Formal Analysis: Private information and war

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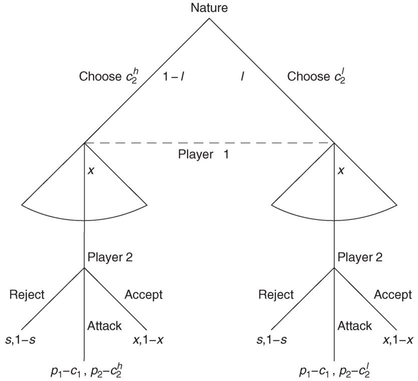
Week 4 Session 2

Bargaining with incomplete information

Complete information version

Consider the crisis bargaining game where Player 2's cost of conflict is either c_2^l or c_2^h, where $c_2^l < c_2^h$.

If Player 1 knows Player 2's cost of conflict and payoffs are linear, then what proposal x does Player 1 make? (Assume if war p_1 is Pr(player 1 wins) and $1 - p_1$ is Pr(player 2 wins).)



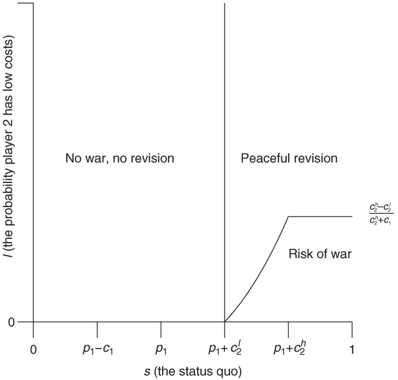
Complete information version (cont'd)

Player 2's *bottom line* is the x such that $1 - x = 1 - p_1 - c_2$, i.e. $b_2 = p_1 + c_2$.

So player 1 should propose $p_1 + c_2^h$ to the high-cost type and $p_1 + c_2^l$ to the low-cost type. War never occurs.

Quiz

Now suppose Player 1 does not know Player 2's cost of conflict: $c_2 = c_2^l$ with probability l and with probability $c_2 = c_2^h$ with probability $1 - l$.



The figure shows areas of the parameter space (in terms of s and l) for which each outcome is an equilibrium.

Question: Explain in 2-4 sentences why there is a risk of war in equilibrium when s is high and l is low.

Suggested quiz answers

Long answer: Generally, player 1 must choose whether to make a generous offer that satisfies the low-cost type of player 2 (but gives an unnecessary surplus to the high-cost type) or make a stingier offer that would satisfy the high-cost type of player 2 but lead to war with the low-cost type. When $s > p_1 + c_2^h$, both the high-cost type and the low-cost type would fight player 1; the flat line indicates the minimum l for which player 1 makes an offer that would appease both types. When s is between $p_1 + c_2^l$ and $p_1 + c_2^h$, only the low-cost type would fight (the high-cost type would prefer the status quo to fighting); making the more generous offer sacrifices little when s is close to the left side of this interval and more when s is close to the right side, which is why the boundary between "Risk of war" and "Peaceful revision" is increasing in s.

Shorter but still acceptable answer: When s is high and l is low, the optimal proposal by player 1 only avoids war if player 2 has a high cost of war. Thus there is a risk of war in equilibrium.

Fundamental theorem of crisis bargaining

Ramsay (2017) writes that bargaining theory has yielded

> *what might be called the fundamental theorem of crisis bargaining: The optimal diplomatic or bargaining strategy is not the one with no risk of war. That is, there is a prevalent risk–reward trade-off in the negotiation process when countries face uncertainty about their rival's willingness to fight.*

Same idea in forming a coalition, buying a house, seeking romantic partners (?), . . .

Modeling mistrust

Prisoner's dilemma, Assurance game, or other?

Preventive war game with mistrust

		Player 2: Not attack	Player 2: Attack
Player 1	Not attack	s_1, s_2	$p_1^s - c_1, p_2^f - c_2$
	Attack	$p_1^f - c_1, p_2^s - c_2$	$p_1 - c_1, p_2 - c_2$

Private information and mutual optimism

Van Evera 1999:

> *The root cause of war lies in the opacity of the future and in the optimistic illusions that this opacity allows. These illusions lead states to fight in false hope of victory . . . If states agree on their relative power, this test [war] is unnecessary; but if they disagree, a contest of arms can offer the only way to persuade the weaker side that it is the weaker.*

The problem with mutual optimism as an explanation for war

Setup: Alice and Bob play a game of dice. In private, each rolls one fair six-sided dice and decides whether to "fight" or not.

If both choose to fight, each player must pay a cost $c \in (0, 1/6)$. The player with the higher roll wins. The contest payoff is 1 for the winner, -1 for the loser, and 0 for both in the event of a tie.

If either decides not to fight, each get a payoff of 0.

Question:

1. Suppose Bob chooses to fight regardless of the value on the dice. Would "fight if 4 or more" be optimal for Alice?
2. Suppose Bob plays "fight if 4 or more". Would "fight if 4 or more" be optimal for Alice?
3. Suppose Bob plays "fight if 6 or more". Would "fight if 6 or more" be optimal for Alice?

Ask the experts

Always remember, however sure you are that you can easily win, that there would not be a war if the other man did not think he also had a chance. (Winston Churchill, 1930)

But also remember: leaders need to be reminded of this.

