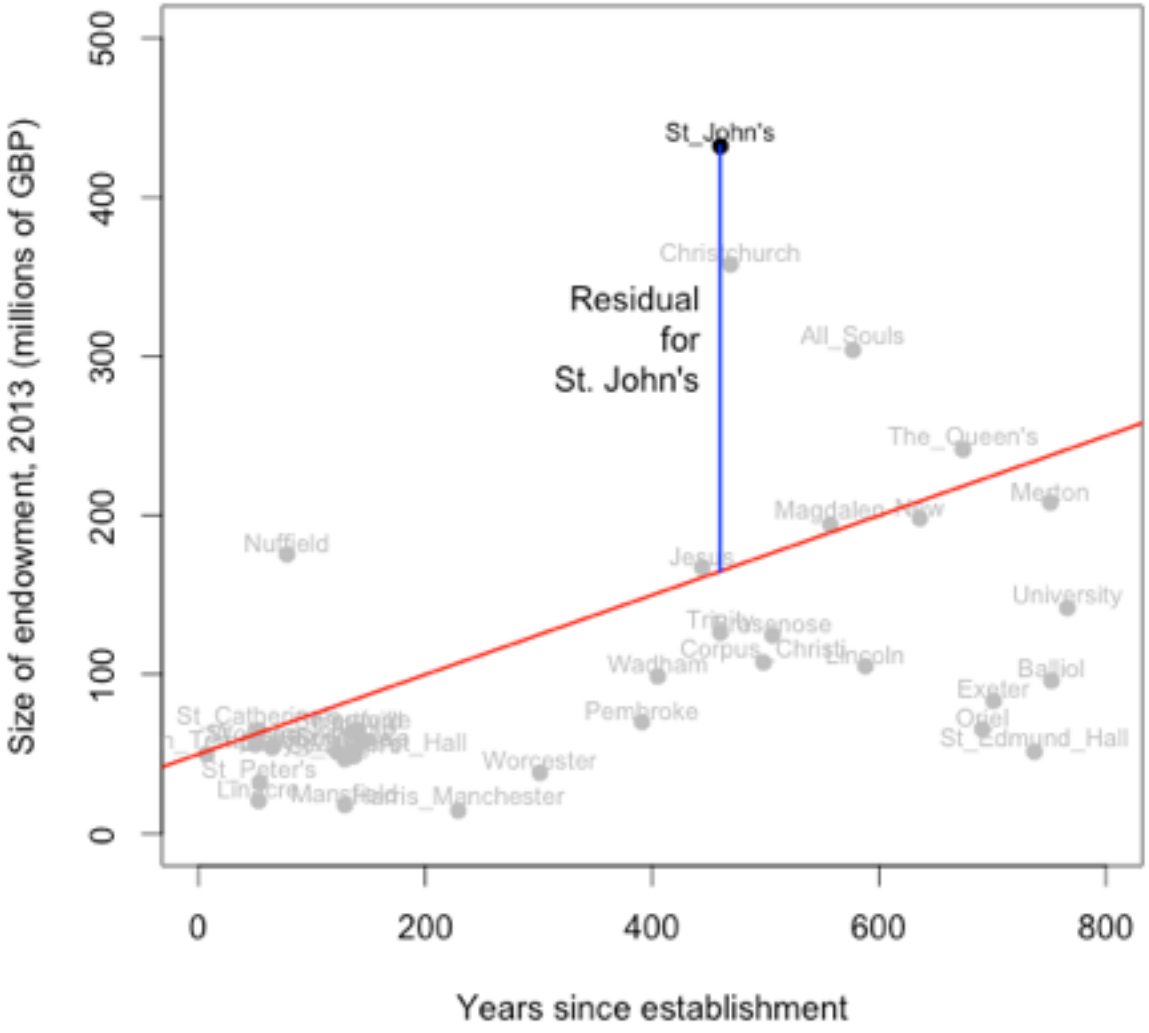

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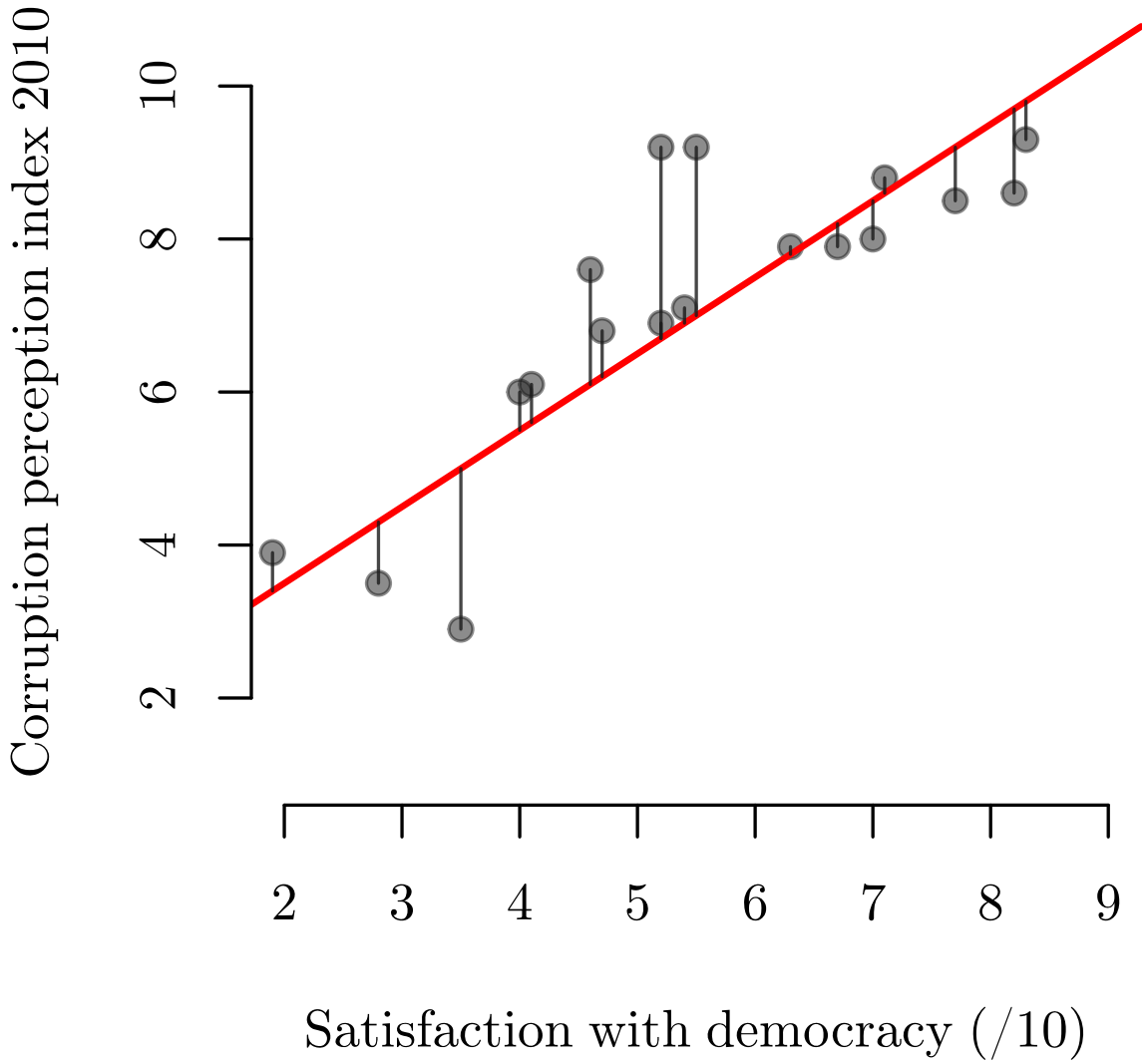

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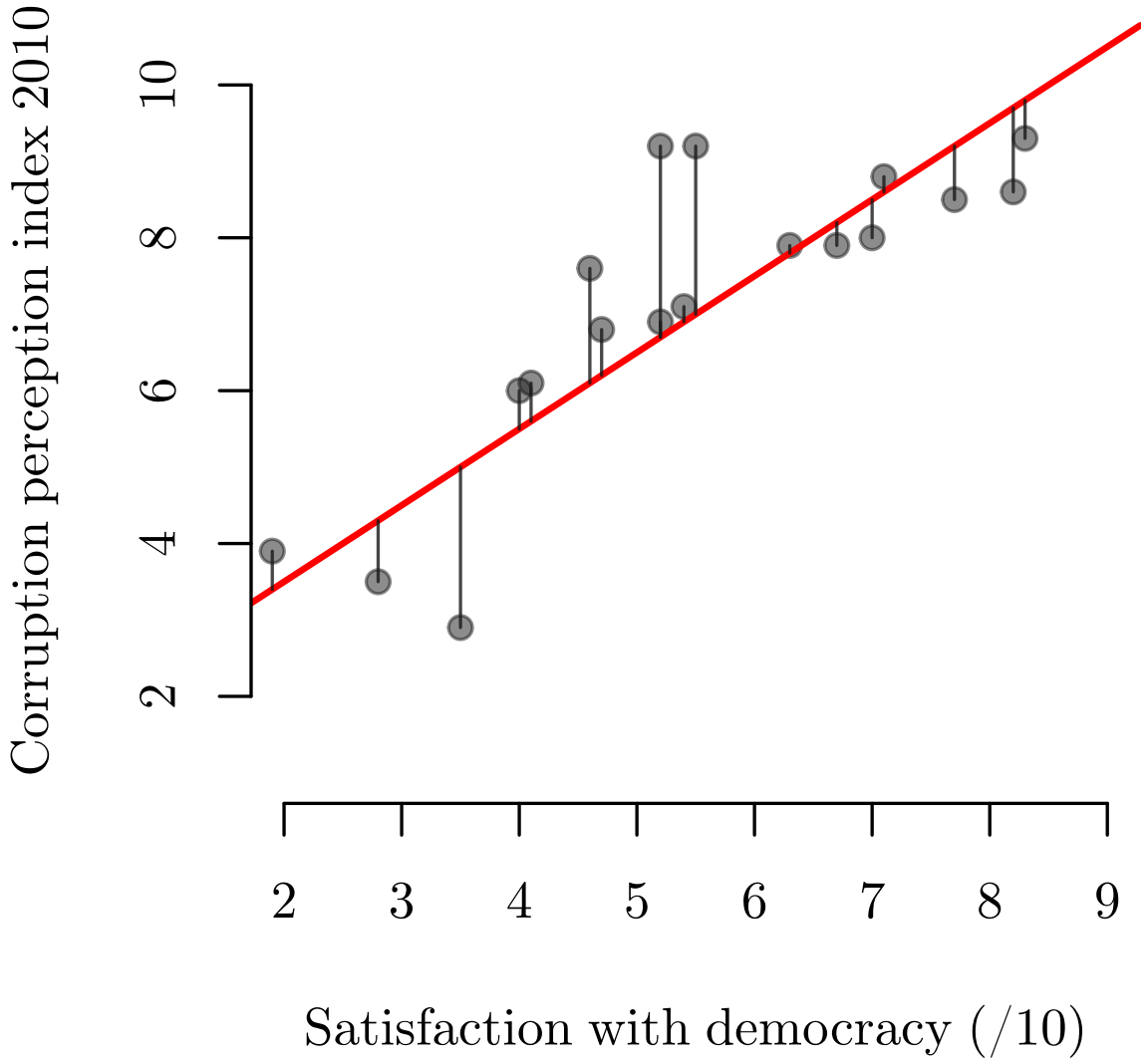

