Statistical Modeling: Intro and Applications (or:What else is there?) **Intermediate Social Statistics** Week 8 (7 March 2017) Andy Eggers

We've seen:

- Regression (OLS)
- RCTs
- Matching
- Instrumental variables
- RDD
- Diff-in-diff/panel

You also saw:

Logistic regression

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What else do we need?

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If your dependent variable is	you need this model.	See this Stata command.		
Continuous (and unbounded)	OLS	regress		
Binary (e.g. join WTO or not)	Logit Probit	logit probit		
A count (e.g. 0, 1, 10 wars)	Poisson Negative binomial	poisson nbreg		
Ordered categories (e.g. "opposed", "neutral", in favor")	Ordinal logit Ordinal probit	ologit oprobit		
Non-ordered categories (e.g.Tory, Labour, Lib Dem; Christian, Muslim, Jewish, atheist)	Multinomial logit, conditional logit	mlogit clogit		
A measure of survival or duration (e.g. cabinet or war duration)	Survival or hazard model	stcox		

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See glm (generalized linear model) package for many of these.

Generalized linear models

Linear regression model:

$$\mathsf{E}(Y) = \alpha + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_k X_k$$

Binary logistic models:

$$\log\left[\frac{P(Y=1)}{P(Y=0)}\right] = \alpha + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_k X_k$$

Multinomial logistic models:

$$\log\left[\frac{P(Y=j)}{P(Y=0)}\right] = \alpha_j + \beta_{j1}X_1 + \beta_{j2}X_2 + \cdots + \beta_{jk}X_k$$

Ordinal logistic models:

$$\log\left[\frac{P(Y \ge j)}{P(Y < j)}\right] = \alpha + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_k X_k$$

Count models:

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Gailmard p. 146: "invertible function of the model parameter is expressed as a linear function of the covariate(s)"

• Syntax (trivial)

Stata: [model name] [outcome] [covariates], [options]

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. oprobit gays	arriage Demain	e partyld his	ghachool	culleged	egree		
Iteration 0:	log likeliho	of = -2050.75	283				
Discation 1:	log likeliho	10 = -2315.4	646				
Decostion 2:	log likelihos	102015.4	564				
Iteration 3:	log likeliho	od = -2015.4	564				
Ordered probit	regression			Punker	of 080	-	2176
				LR ohi	2 (4)	-	26.54
				Prob >	08.12	-	0.0008
Log likelihood	 -2315.4544 			Peeudo	82	-	0.0163
pajmerriege	Coef.	Did. Err.		$\mathbb{D}^{(2)}$	(95%	Conf.	Interval
female	.10073	.0498641	2.02	0.043	.002	1949	.198445
pertyid.	0850757	.0123145	-6.58	0.000	595	2155	
highschool	1800961	.0548493	-3.24	0.001	28	5558	072634
oollegedegnee	.1816416	.0684662	2.65	0.008	.047	4503	-315832
/out1	522102	.0634712				5033	
/out2	.1277551	.0629013			.004	1709	-251039



Vertical bars indicate 99-percent confidence intervals

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 - Why I think OLS is enough for estimating treatment effects (and many other tasks)
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 - Introduction to statistical models based on MLE

Ordinal probit application: Hainmueller and Hiscox 2010

Two economic explanations for (variation in) antiimmigrant sentiment:

- Labor market competition → natives should oppose immigrants with skill levels similar to their own
- Fiscal burden → rich natives should be more opposed to low-skilled immigrants than poor natives (especially where immigrants use a lot of public services)





(Random whether respondent gets A or B) Hainmueller and Hiscox ask a sample of US respondents either

- A. Do you agree or disagree that the US should allow more highly skilled immigrants from other countries to come and live here?
- B. Do you agree or disagree that the US should allow more **low**skilled immigrants from other countries to come and live here?

Hainmueller and Hiscox (2010): why reviewers asked for ordinal probit





Hainmueller and Hiscox (2010): why reviewers asked for ordinal probit (cont'd)

FIGURE 3. Support for Highly Skilled and Low-skilled Immigration by Respondents' Skill Level



Allow more highly skilled immigration?

