Formal Analysis: Costly signaling (tying hands)

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Week 7 Session 1

Tying hands

Throwing out the steering wheel

Game of chicken

		Player 2	
		Swerve	Straight
Player 1	Swerve	3, 3	2, 4
	Straight	4, 2	1, 1

Game of chicken after 1 removes steering wheel

		Player 2	
		Swerve	Straight
Player 1	Straight	4, 2	1, 1

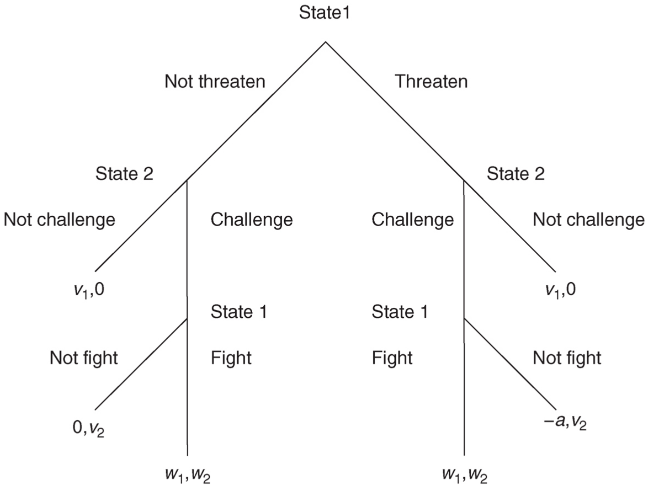
Burning ships/bridges

There are many accounts of military conquest in which the conqueror is said to have eliminated options of escape.

William the Conqueror (England, 1066) and Hernán Cortés (Mexico, 1519-1521) are said to have **burned their ships** on arrival to make escape impossible.

What could motivate this behavior? How could we use a model to explore the possible logic?

Actions that change future payoffs



An alternative approach

Assume probability of conflict is CSF, where e_i is i's effort:

$$\Pr(1 \text{ wins}) \equiv p_1 \equiv \frac{e_1}{e_1 + e_2}$$

The value of winning is 1. The value of losing is v_l. Expending effort e_1 costs γe_1.

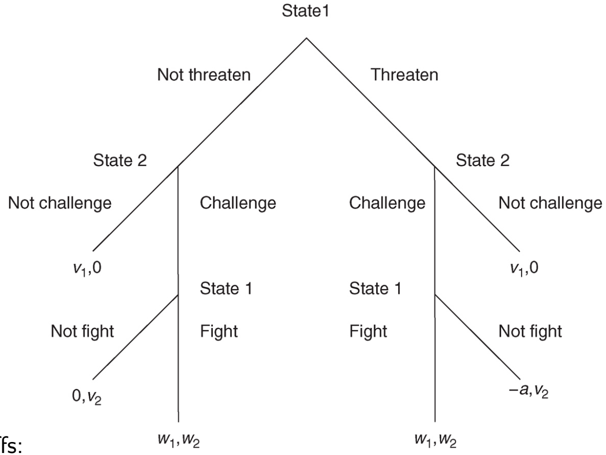
Then expected utility is:

$$\frac{e_1}{e_1 + e_2} + \left(1 - \frac{e_1}{e_1 + e_2}\right) v_l - e_1$$

What can player 1 accomplish by reducing v_l (e.g. by making escape impossible)?

Analysis of the costly signaling game in Kydd: complete information case

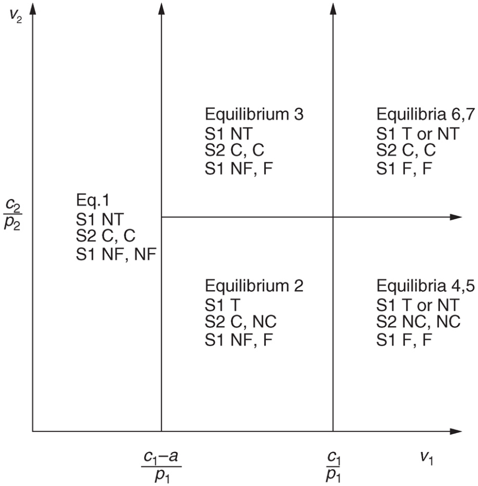
The complete information case



War payoffs:

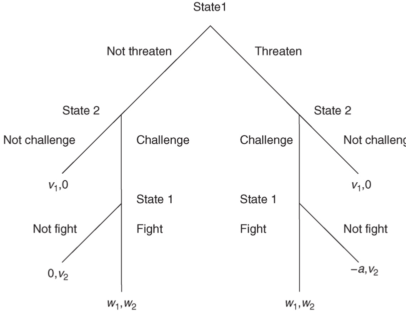
- **Player 1**: $w_1 = p_1 v_1 - c_1 \implies$ if challenged, 1 fights if
  - $v_1 > \frac{c_1}{p_1}$, assuming **did not** issue threat
  - $v_1 > \frac{c_1}{p_1} - a$, assuming **did** issue threat
- **Player 2**: $w_2 = p_2 v_2 - c_2 \implies$ if 1 will fight, 2 challenges if $v_2 > \frac{c_2}{p_2}$