Mechanism design and “game-free results”

“Game-free results”

[Crisis bargaining literature] resembles more a collection of theoretical anecdotes than a systematic body of organized reasoning linking uncertainty to the risk of costly war. The formal literature on international conflict contains a wide variety of modeling approaches. . . . [O]ur collective knowledge regarding the relationship between uncertainty, the incentive to misrepresent, and war is entangled with countless other assumptions about the type of uncertainty, the timing of actions, the bargaining protocol, and various other assumptions made for either practical or substantive reasons. While this diversity of models is not necessarily a cause for alarm, with some regularity we discover that central conclusions reached from the study of one particular model are overturned when new game forms are considered. (Fey and Ramsay, 2011)

Mechanism design approach

Revelation principle (Myerson, 1979): Suppose s^* is a Bayesian-Nash equilibrium of a (crisis bargaining) game. Then there exists an *incentive-compatible direct mechanism* yielding the same outcome.

Definitions: In the case of crisis bargaining games,

- a **direct mechanism** is a game in which each player's only action is to report a **type** (e.g. "My cost of war is low"); the game assigns a probability of war and a payoff to each profile of actions/types
- an **incentive-compatible direct mechanism** is a direct mechanism in which the players report their true types

Revelation principle (RP) restated: If there is no incentive-compatible direct mechanism yielding a given outcome, then there is no game with that equilibrium.

How Fey and Ramsay (2011) use RP (1)

Suppose each player's cost of fighting can be high or low (private info), but player 1's probability of winning (p) is common knowledge.

Consider this direct mechanism: "Say whether you have high or low costs. Now, player 1 gets $x = p$, player 2 gets $1 - x = 1 - p$, and no war occurs."

- Is this direct mechanism incentive-compatible?

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- Is this direct mechanism incentive-compatible?

Yes (because payoff doesn't depend on costs).

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- Is this direct mechanism incentive-compatible?

Yes (because payoff doesn't depend on costs).

So by RP there could be a game form that yields this as an equilibrium.

Direct mechanism (1)

		Player 2	
		"My costs are Low"	"My costs are High"
Player 1	"My costs are Low"	$p, 1-p$	$p, 1-p$
	"My costs are High"	$p, 1-p$	$p, 1-p$

(Trivially incentive-compatible.)

How Fey and Ramsay (2011) use RP (2)

Again suppose each player's cost of fighting can be high or low (private info) and player 1's probability of winning (p) is common knowledge.

Consider this direct mechanism: "Say whether you have high or low costs. Now, split the resource equally if you announce the same costs; otherwise the lower-cost player gets the whole thing, and no war occurs."

- Is this direct mechanism incentive-compatible?

How Fey and Ramsay (2011) use RP (2)

Again suppose each player's cost of fighting can be high or low (private info) and player 1's probability of winning (p) is common knowledge.

Consider this direct mechanism: "Say whether you have high or low costs. Now, split the resource equally if you announce the same costs; otherwise the lower-cost player gets the whole thing, and no war occurs."

- Is this direct mechanism incentive-compatible?

No, so by RP there cannot be a game form that yields this as an equilibrium.

Direct mechanism (2)

		Player 2	
		"My costs are Low"	"My costs are High"
Player 1	"My costs are Low"	$\frac{1}{2}, \frac{1}{2}$	$1, 0$
	"My costs are High"	$0, 1$	$\frac{1}{2}, \frac{1}{2}$

(Not incentive-compatible.)

How Fey and Ramsay (2011) use RP (3)

Again suppose each player's cost of fighting can be high or low (private info), and player 1's probability of winning (p) is common knowledge.

Consider this direct mechanism: "Say whether you have high or low costs. If both declare low, players fight a war. If 2 declares H, 1 gets everything; if 1 declares H and 2 declares L, 1 gets everything."

- Is this direct mechanism incentive-compatible?

How Fey and Ramsay (2011) use RP (3)

Again suppose each player's cost of fighting can be high or low (private info), and player 1's probability of winning (p) is common knowledge.

Consider this direct mechanism: "Say whether you have high or low costs. If both declare low, players fight a war. If 2 declares H, 1 gets everything; if 1 declares H and 2 declares L, 1 gets everything."

- Is this direct mechanism incentive-compatible?

Could be (depends on parameters), so by RP there could be a game form that yields this as an equilibrium. (See Fey and Ramsay Fig 1.)

Direct mechanism (3)

		Player 2	
		"My costs are Low"	"My costs are High"
Player 1	"My costs are Low"	$p - c_1, 1 - p - c_2$	$1, 0$
	"My costs are High"	$0, 1$	$1, 0$

Let π be probability a given player has low costs. Then this direct mechanism is incentive compatible for player 1 if player 1 would want to report L if costs really were low ($(1-\pi)(p-c_L)+\pi \geq \pi$) and would want to report H if costs really were high ($(1-\pi)(p-c_H)+\pi \leq \pi$), which corresponds to $c_L \leq p \leq c_H$.

Corresponding condition for 2 is $c_L \leq \frac{1}{1-\pi} - p \leq c_H$ (e.g. $p = .5, c_L = .25, c_H = .8$).

Fey and Ramsay key points

- if the payoffs in an incentive-compatible direct mechanism depend on private info, there must be war, because only war penalizes the high-cost type (or low-power type) for lying
- therefore, any crisis-bargaining game form and equilibrium in which payoffs depend on private info has a positive probability of war
- if private info is costs, trivial peaceful eqm in which players get $p, 1 - p$
- if private info is power, harder to find peaceful eqm

if the payoffs depend on private information, there must be war in equilibrium

<!– so the deeper point from the RP is that there can't be a peaceful eqm in which payoffs depend on private info, because war is the only way to make players pay for lying about type in the DM.