Ordered probit

Motivations:

- **Predict** ordered outcomeY
- Characterize the determinants of a latent variable Y* (e.g. support for immigration) underlying ordered outcome Y

- I. Strongly disagree
- 2. Disagree
- 3. Neither agree nor disagree
- 4. Agree
- 5. Strongly agree

Ordered probit: theory

Suppose we observed Y* (support for immigration), which in conjunction with cutpoints T_1 , T_2 etc perfectly predicts the response given:

$$Y = \begin{cases} 1, & \text{if } Y^* \leq \tau_1 \\ 2, & \text{if } Y^* \in (\tau_1, \tau_2] \\ 3, & \text{if } Y^* \in (\tau_2, \tau_3] \\ 4, & \text{if } Y^* \in (\tau_3, \tau_4] \\ 5, & \text{if } Y^* > \tau_4 \end{cases}$$

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Ordered probit: theory (continued)

We don't observe Y*, but we postulate that it is a linear function of covariates, plus random error (standard normal):





Ordered probit: visualization

That implies that given τ_1 , τ_2 , τ_3 , τ_4 and $\mu_i = x_i\beta$ we know the probability of each outcome:

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Ordered probit: visualization (2)



Ordered probit: visualization (3)



Binary probit: a special case with single threshold at 0



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Back to Hainmueller and Hiscox

To explicitly test the labor market competition argument, we estimate the systematic component of the ordered probit model with the specification.

 $\mu_i = \alpha + \gamma \text{HSKFRAME}_i + \delta (\text{HSKFRAME}_i)$

 \cdot EDUCATION_i) + θ EDUCATION_i + $Z_i \psi$,

where the parameter γ is the lower-order term on the treatment indicator that identifies the premium that natives attach to highly skilled immigrants relative to low-skilled immigrants. The parameter δ captures how the premium for highly skilled immigration varies conditional on the skill level of the respondent.

Z_i contains controls: 7 age bracket dummies, gender dummy, 4 race dummies

 $(\mu_i = Y^* = x_i\beta)$

"Notice that because the randomization orthogonalized HSKFRAME with respect to Z, the exact covariate choice does not affect the results of the main coefficients of interest." p.70

Ordered probit: estimation

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How do we estimate β and $\tau_1, \tau_2, \tau_3, \tau_4$?

Stata:oprobit depvar [indepvars] [weight] [, options]

Ordered probit: estimation

How do we estimate β and $\tau_1, \tau_2, \tau_3, \tau_4$?

Stata: oprobit depvar [indepvars] [weight] [, options]

. oprobit sh_both hskframe ppeducat hskeduc xx* [pweight=weight1]

Iteration	0:	log	pseudolikelihood	=	-2418.2933
Iteration	1:	log	pseudolikelihood	=	-2306.2688
Iteration	2:	log	pseudolikelihood	=	-2306.1887
Iteration	3:	log	pseudolikelihood	=	-2306.1887

Ordered probit regression	Number of obs	=	1,589
	Wald chi2(8)	=	158.52
	Prob > chi2	=	0.0000
Log pseudolikelihood = -2306.1887	Pseudo R2	=	0.0464

		Robust				
sh_both	Coef.	Std. Err.	z	P> z	[95% Conf.	Interval]
hskframe	.7261249	.2025688	3.58	0.000	.3290974	1.123152
ppeducat	.2683796	.0484328	5.54	0.000	.1734531	.3633061
hskeduc	0653202	.0667142	-0.98	0.328	1960777	.0654373
xxfemale	1771998	.0644352	-2.75	0.006	3034904	0509092
xxppagecat	0110243	.0196088	-0.56	0.574	0494569	.0274083
xxWhite	374742	.0990717	-3.78	0.000	5689189	1805651
xxBlack	4720909	.1352577	-3.49	0.000	7371911	2069907
xxHispanic	.0627729	.2058409	0.30	0.760	3406679	.4662136
/cut1	114744	.1910944			4892822	.2597941
/cut2	.5613041	.1905945			.1877457	.9348625
/cut3	1.254911	.1907666			.8810152	1.628807
/cut4	2.258038	.2003352			1.865388	2.650688

Hainmueller and Hiscox: ordered probit results

TABLE 1. Individual Support for Highly Skilled and Low-skilled Immigration—Test of the Labor Market Competition Model

	In Fav						
	High Skilled Immigration	Low-skilled Immigration			In Favor of: Immigration		
	(1)	(2)	(3)	(4)	(5)	(6) labor	(7) force
Dependent Variable						in	out
EDUCATION	0.21 (0.05)	0.27 (0.05)		0.27 (0.05)		0.33 (0.06)	0.19 (0.07)
HSKFRAME			0.54 (0.07)	0.73 (0.20)	0.56 (0.12)	0.73 (0.28)	0.64 (0.29)
HSKFRAME EDUCATION				-0.07 (0.07)		-0.08 (0.09)	0.00 (0.11)
HS DROPOUT				(,	-0.41	(,	(,
HSKFRAME-HS DROPOUT					0.24		
HIGH SCHOOL					-0.16		
HSKFRAME-HIGH SCHOOL					-0.05		
BA DEGREE					0.41		
HSKFRAME-BA DEGREE					(0.12) -0.08 (0.16)		
(N)	798	791	1589	1589	1589	946	643
Covariates	x	x	x	x	x	x	x

Order Probit Coefficients shown with standard errors in parentheses. All models include a set of the covariates age, gender, and race (coefficients not shown here). The reference category for the set of education dummies is SOME COLLEGE (respondents with some college education).