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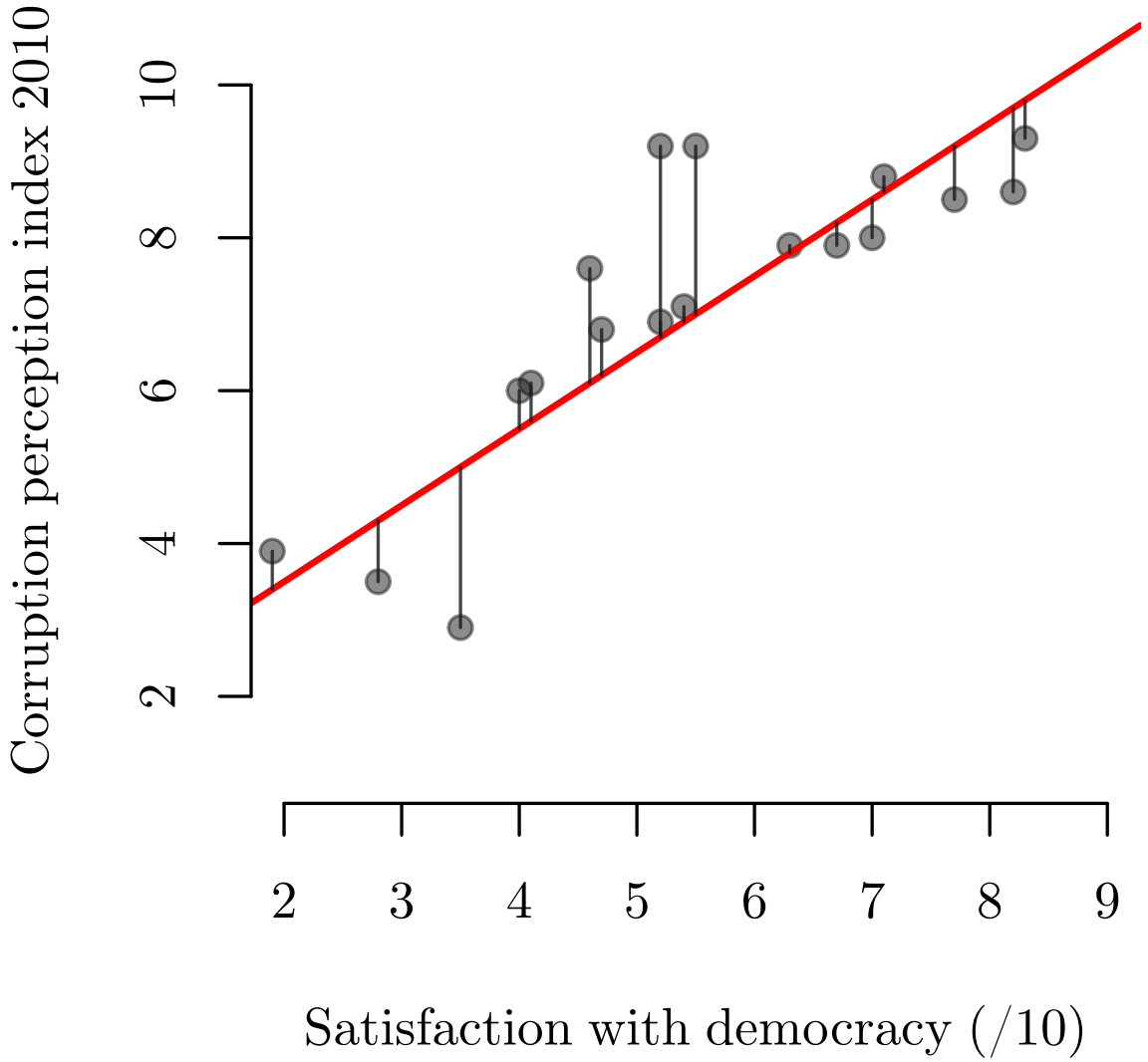

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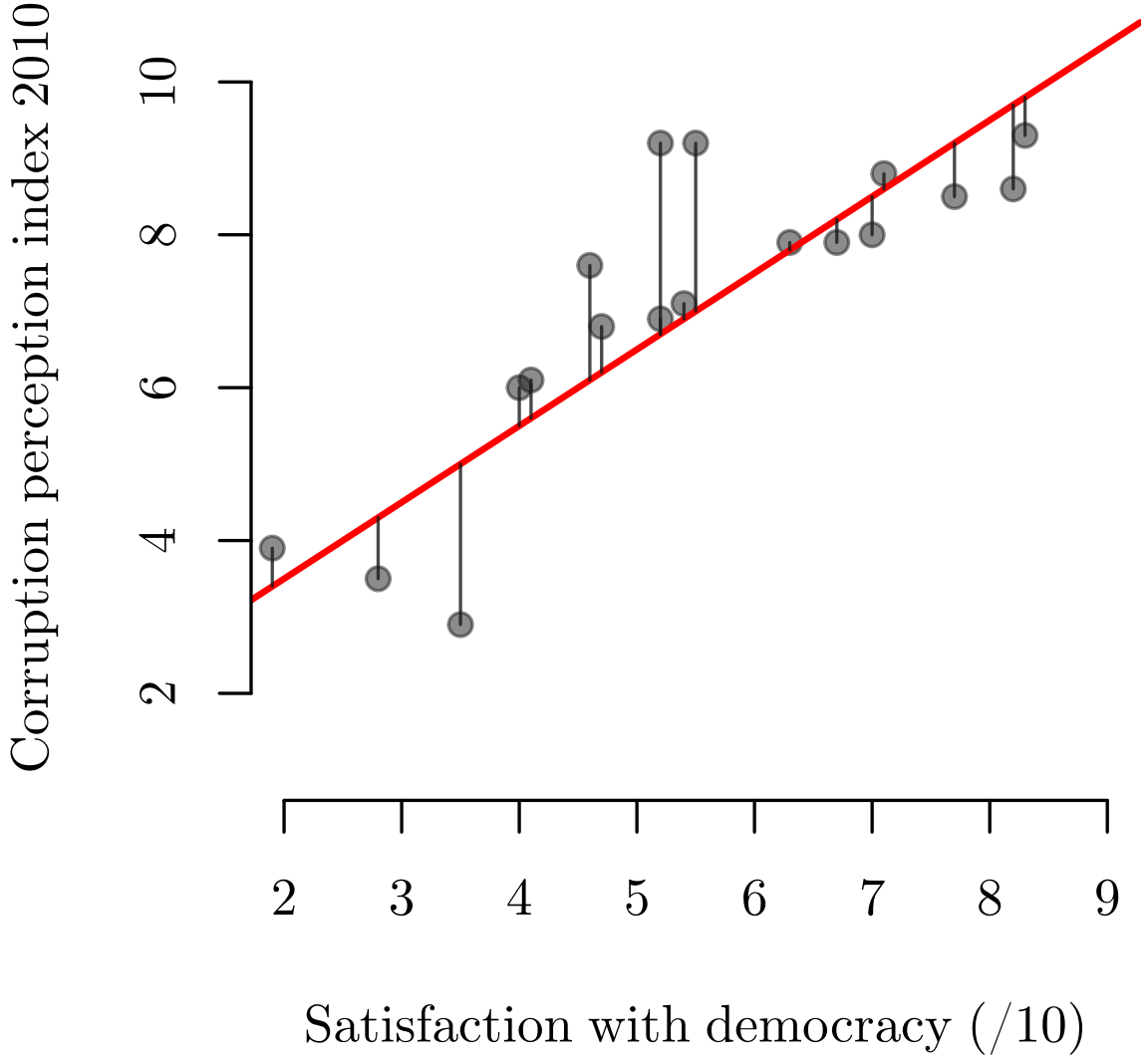

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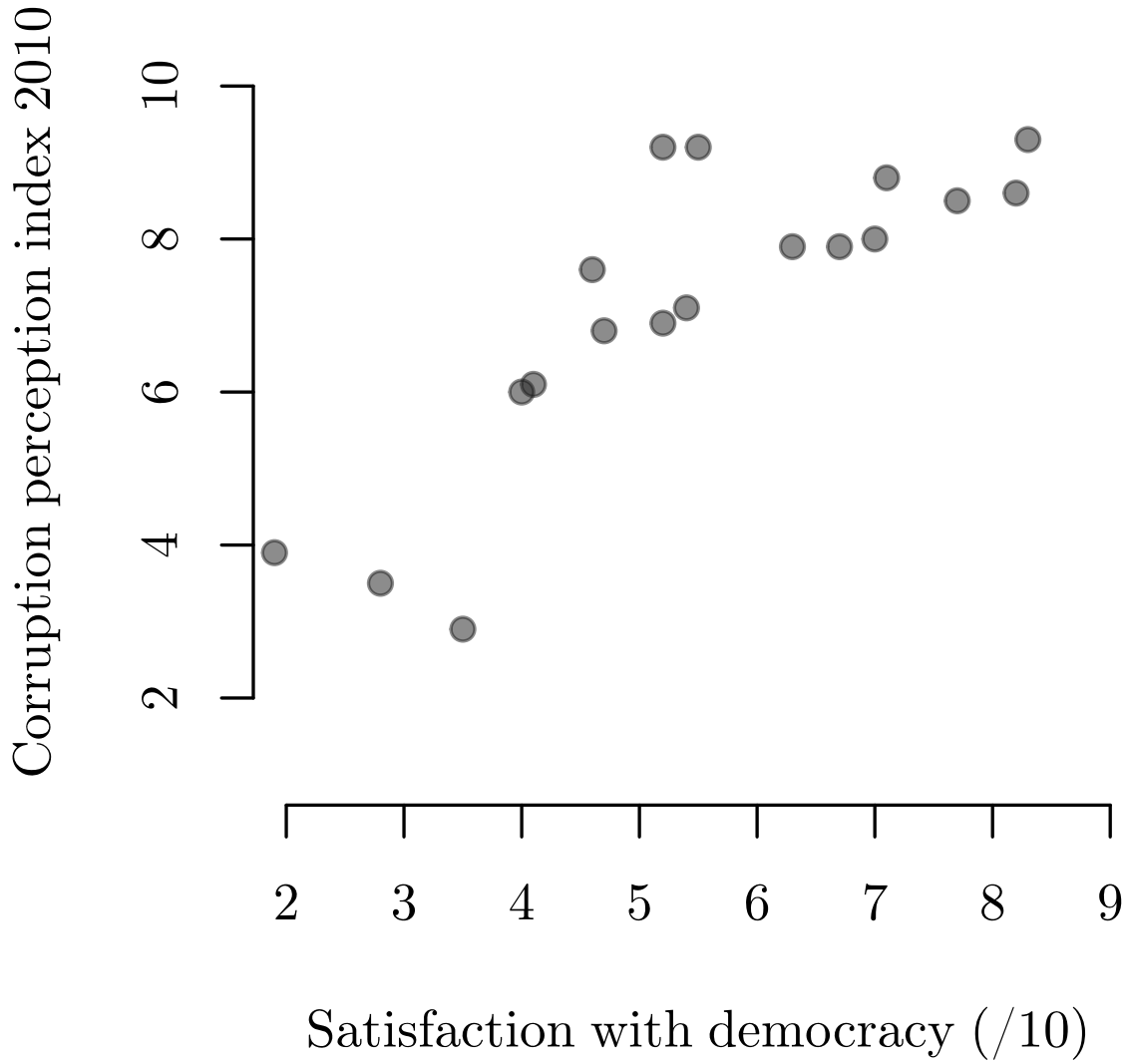