The complete information case (2)



The interesting case

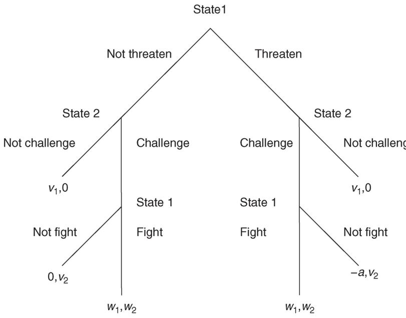
Q: Under what conditions would it be valuable to state 1 to be able to impose a cost *a* on backing down from a threat?



The interesting case

Q: Under what conditions would it be valuable to state 1 to be able to impose a cost *a* on backing down from a threat?

A: When, by issuing the threat, state 1 could convince state 2 not to challlenge.



The interesting case

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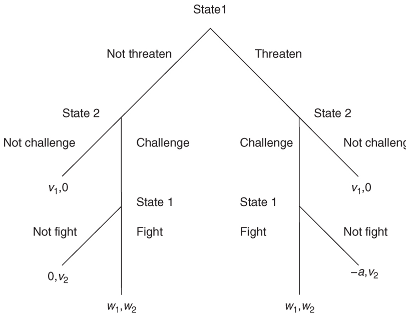
Q: When would that be the case?



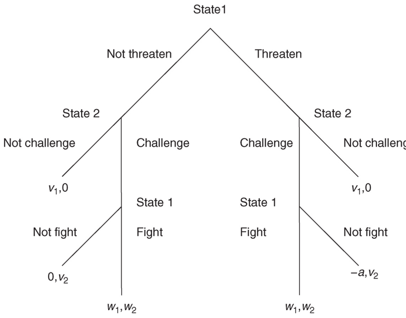
The interesting case

Q: Under what conditions would it be valuable to state 1 to be able to impose a cost *a* on backing down from a threat?

A: When, by issuing the threat, state 1 could convince state 2 not to challlenge.

Q: When would that be the case?

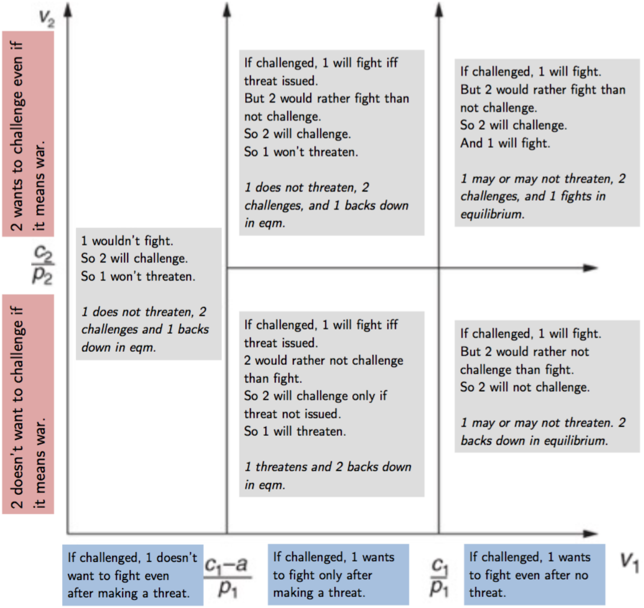
A: When state 1 is the type who would **Fight** only if it has made a threat, and state 2 is the type who would **Not challenge** only if it knew state 1 would **Fight**.



The interesting case (2)

Making a threat *(creating a cost for backing down)* is valuable:

- "When state 1 is the type who would **Fight** only if it has made a threat ...":
  - **would not fight** if no threat $\implies p_1 v_1 - c < 0$, i.e. $v_1 < \frac{c_1}{p_1}$
  - **would fight** if threat $\implies p_1 v_1 - c > -a$, i.e. $v_1 > \frac{c_1 - a}{p_1}$
  - together: $v_1 \in \left(\frac{c_1 - a}{p_1}, \frac{c_1}{p_1}\right)$
- "...and state 2 is the type who would **Not challenge** only if it knew state 1 would **Fight**:
  - **would challenge** if it knew state 1 **would not fight** $\implies$ $v_2 > 0$
  - **would not challenge** if it knew state 1 **would fight** $\implies$ $p_2 v_2 - c_2 < 0$, i.e. $v_2 < \frac{c_2}{p_2}$
  - together: $v_2 \in \left(0, \frac{c_2}{p_2}\right)$