Hainmueller and Hiscox: why ordered probit?

Conventional view: "Your outcome is an ordered categorical variable, so you must estimate an ordered probit model! (Although I don't remember exactly why.)"

But the authors don't use the model for prediction (e.g. estimated proportion of respondents answering category 4 given treatment status, education, gender.)

They report the coefficients (and not the cutoffs!), and move on to logit for a different outcome: **support more immigration**.

Hainmueller and Hiscox: logit results

To give some sense of the substantive magnitudes involved, we simulate the predicted probability of supporting an increase in immigration (answers "somewhat agree" and "strongly agree" that the U.S. should allow more immigration) for the median respondent (a white woman aged 45) for all four skill levels and both immigration types based on the least restrictive model (model five in Table 1).

FIGURE 4. Support for Highly Skilled and Low-skilled Immigration by Respondents' Skill Level



Why logit?

Conventional view: "Outcome is a binary variable, so you must use logit! (Although I don't remember exactly why.)"

pport for Highly Skilled and Low-skilled Immigration by Respondents' S



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Why not estimate a linear probability model (LPM) — i.e. OLS despite binary outcome?



pport for Highly Skilled and Low-skilled Immigration by Respondents' S

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The usual case against the linear probability model (LPM)



Explanatory Variable (X)

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Explanatory Variable (X)

- Predictions outside the range of dependent variable
- Heteroskedasticity (violates OLS assumption)
- Non-normal errors (violates OLS assumption*)
- Unrealistic for probability to be linear in X

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 - Yes, especially when probabilities are near 1 or 0 (ceiling and floor effects); but is probit the right form?

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 - Logit vs LPM matters only if particular kind of covariate imbalance

Gailmard pp 171-2

"If the CEF is linear, as it is for a saturated model, [OLS] gives the CEF.... If the CEF is non-linear, [OLS] approximates the CEF. Usually it does it pretty well. Obviously, the LPM won't give the true marginal effects from the right nonlinear model. But then, the same is true for the 'wrong' nonlinear model! The fact that we have a probit, a logit, and the LPM [shows] that we don't know what the 'right' model is. Hence, there is a lot to be said for sticking to a linear regression function as compared to a fairly arbitrary choice of a non-linear one! Nonlinearity per se is a red herring."



Steve Pischke

from MHE blog http://www.mostlyharmlesseconometrics.com/2012/07/probit-better-than-lpm/





Respondent educational attainment
The defense of the LPM: continued

on LPM)



Respondent educational attainment

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 (Will statistical modeling help with explanation?)

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So let's get a taste of statistical modeling more generally.

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More formally, a statistical model describes a **set of probability distributions** for a random variable (Y).

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In any interesting statistical model, different units have different distributions, depending on the features of the unit (e.g. exposure to treatment vs. control, values of covariates).

Gailmard 4.3

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Gailmard 4.4 The probability distribution of a random variable Y can be summarized by

- a cumulative distribution function (CDF) gives $Pr(Y \le y)$
- (if discrete) a probability mass function (PMF) gives Pr(Y=y)

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Gailmard 4.4

A random variable Y takes one of multiple possible (numerical) values depending on the outcome of an "experiment".

Conventional notation: Y is the RV; y is a particular value.

The probability distribution of a random variable Y can be summarized by

- a cumulative distribution function (CDF) gives $Pr(Y \le y)$
- (if discrete) a probability mass function (PMF) gives Pr(Y=y)
- (if continuous) a probability density function (PDF) gives the derivative of the CDF at y

Normal PDF and CDF

















...that characterize a data generating process (DGP)...







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35

How probability works

Given a set of probability distributions...



 $\mu = 3, \, \sigma = 1 \ \mu = 6, \, \sigma = 2.5$

- -

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Gailmard 4

















У





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37

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- events are independent
- rate of occurrence (probability per unit time) is constant (λ)



Gailmard 6.3

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Single count

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Single count (2)



y: Number of occurrences

Single count (2)

Suppose we view the number of students sitting in row 3 as a Poisson random variable.