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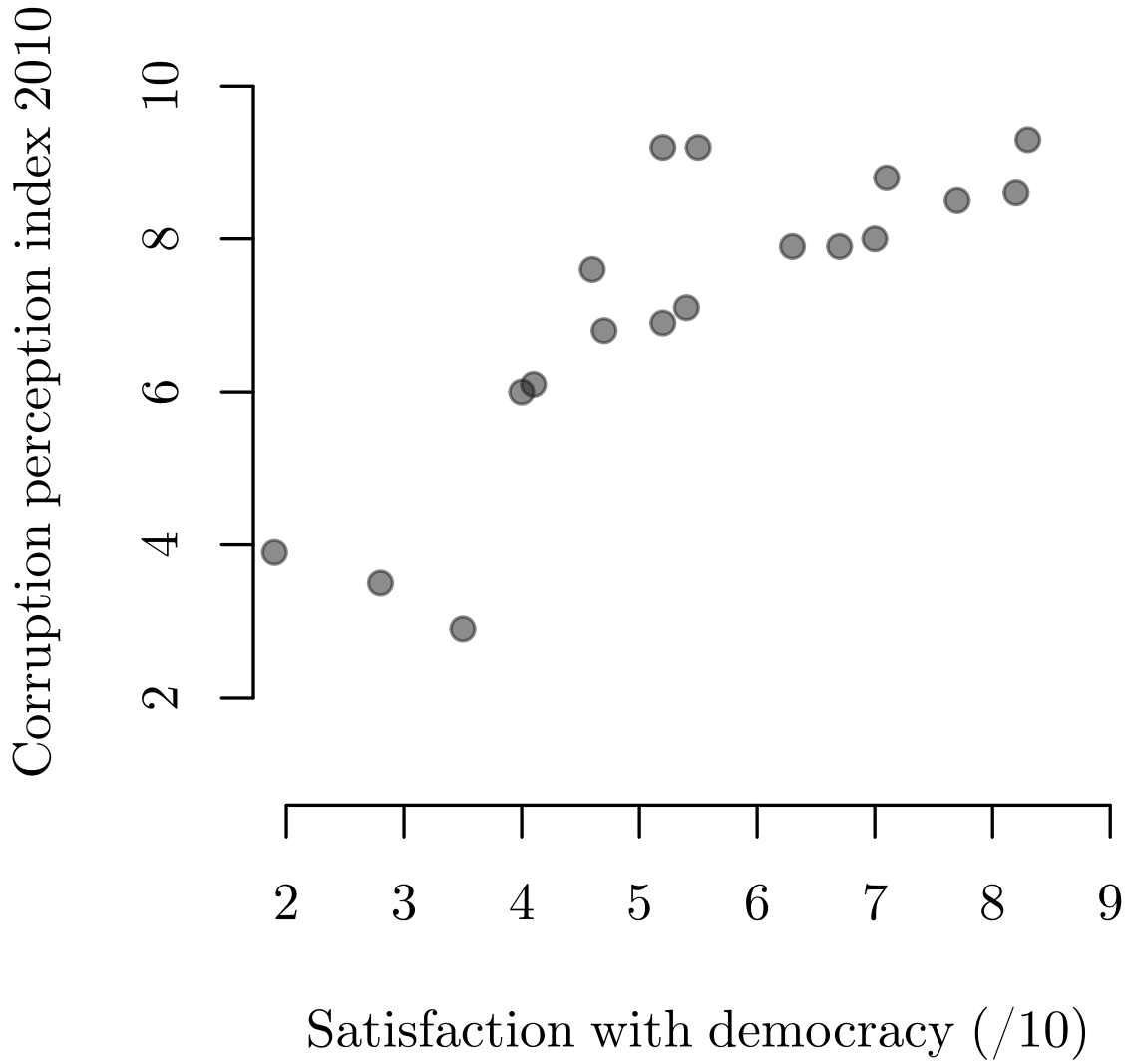
For each possible predictive line,

- calculate residuals
- square each residual
- add squared residuals

Choose the line that minimizes that sum.

OLS: Ordinary least squares



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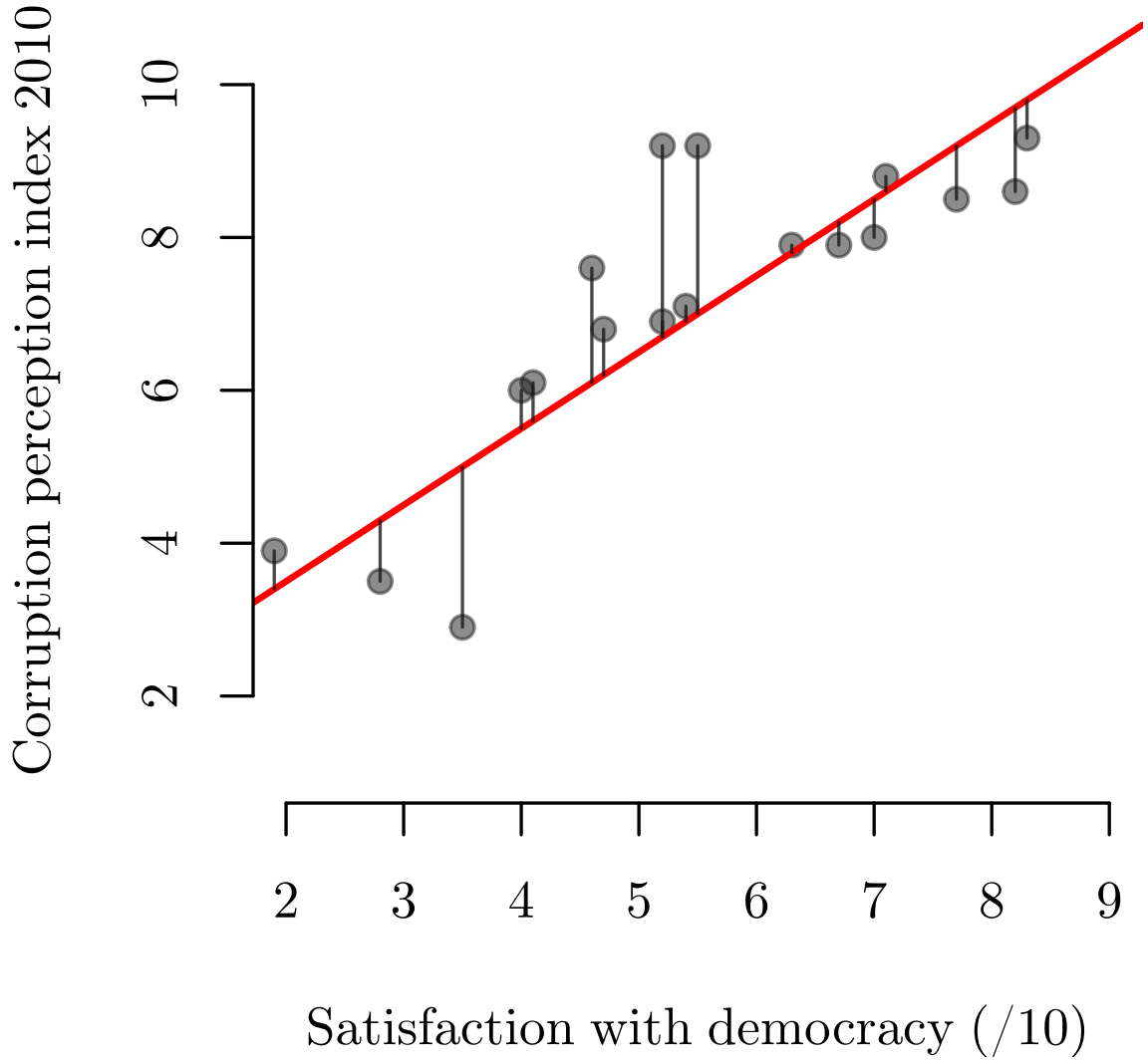

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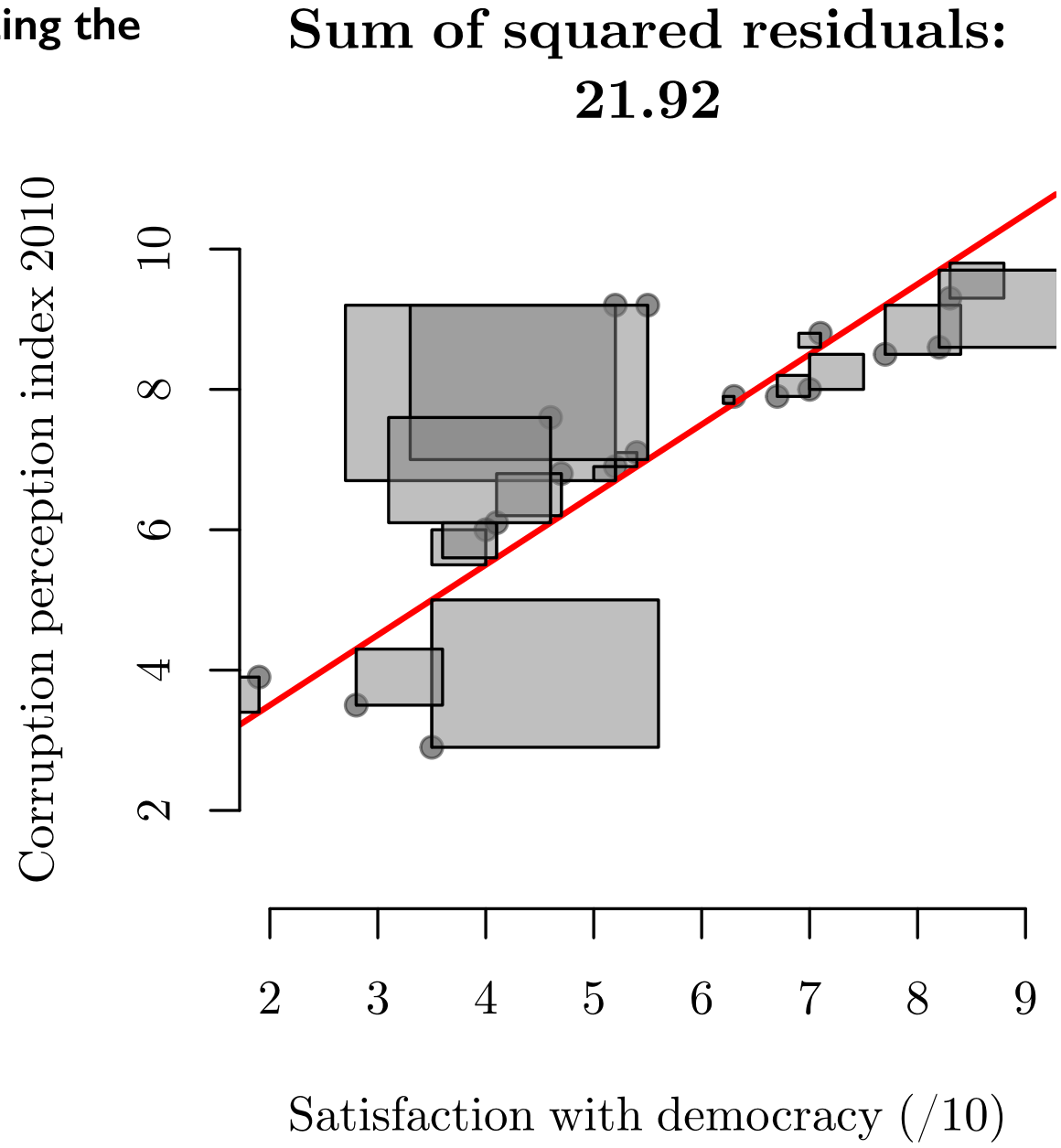