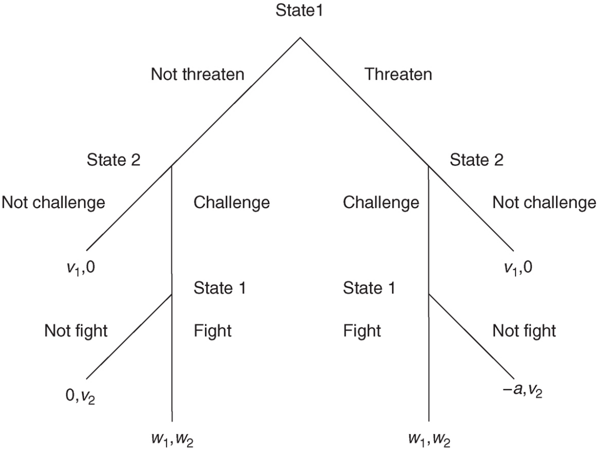
V_2
2 wants to challenge even if it means war.
$\frac{c_2}{p_2}$
2 doesn't want to challenge if it means war.
1 wouldn't fight.
So 2 will challenge.
So 1 won't threaten.
1 does not threaten, 2 challenges and 1 backs down in eqm.
If challenged, 1 will fight iff threat issued.
But 2 would rather fight than not challenge.
So 2 will challenge.
So 1 won't threaten.
1 does not threaten, 2 challenges, and 1 backs down in eqm.
If challenged, 1 will fight.
But 2 would rather fight than not challenge.
So 2 will challenge.
And 1 will fight.
1 may or may not threaten, 2 challenges, and 1 fights in equilibrium.
If challenged, 1 will fight iff threat issued.
2 would rather not challenge than fight.
So 2 will challenge only if threat not issued.
So 1 will threaten.
1 threatens and 2 backs down in eqm.
If challenged, 1 will fight.
But 2 would rather not challenge than fight.
So 2 will not challenge.
1 may or may not threaten. 2 backs down in equilibrium.
If challenged, 1 doesn't want to fight even after making a threat.
$\frac{c_1-a}{p_1}$
If challenged, 1 wants to fight only after making a threat.
$\frac{c_1}{p_1}$
If challenged, 1 wants to fight even after no threat.
V_1

Analysis of the costly signaling game in Kydd: incomplete information case

What does incomplete information mean here?

The values v_1 & v_2 are distributed according to f_1 & f_2 (F_1 & F_2).

Backward induction harder when you don't know the other player's type: how?



Approach to solving the incomplete information game

We posit a cutoff v_1^*: types of state 1 with $v_1 > v_1^*$ threaten.

The state with $v_1 = v_1^*$ must be indifferent between threatening and not threatening.

Two cases:

- **no-bluffing equilibrium**: cutoff is between $\frac{c_1 - a}{p_1}$ and $\frac{c_1}{p_1}$, so the type at the threshold is one who would fight if challenged
- **bluffing equilibrium**: cutoff is below $\frac{c_1 - a}{p_1}$, so the type at the threshold is one who would not fight if challenged

Approach to solving the incomplete information game: no-bluff equilibrium

No-bluffing equilibrium: threat cutoff is between $\frac{c_1 - a}{p_1}$ and $\frac{c_1}{p_1}$, so the type at the threshold is one who would fight if challenged

State 2 knows it will have a war if it challenges a state who has threatened $\implies$ state 2 challenges only if $v_2 > \frac{c_2}{p_2} \equiv v_2^*$.

Where should state 1's threat cutoff v_1^* be?

Approach to solving the incomplete information game: no-bluff equilibrium (2)

Where should state 1's threat cutoff v_1^* be?

If threaten, then two possibilities:

- $v_2 < v_2^*$: state 2 does not challenge, state 1 gets v_1
- $v_2 > v_2^*$: state 2 challenges, state 1 gets $p_1 v_1 - c_1$

If not threaten, then state 2 will challenge and state 1 will get 0.

How do we solve for the optimal v_1^*?

Approach to solving the incomplete information game: no-bluff equilibrium (3)

The probability that 2 challenges is $F(v_2^*)$.

So state 1's payoff from threatening is

$$F_2(v_2^*)v_1 + (1 - F_2(v_2^*))\,(p_1 v_1^* - c_1)$$

Set this equal to 0 and solve for v_1.

The bluffing equilibrium

Bluffing equilibrium: threat cutoff is between 0 and $\frac{c_1 - a}{p_1}$, so the type at the threshold is one who would **not** fight if challenged

Can solve for v_1^* the same way (given v_2^*):

- if threaten,
  - with probability $F_2(v_2^*)$ will not be challenged (and so get v_1)
  - with probability $1 - F_2(v_2^*)$ will be challenged and back down (and so get $-a$)
- if don't threaten, will be challenged and back down (and so get 0)

But 2's decision about what to do when threatened (i.e. choice of cutoff v_2^*) is more complicated than in no-bluffing case: threat might be a bluff, so response to threat depends on v_1^*

Costly signaling game: interpretation

- Think of no-bluffing equilibrium. Why don't types of state 1 with $v_1 < v_1^*$ threaten?
- Think of the bluffing equilibrium. Why don't types of state 1 with $v_1 < v_1^*$ threaten?
- The threat in this game can be read as, "I will fight if you challenge me." Why should this affect state 2's behavior at all?
- How does this relate to the signaling model of education we discussed?