For what value of λ is the observed outcome most likely?

This is the most basic illustration of Maximum Likelihood Estimation (MLE) for λ .



y: Number of occurrences

For two events E and F, the probability of both events happening is written

$$P(E,F)$$
 or $P(E \cap F)$ joint probability

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If E and F are independent,

$$\mathcal{P}(E|F) = \mathcal{P}(E)$$

and:

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y: Number of occurrences

Suppose we have 5, 2, 7, 4 students in these rows.

	λ = 2	λ = 5
5	0.04	0.18
2	0.27	0.08
7	0.003	0.10
4	0.10	0.18
Likelihood = column product	3.03/IM	270.84/IM

Suppose we have 5, 2, 7, 4 students in these rows.



Vector of counts (continued)

We can of course try this for more values of λ :

	λ = 2	λ = 3	λ = 4	λ = 5	λ = 6
5	0.04	0.10	0.16	0.18	0.16
2	0.27	0.22	0.15	0.08	0.04
7	0.003	0.02	0.06	0.10	0.14
4	0.10	0.17	0.20	0.18	0.13
Likelihood = column product (× IM)	3.03	82.00	266.39	270.84	132.07

Vector of counts (continued)

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	λ = 2	λ = 3	λ = 4	λ = 5	λ = 6	
5	0.04	0.10	0.16	0.18	0.16	$\begin{array}{c c} & & & & \\ \vdots \\ \vdots$
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7	0.003	0.02	0.06	0.10	0.14	Pr(y λ)
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The pmf can be written $f(y|\lambda)$: a function of y (the observed data) whose shape depends on λ (the parameter).

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 λ : Rate parameter

Using θ to refer to parameters, consider:

$$\hat{\theta}(\mathbf{y}) = \operatorname{argmax}_{\theta} L(\theta | \mathbf{y})$$

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A general approach to statistical modeling:

- write down $f(y|\theta)$ (pdf/pmf: probability of outcomes conditional on parameters), which is also $L(\theta|y)$
- observe data (y: actual outcomes)
- find parameters that maximize $L(\theta|y)$: the MLE!

Maximum likelihood (common notation)

$$\mathcal{L}(\theta|\mathbf{Y}) = f(y_1, y_2, \dots, y_n|\theta)$$

= $f(y_1|\theta)f(y_2|\theta)\dots f(y_n|\theta)$
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Suppose we observe 5, 2, 7, and 4 students. $\lambda = row$ # students row 5 3 0.1 2 4 0.15 5 7 0.10 0.13 6 4 Likelihood = column 206.52 product (× IM)



y: Number of occurrences
Vector of counts with a covariate

Suppose we observe 5, 2, 7, and 4 students. # students $\lambda = row$ row 3 5 0.1 4 2 0.15 5 7 0.10 0.13 6 4 Likelihood = column 206.52 product (× IM)

Now suppose $\lambda = \beta_0 + \beta_1 \times row$, and find β_0 , β_1 that maximize the likelihood.



y: Number of occurrences

$$\mathcal{L}(\theta|\mathbf{Y}) = f(y_1, y_2, \dots, y_n|\theta)$$

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The likelihood function for the last MLE problem you just solved was:

$$\begin{split} \mathcal{L}(\theta|\mathbf{y}) &= f(y_3, y_4, y_5, y_6|\theta) \\ &= f(y_3|\theta) f(y_4|\theta) f(y_5|\theta) f(y_6|\theta) \\ &= \prod_{i=3}^6 f(y_i|\theta) \\ &= \prod_{i=3}^6 \frac{\lambda^{y_i} e^{-\lambda}}{y_i!} = \prod_{i=3}^6 \frac{(\beta_0 + x_i \beta_1)^{y_i} e^{-\beta_0 - x_i \beta_1}}{y_i!} \end{split}$$

How statistical models look in research papers

A Statistical Method for Empirical Testing of Competing Theories

Kosuke Imai Princeton University Dustin Tingley Harvard University the model specified in equation (1) yields the following observed-data likelihood function where the latent variable Z_i has been integrated out,

$$L_{obs}(\Theta, \Pi | \{X_i, Y_i\}_{i=1}^N) = \prod_{i=1}^N \left\{ \sum_{m=1}^M \pi_m f_m(Y_i | X_i, \theta_m) \right\}.$$
 (2)

Comparing Interest Group Scores across Time and Chambers: Adjusted ADA Scores for the U.S. Congress