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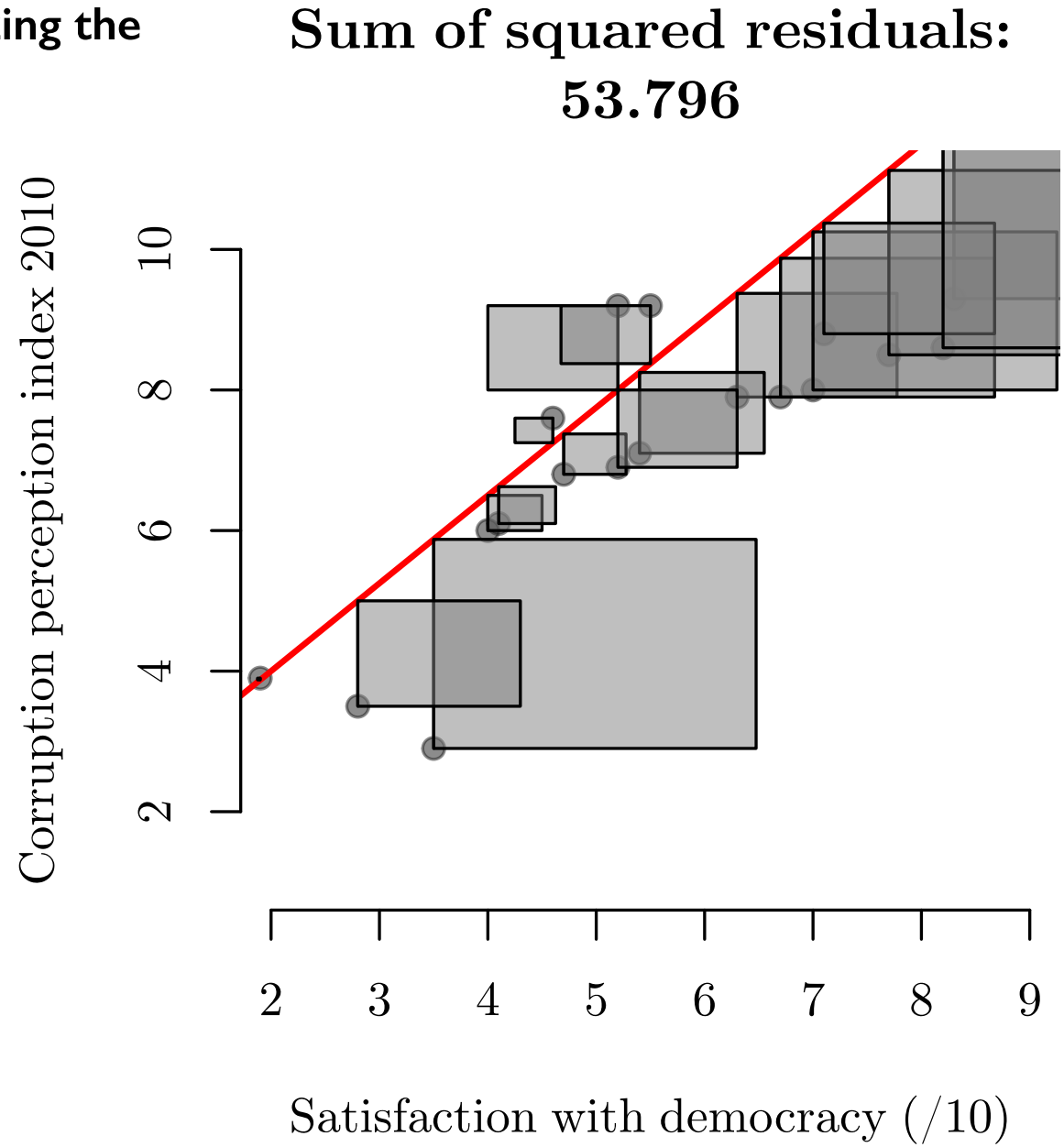

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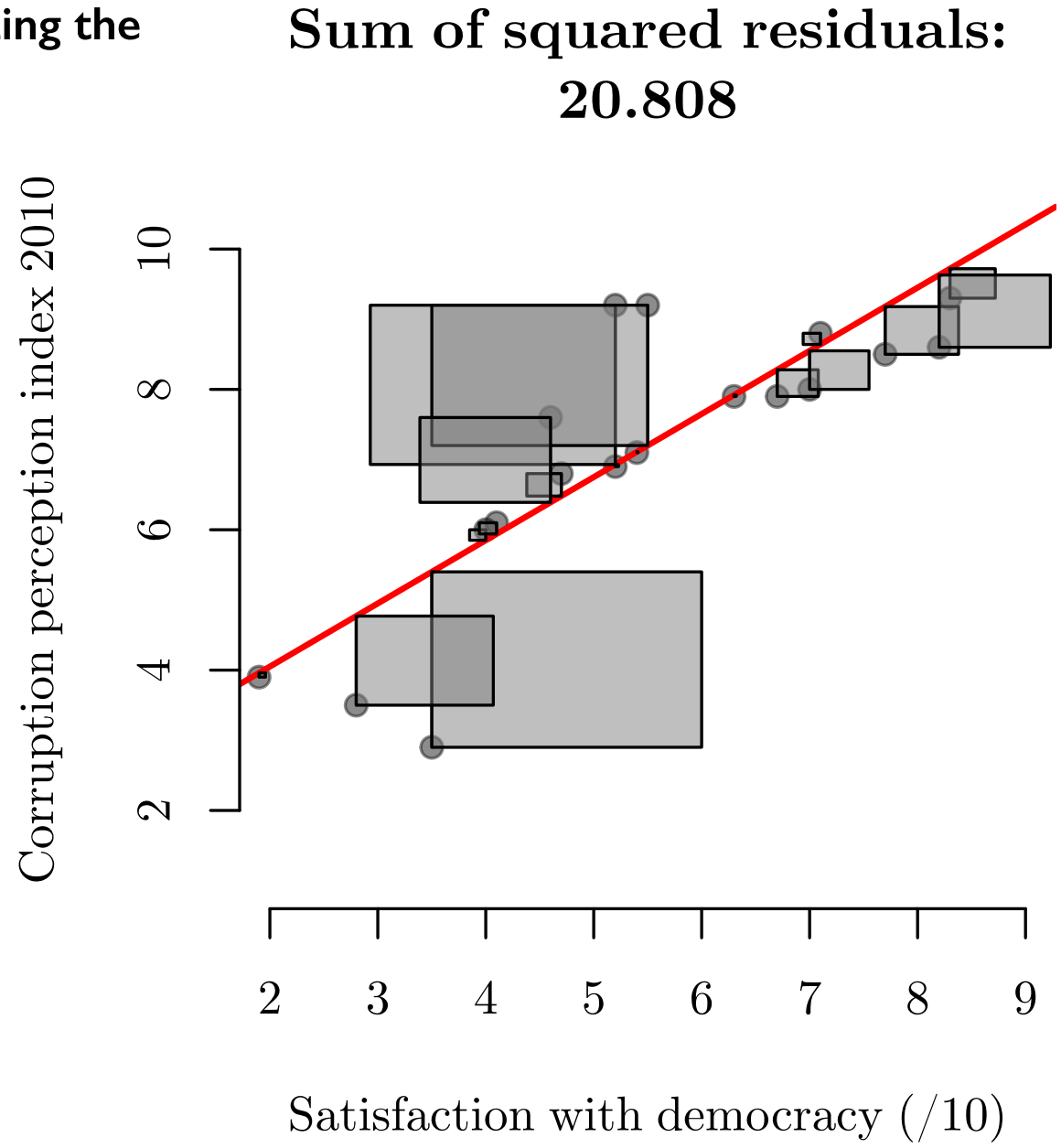

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```
> lm(D$corruption_perception_index_2010 ~ D$swd10)

Call:
lm(formula = D$corruption_perception_index_2010 ~ D$swd10)

Coefficients:
(Intercept)      D$swd10
      2.245        0.894
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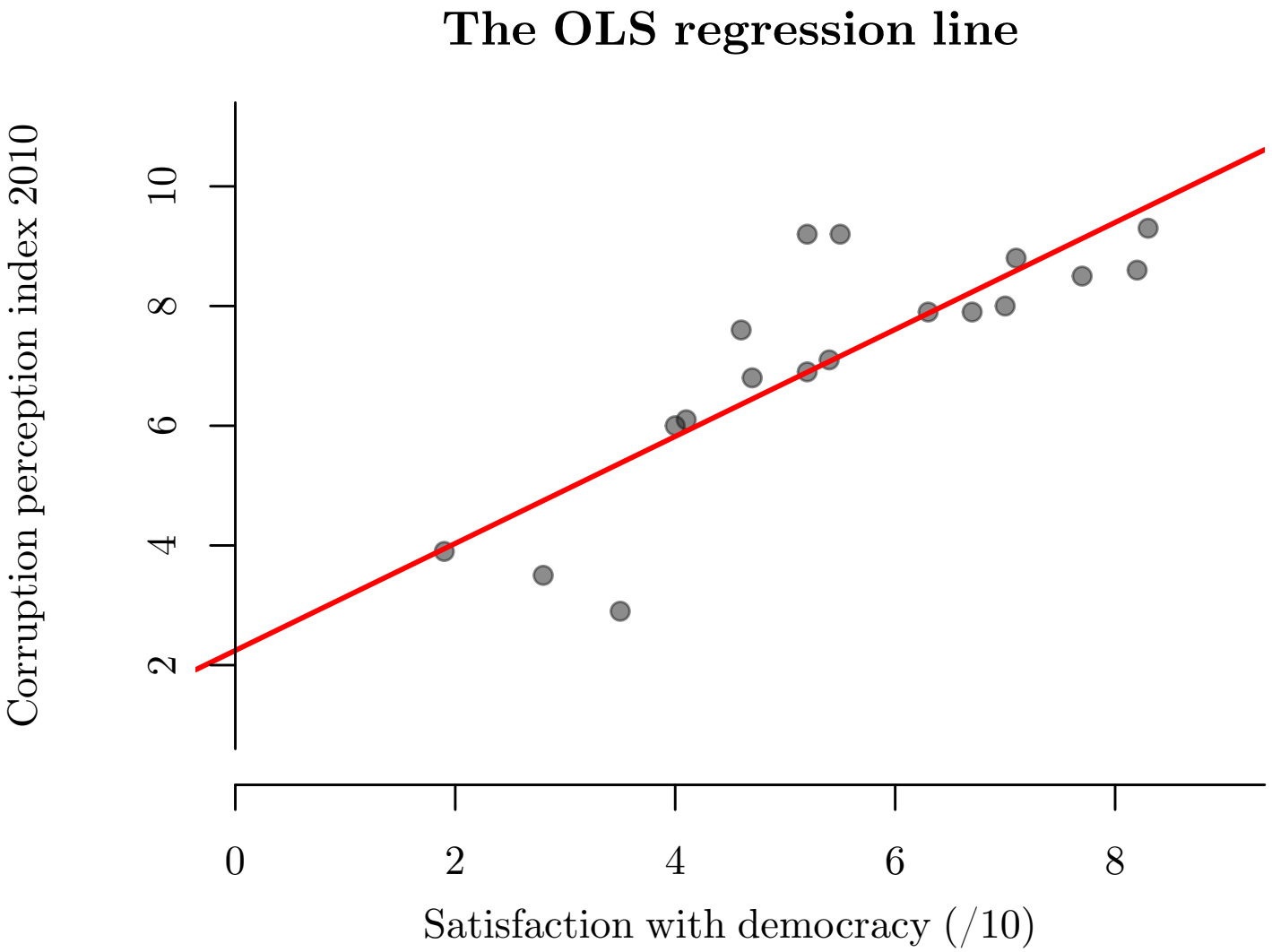
Intercept of regression line

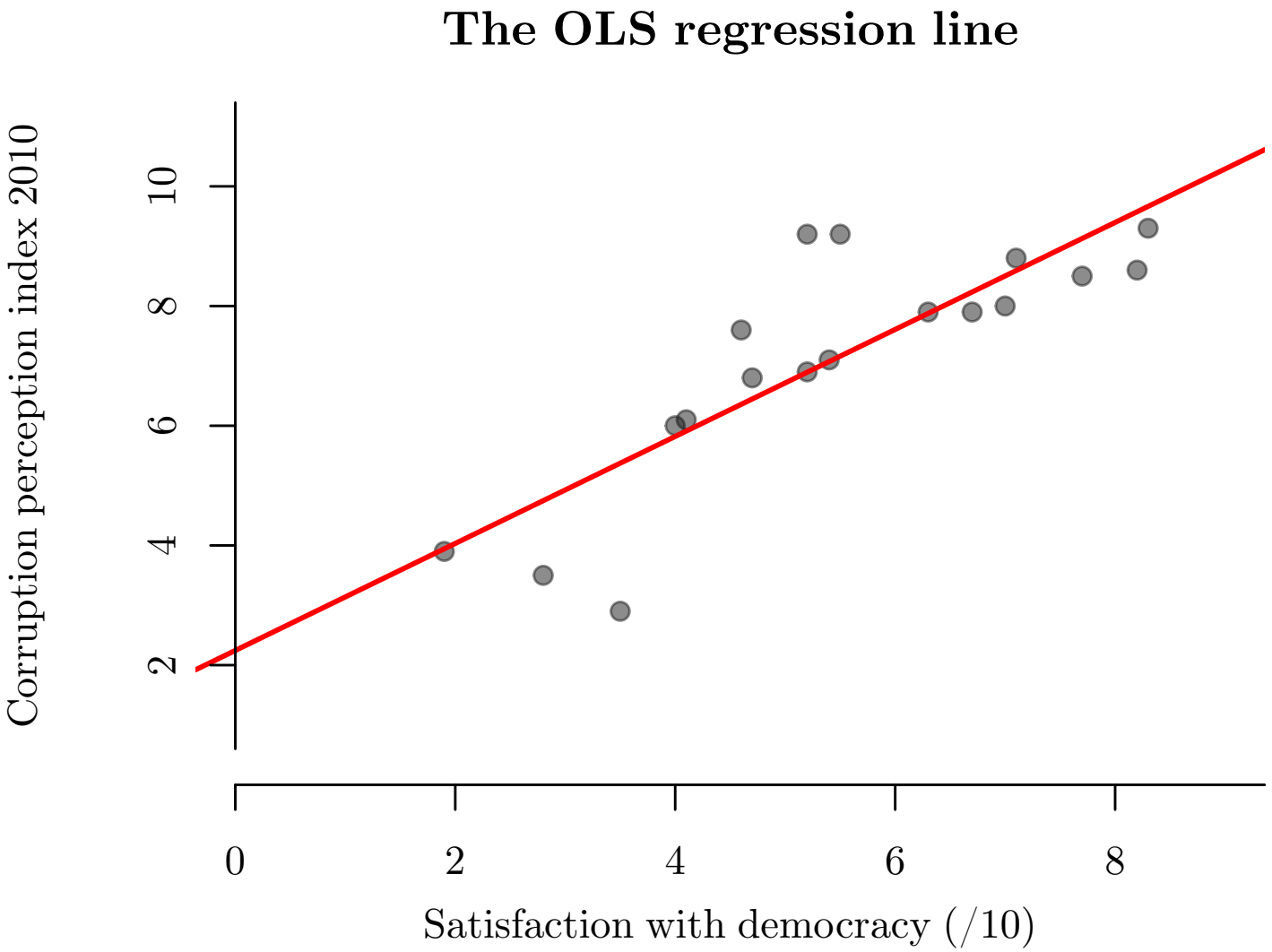
Slope (gradient) of regression line

Slope (gradient) and intercept: remember y = mx + c?



Slope (gradient) and intercept: remember y = mx + c?

```
Coefficients:
(Intercept)      D$swd10
      2.245        0.894
```



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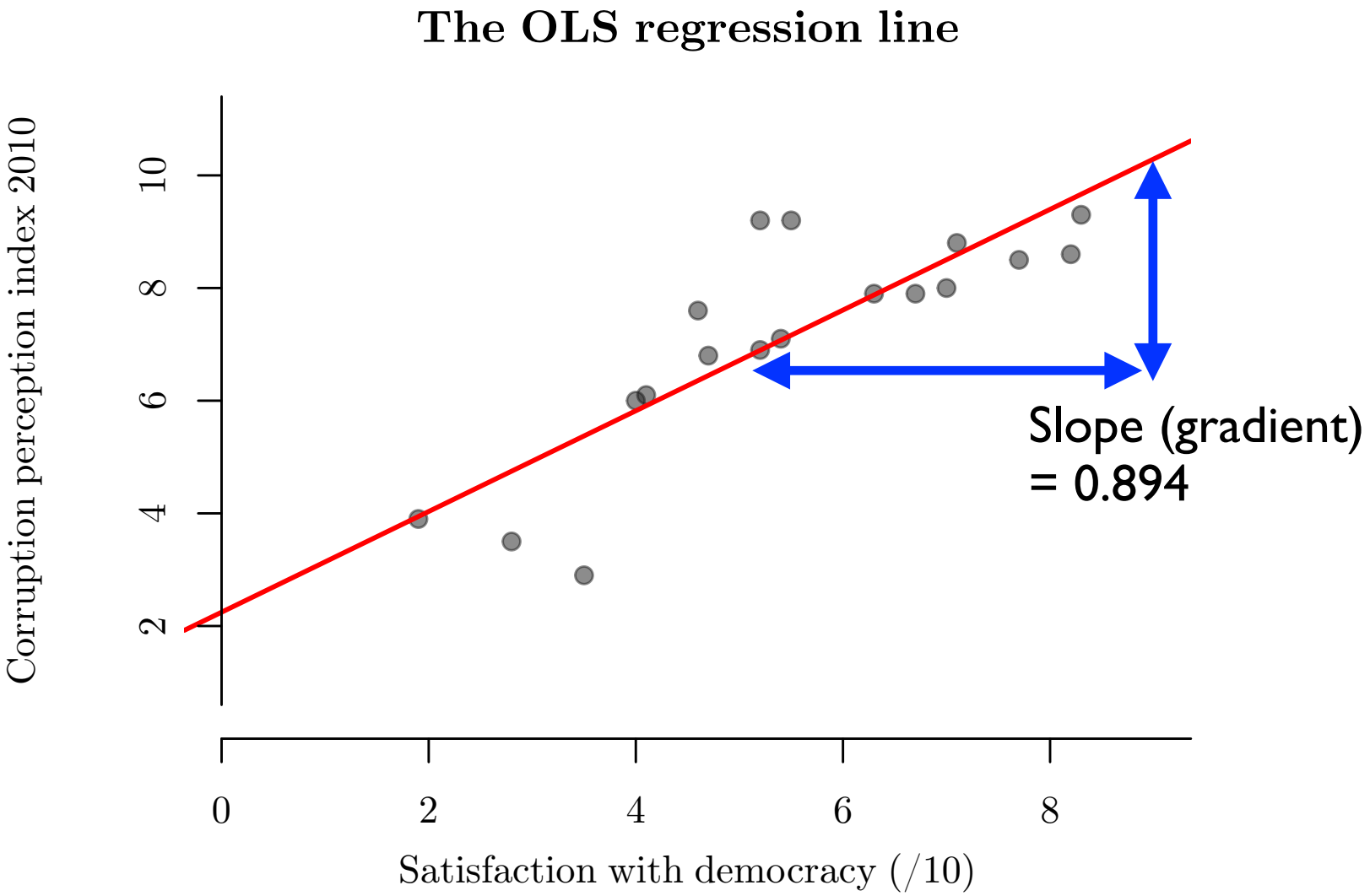

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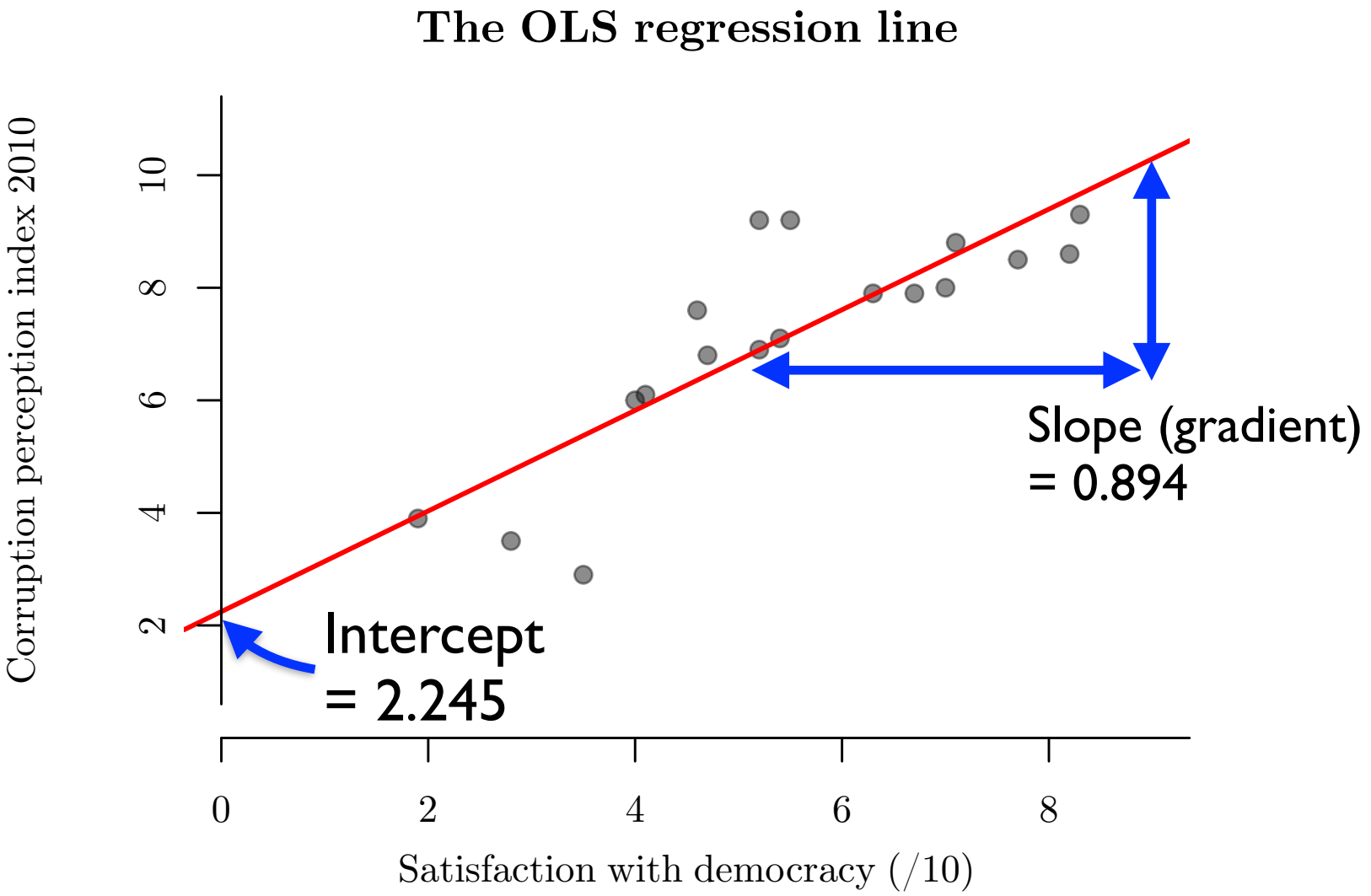