TIM GROSECLOSE Stanford University STEVEN D. LEVITT University of Chicago and JAMES M. SNYDER, JR. Massachusetts

Given this representation, we can estimate a_t^c 's, b_t^c 's, and x_i 's by maximizing the following likelihood function:

$$L(\bar{a}, \bar{b}, \bar{x}, \sigma; \bar{y}) = \prod_{t \in T} \prod_{c \in \{\mathrm{H}, \mathrm{S}\}} \prod_{i \in I_t^c} \phi\left(\frac{y_{it} - a_t^c - b_t^c x_i}{\sigma}\right) \frac{1}{\sigma},$$

How statistical models look in research papers

Ideology and Interests in the Political Marketplace

Adam Bonica Stanford University

Assuming independence across candidates and contributors, the log-likelihood to be maximized is,

$$LL(Y|\lambda,\sigma) = \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{t=1}^{T} \sum_{g=0}^{1} (1 - d_{ijt_g}) \ln (NB)$$
$$\times (y_{ijt_g}|\lambda_{ijt_g},\sigma_{it_g})) + (d_{ijt_g}) \quad (3.3)$$
$$\ln \left(1 - \sum_{k=0}^{9} NB(k|\lambda_{ijt_g},\sigma_{it_g})\right)$$

where Y is an $n \times m$ matrix of observed contribution counts with y_{ijt_g} being the contribution amount of PAC *i* to candidate *j* in period t_g .

How statistical models look in research papers

How to Analyze Political Attention with Minimal Assumptions and Costs

Kevin M. Quinn University of California, Berkeley Burt L. Monroe The Pennsylvania State University Michael Colaresi Michigan State University Michael H. Crespin University of Georgia Dragomir R. Radev University of Michigan As will become apparent later, it will be useful to write this sampling density in terms of latent data $\mathbf{z}_1, \ldots, \mathbf{z}_D$. Here \mathbf{z}_d is a *K*-vector with element z_{dk} equal to 1 if document *d* was generated from topic *k* and 0 otherwise. If we could observe $\mathbf{z}_1, \ldots, \mathbf{z}_D$ we could write the sampling density above as

$$p(\mathbf{Y}, \mathbf{Z} \mid \boldsymbol{\pi}, \boldsymbol{\theta}) \propto \prod_{d=1}^{D} \prod_{k=1}^{K} \left(\pi_{s(d)k} \prod_{w=1}^{W} \theta_{kw}^{y_{dw}} \right)^{z_{dk}}$$

Surveying a suite of algorithms that offer a solution to managing large document archives.

BY DAVID M. BLEI

Probabilistic Topic Models

With this notation, the generative process for LDA corresponds to the following joint distribution of the hidden and observed variables,

$$p(\beta_{1:K}, \theta_{1:D}, z_{1:D}, w_{1:D})$$

$$= \prod_{i=1}^{K} p(\beta_i) \prod_{d=1}^{D} p(\theta_d)$$

$$\left(\prod_{n=1}^{N} p(z_{d,n} | \theta_d) p(w_{d,n} | \beta_{1:K}, z_{d,n}) \right). (1)$$

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and we assume word use is iid (conditional on λ_{ijt}). Can we estimate $\alpha_{it}, \psi_j, \beta_j$, and ω_{it} with MLE? OLS?

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For the word "deficit":

- lower Ψ_j
- larger (in magnitude) β_j; for example, if the right talks about "deficits" more frequently and party positions are oriented so that right is positive, β_j should be large and positive.

Eiffel Tower of words

Slapin and Proksch, 2008 FIGURE 2 Word Weights vs. Word Fixed Effects. Left-Right Dimension, Germany 1990–2005 (Translations given in text)



Word Weights

Estimated party positions in Germany

Slapin and Proksch, 2008



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But the OLS coefficients are also the MLE in a statistical model where $\Upsilon \sim N(\alpha + \beta X, \sigma^2)$ (i.e. mean of $\alpha + \beta X$, normal error with variance σ^2).

*This is main message of MHE 3.1 and Gailmard 132-135; see also Gailmard 314 ff. 57

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General advice

- Keep it simple
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- Keep it visual: before (and after) running a model, look at the data!
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- Get good at Stata, R, or both
- There are many ways to contribute. Choose some combination of:
 - better data
 - better design (e.g. causal inference)
 - better measurement
 - better theory

Often one of these makes possible another.



Jones's "New Portable Orrery" (1794)



Jones's "New Portable Orrery" (1794)

"All models are wrong, but some are useful." George Box