How well does our regression line predict the outcome? R^2

```
> summary(lm(D$corruption_perception_index_2010 ~ D$swd10))

Call:
lm(formula = D$corruption_perception_index_2010 ~ D$swd10)

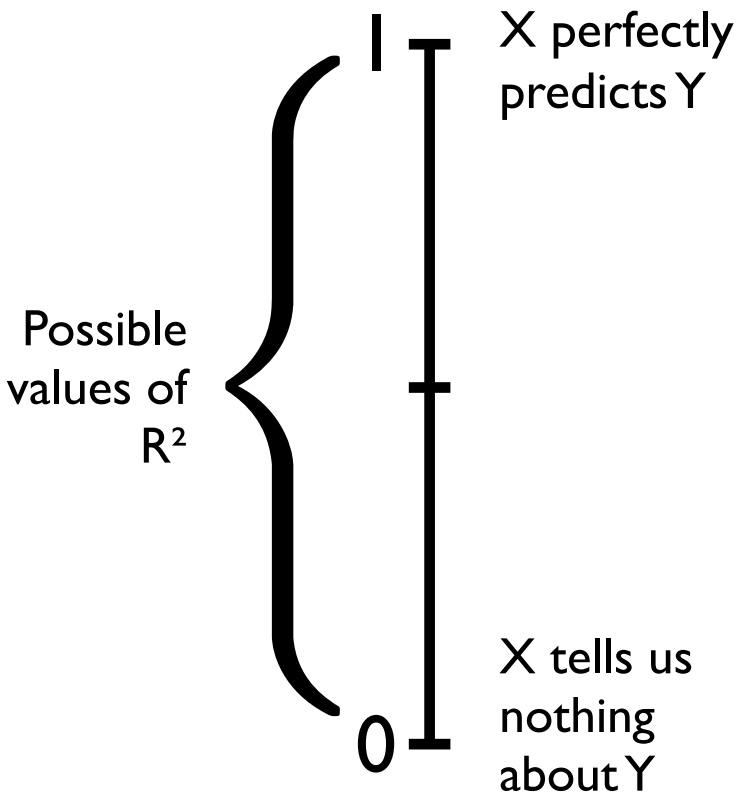
Residuals:
     Min       1Q   Median       3Q      Max
-2.47405 -0.46842  0.01456  0.20319  2.30623

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   2.2452     0.8667   2.590   0.0197 *
D$swd10       0.8940     0.1510   5.918 2.16e-05 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.14 on 16 degrees of freedom
  (18 observations deleted due to missingness)
Multiple R-squared:  0.6864,   Adjusted R-squared:  0.6668
F-statistic: 35.03 on 1 and 16 DF,  p-value: 2.162e-05
```

R^2: intuition

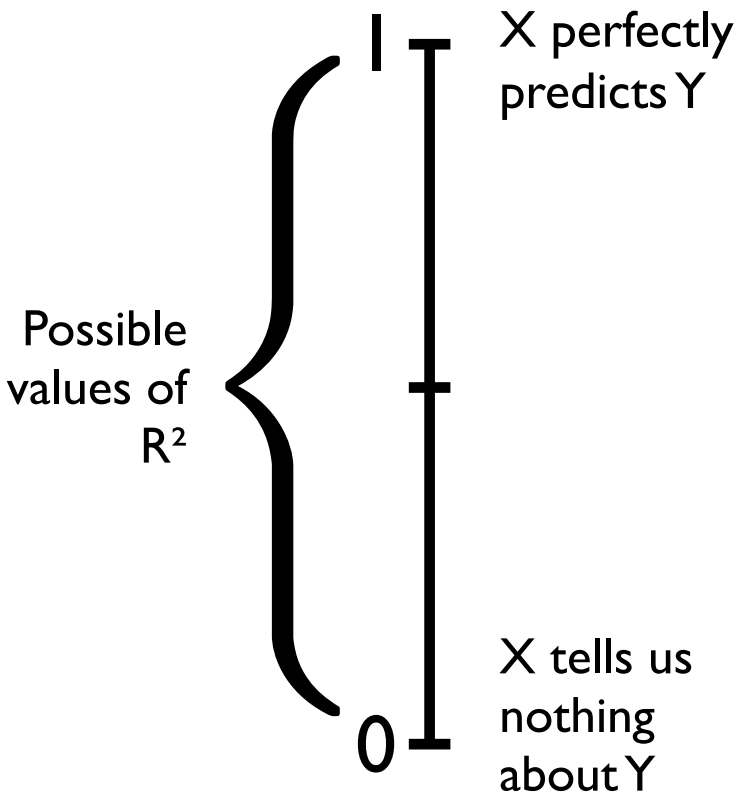
How much better are the predictions from our OLS regression line than the predictions from a flat line (i.e. not using X at all)?



R^2: intuition

How much better are the predictions from our OLS regression line than the predictions from a flat line (i.e. not using X at all)?

How much of the variation in Y is "explained" by the variation in X?



R^2: calculation

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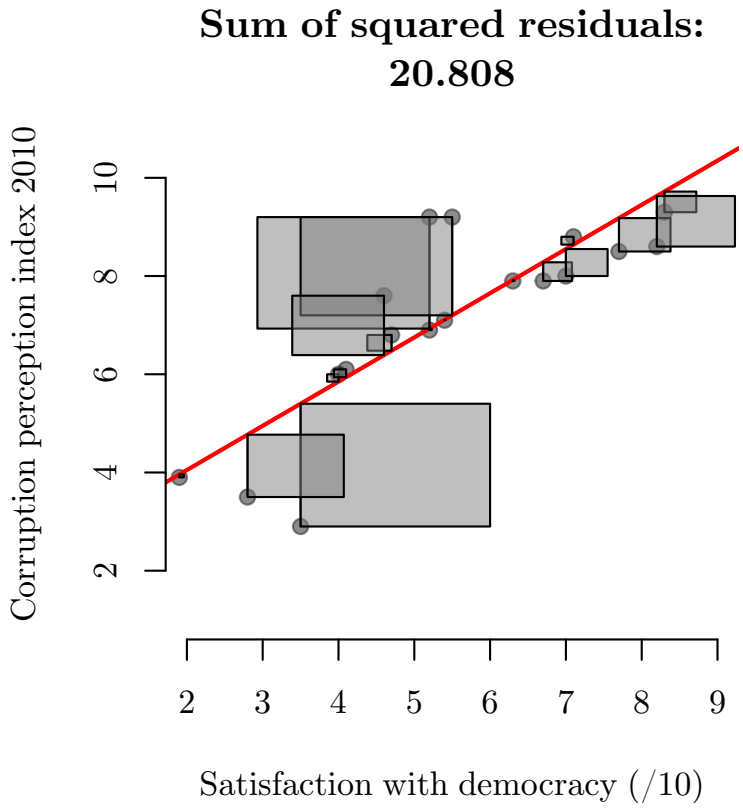

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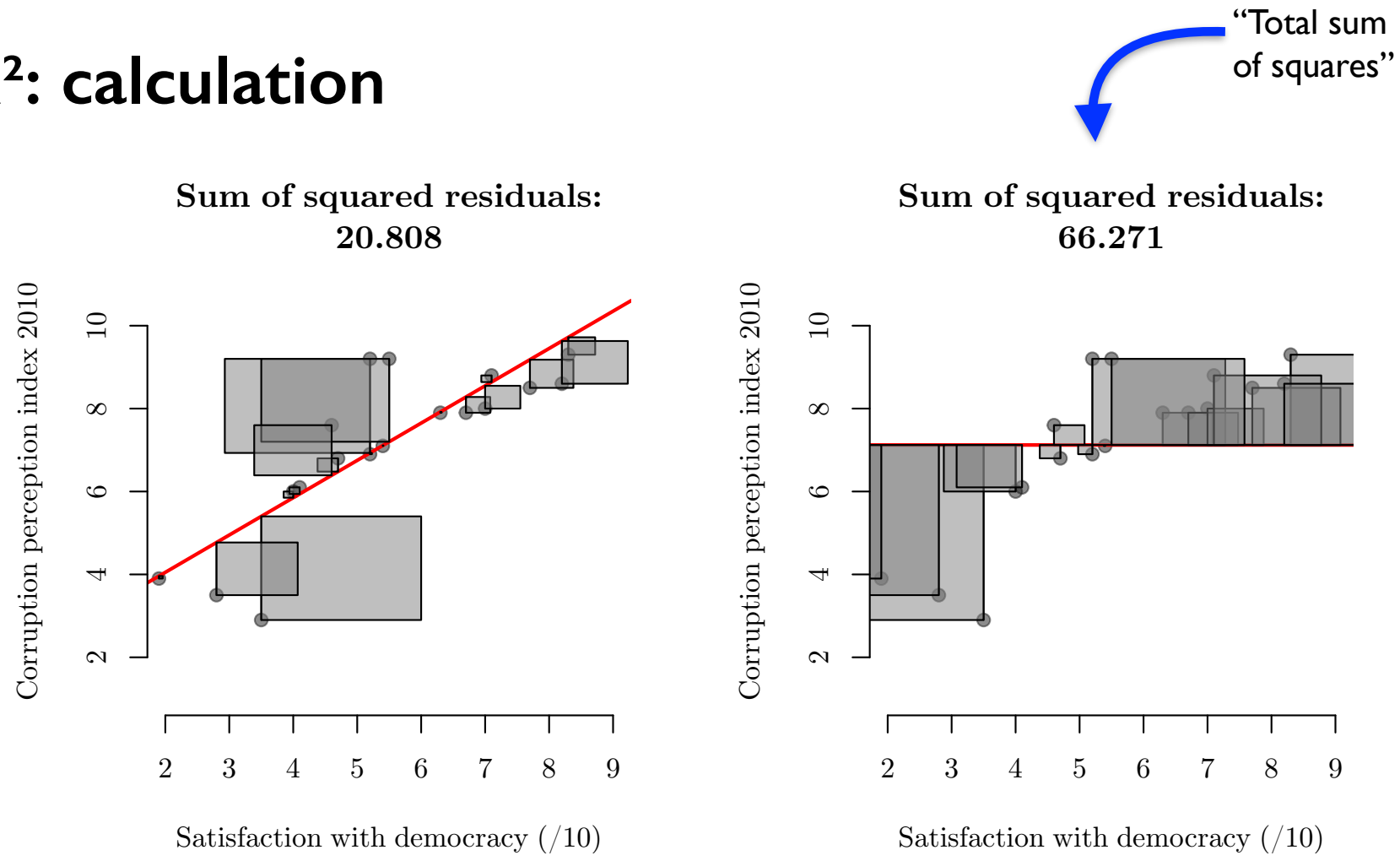
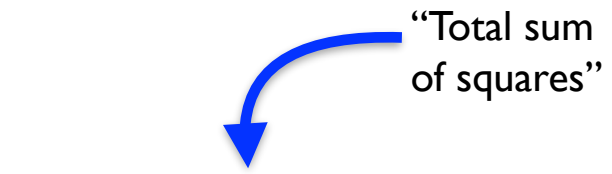
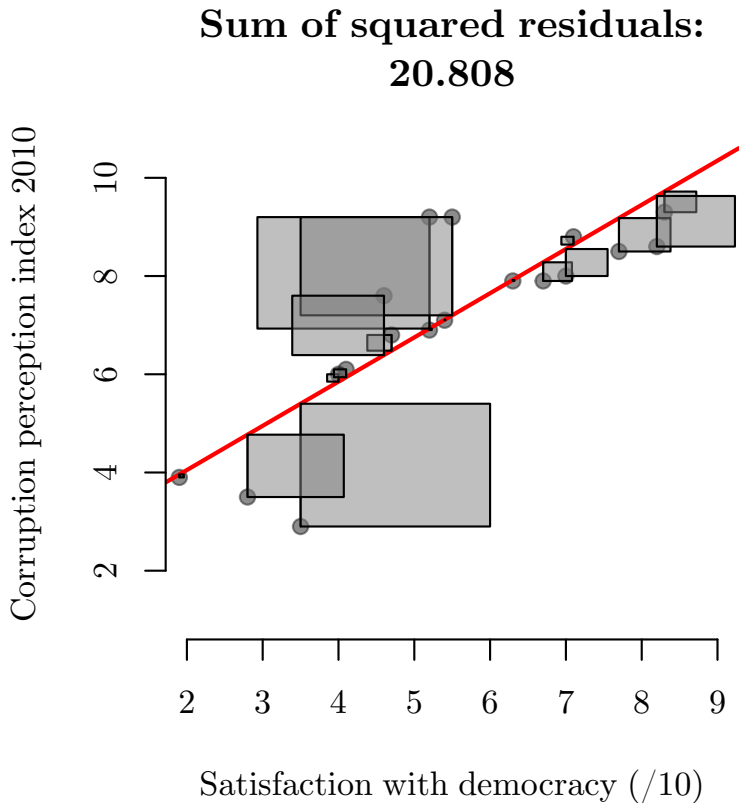
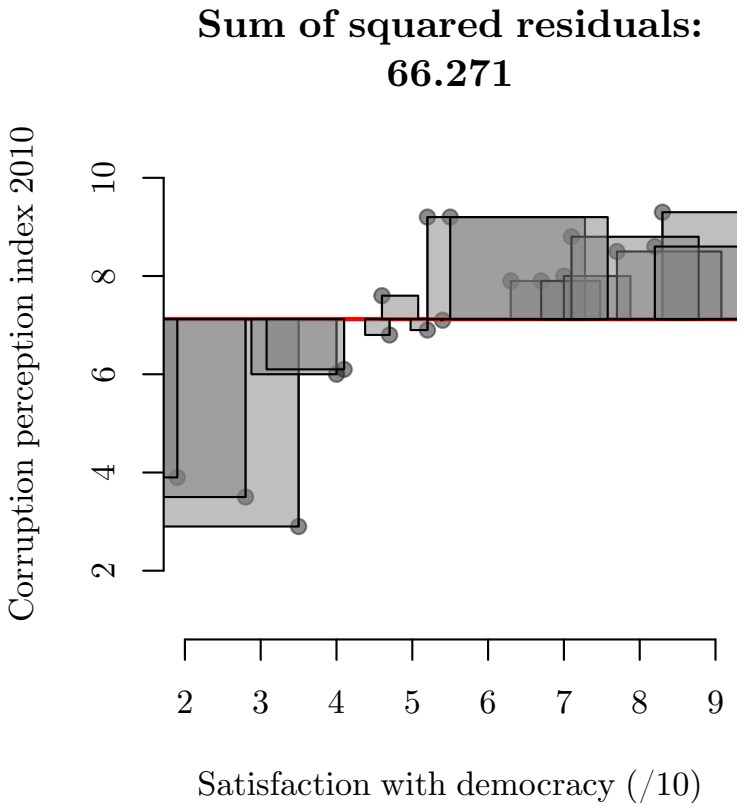


R^2: calculation

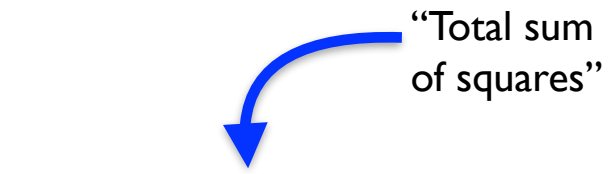


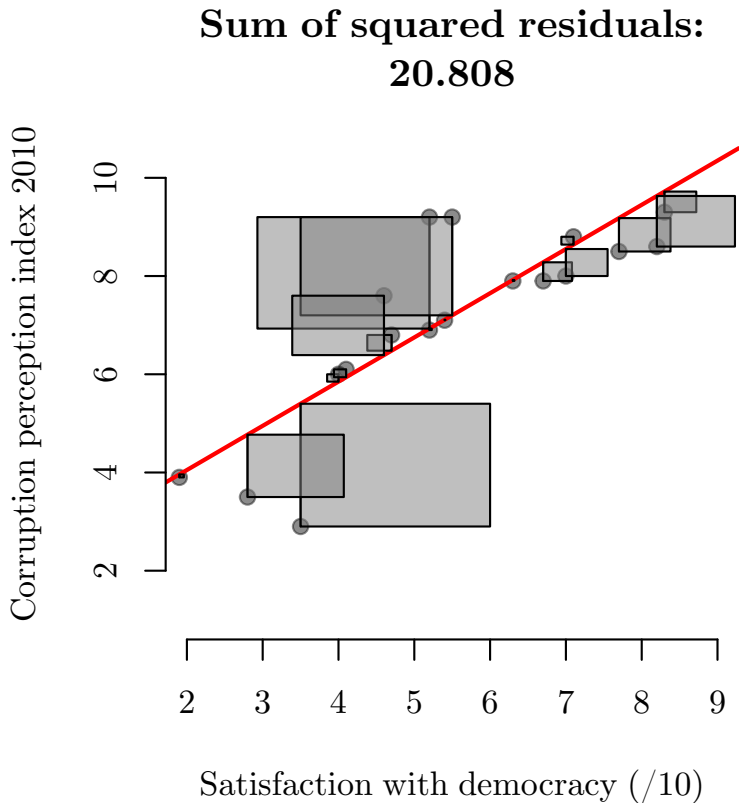


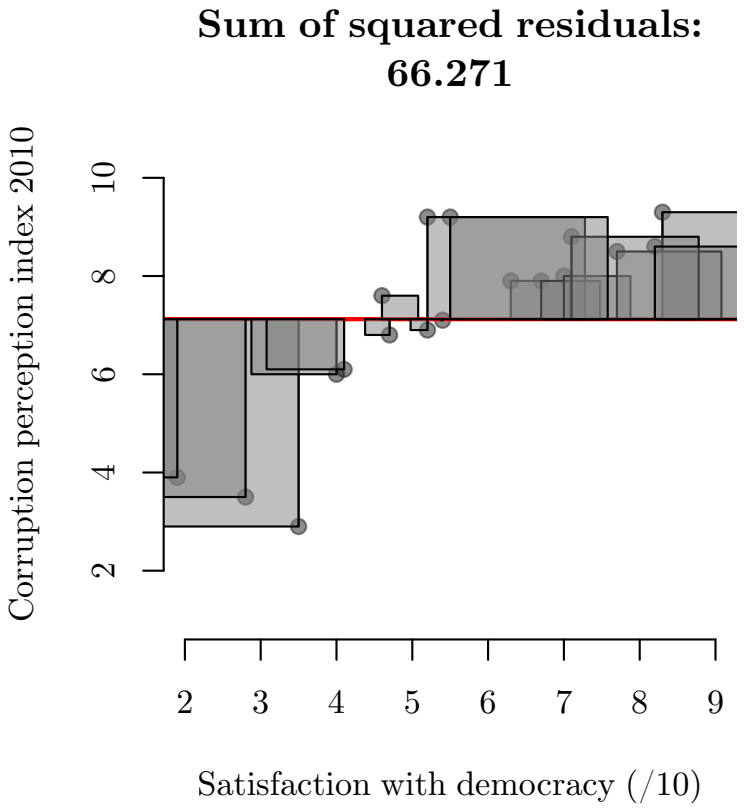


$$1 - \frac{20.808}{66.271} =$$

R^2: calculation







$$1 - \frac{20.808}{66.271} = 0.6864$$

Connections between measures of bivariate relationships

Key measures:

- covariance
- correlation
- OLS regression output:
  - intercept
  - slope
  - R^2

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Regression slope (but not covariance or correlation) depends on which is Y and which is X

Covariance and regression slope (but not correlation) depend on the units

Looking ahead

- Next week: data labs, part 3!
- Next two weeks: multivariate regression and statistical inference

The roots of regression

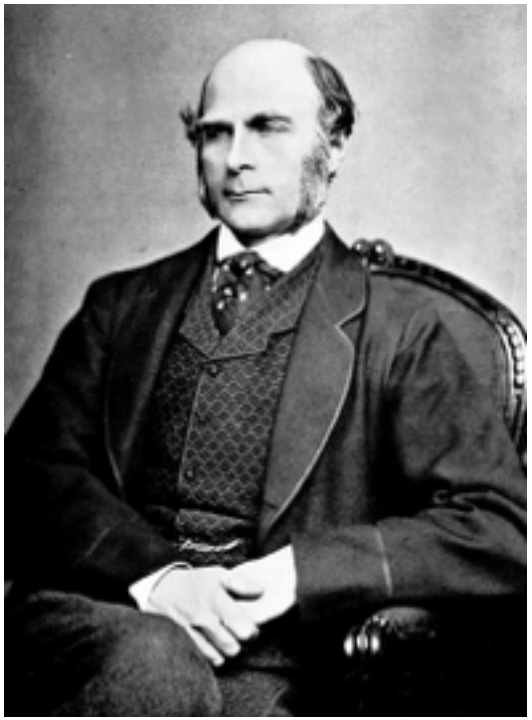
ANTHROPOLOGICAL MISCELLANEA.

REGRESSION *towards* MEDIOCRITY *in* HEREDITARY STATURE.

By FRANCIS GALTON, F.R.S., &c.

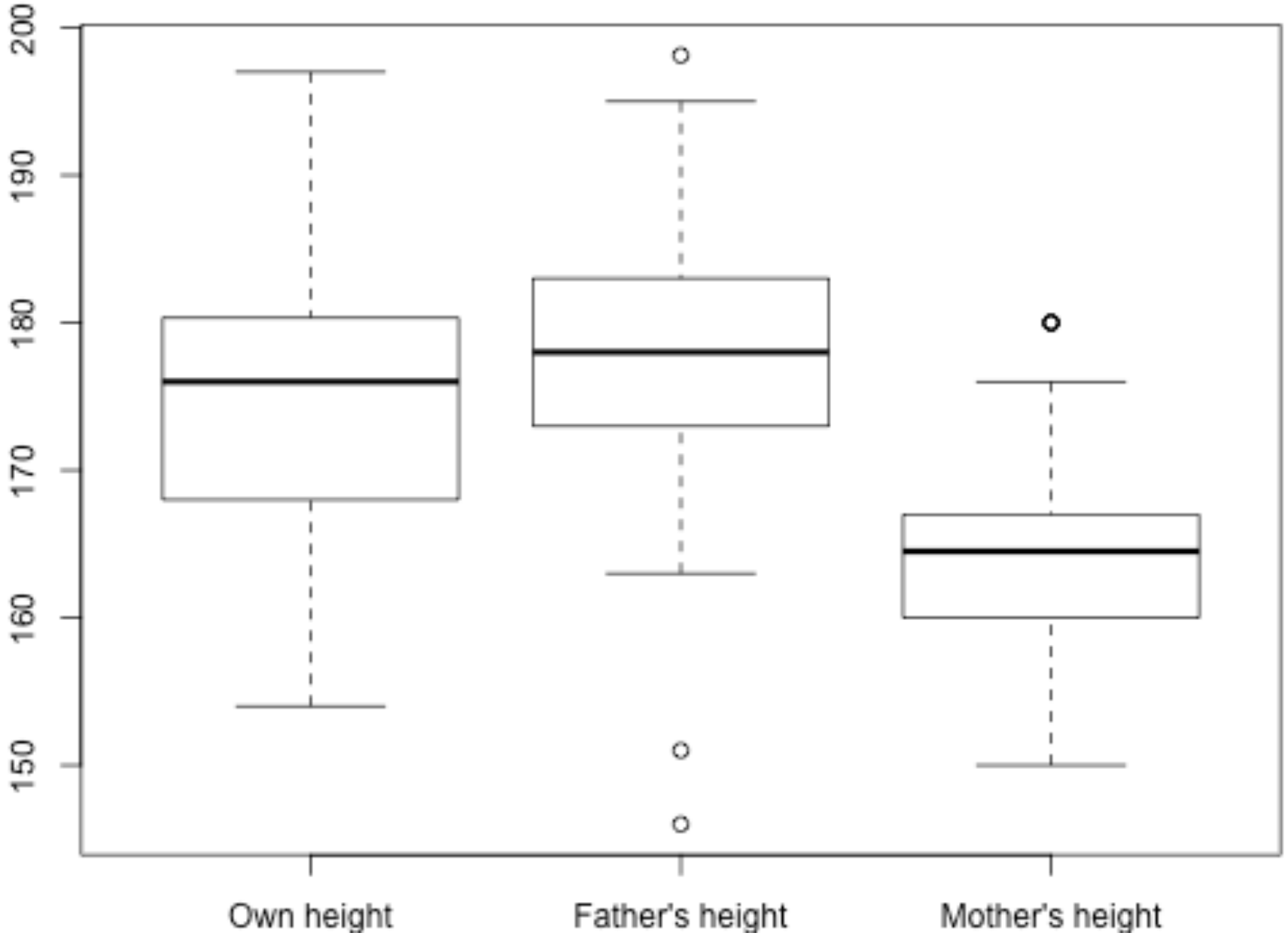
[WITH PLATES IX AND X.]

THIS memoir contains the data upon which the remarks on the Law of Regression were founded, that I made in my Presidential Address to Section H, at Aberdeen. That address, which will appear in due course in the Journal of the British Association, has already been published in "Nature," September 24th. I reproduce here the portion of it which bears upon regression, together with some amplification where brevity had rendered it obscure, and I have added copies of the diagrams suspended at the meeting, without which the letterpress is necessarily difficult to follow. My object is to place beyond doubt the existence of a simple and far-reaching law that governs the hereditary transmission of, I believe, every one of those simple qualities which all possess, though in unequal degrees. I once before ventured to draw attention to this law on far more slender evidence than I now possess.

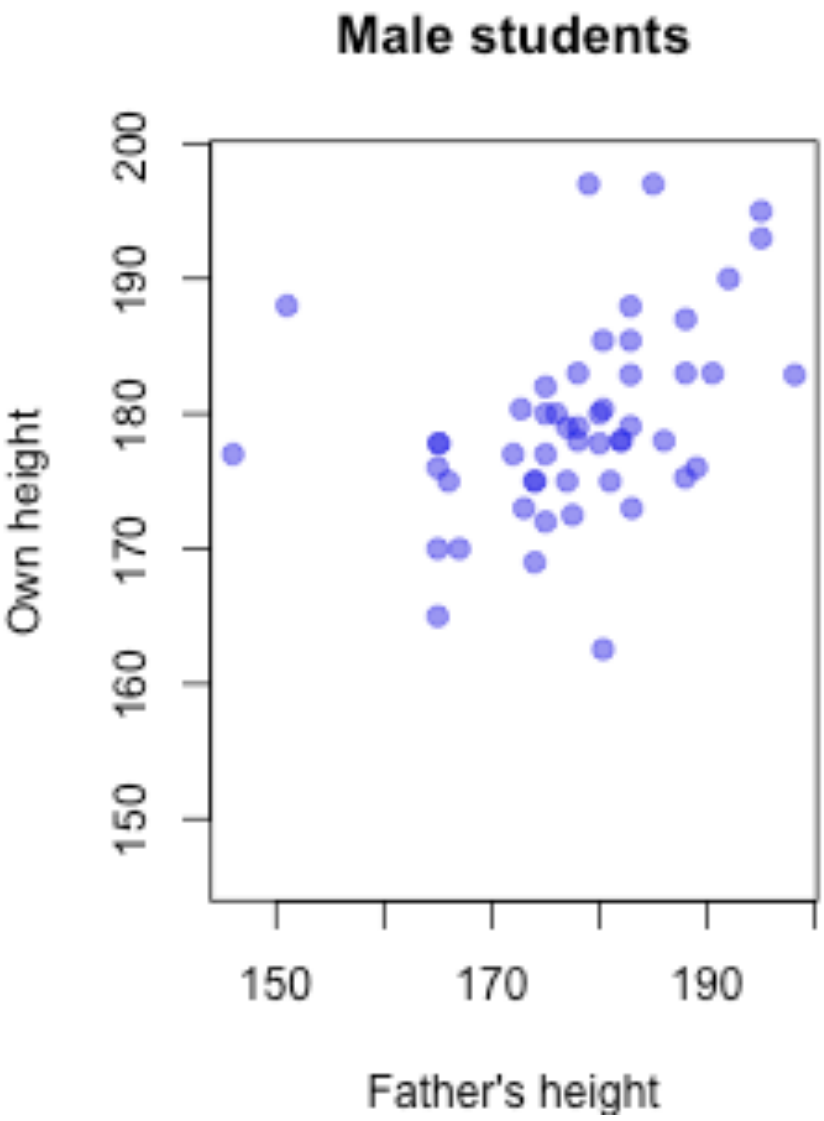


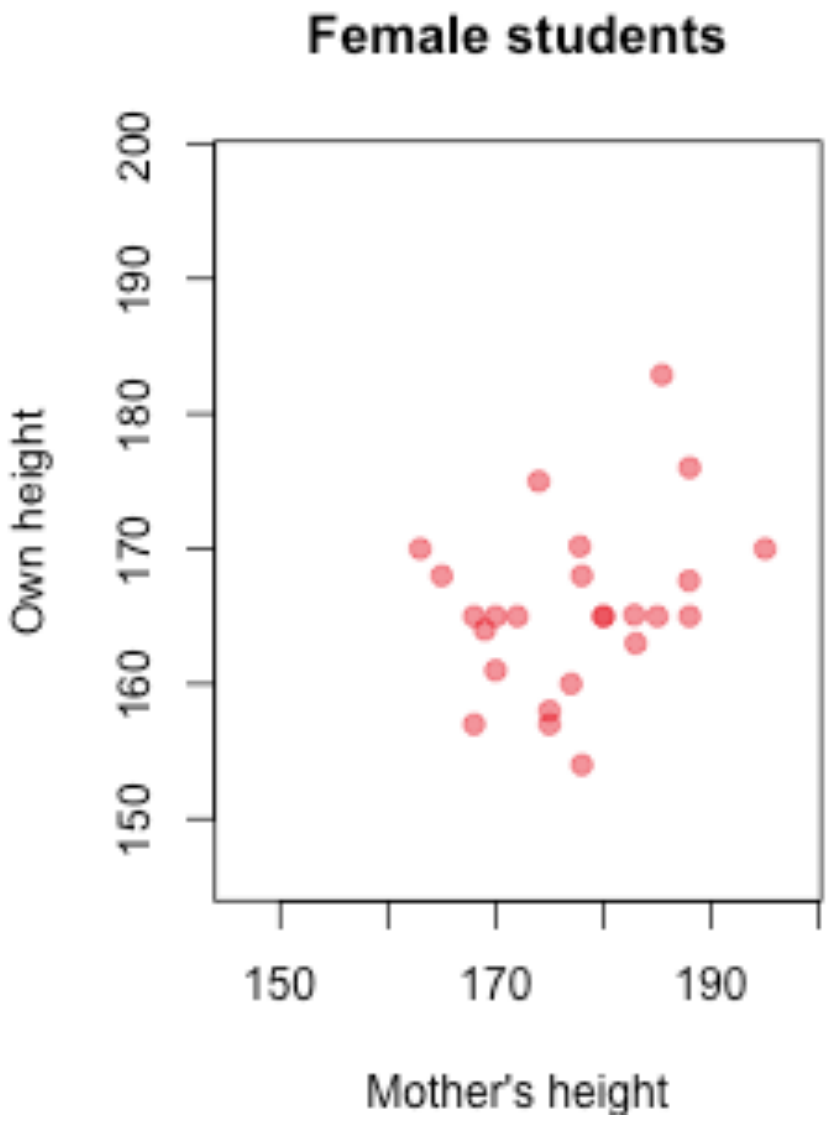
Francis Galton, *Journal of the Anthropological Institute of Great Britain and Ireland*, 1886.

Height data from 2015 survey

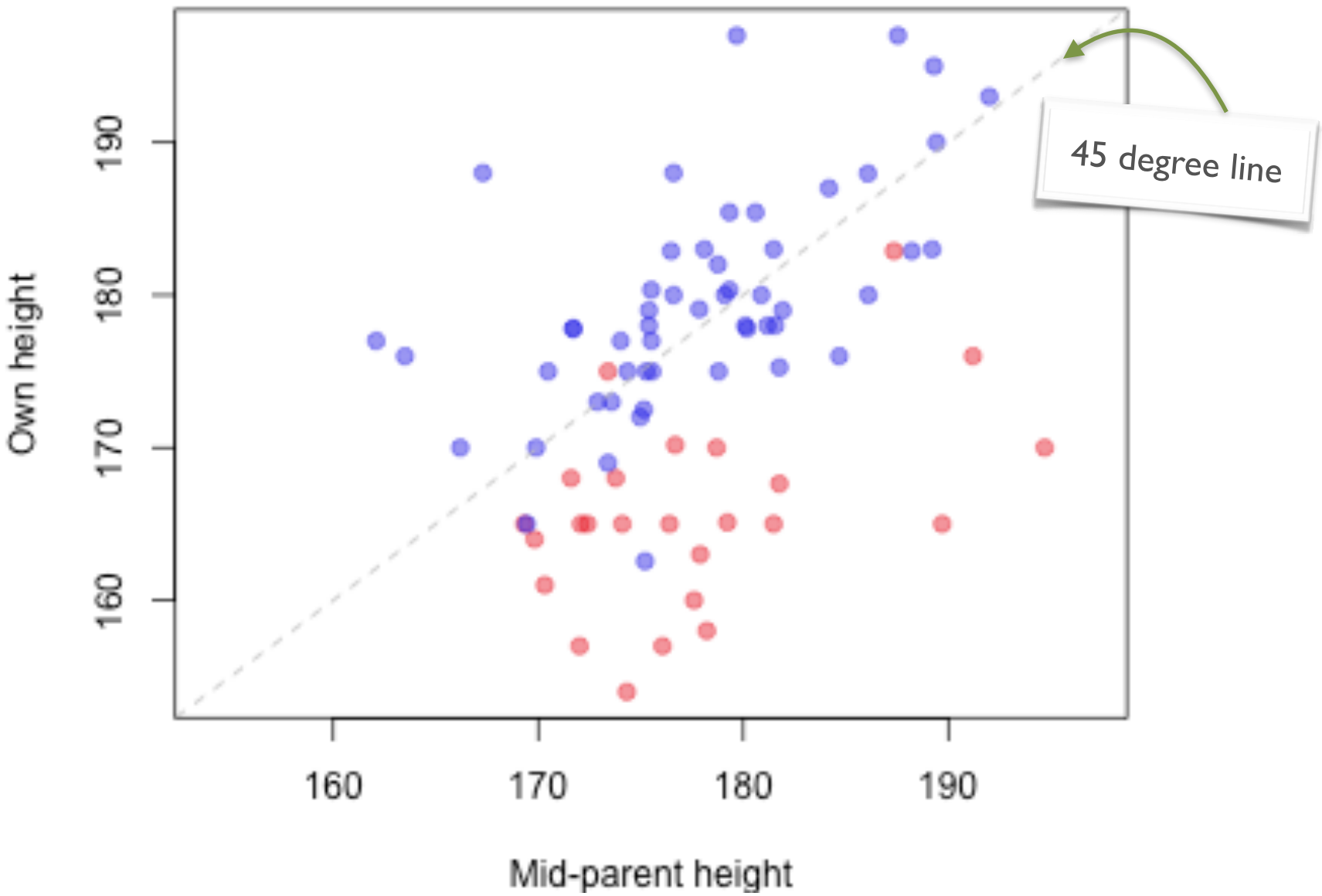


Height data, by gender





Own height and "mid-parent" height



Regression line shows "regression to the mean"!