Formal Analysis: Electoral competition under uncertainty

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Week 3 Session 1

Distributional politics

Distributional politics without uncertainty

Setup

- Two groups in society, 1 and 2; share $\alpha > 1/2$ of voters are in group 1; y_1 and y_2 are the groups' exogenous per-person income
- Two parties A and B compete by proposing per-voter net transfers t_1 and t_2 (t_{1A}, t_{2A} and t_{1B}, t_{2B})
- Party proposals must balance the budget ($\alpha t_1 + (1-\alpha)t_2 = 0$) and cannot take more than groups have (i.e. $y_1 + t_1 \geq 0$, $y_2 + t_2 \geq 0$)
- All voters in a group vote for the party offering them more, e.g. voters in 1 vote A if $t_{1A} > t_{1B}$; if tie, split evenly

What is the Nash equilibrium of this game?

Distributional politics with uncertainty

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- Party proposals must balance the budget: $\alpha t_1 + (1-\alpha)t_2 = 0$
- Voter i in group g votes for party A if

$$v(y_g + t_{gA}) > v(y_g + t_{gB}) + \eta_i,$$

where v is a monotonically increasing and concave function and η_i distributed uniformly on $\left[-\frac{1}{2}, \frac{1}{2}\right]$

Handout solves this. Add uncertainty $\rightarrow$ convergence at a different point.

Distributional politics without uncertainty (2)

Setup: same as first scenario, except multiple groups:

- Many groups in society; share $\alpha_g < 1/2$ of voters in group g; y_g is group g's exogenous per-person income
- Two parties A and B compete by proposing per-voter net transfers for each group, where t_{gP} is net transfer to group g from party $P \in \{A, B\}$
- Party proposals must balance the budget ($\sum_g \alpha_g t_{gP} = 0$) and cannot take more than groups have (i.e. $y_g + t_{gP} \geq 0$) for $P \in \{A, B\}$
- All voters in a group vote for the party offering them more, e.g. voters in group g vote A if $t_{gA} > t_{gB}$; if tie, split evenly

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See 2.1.1. Add uncertainty → Nash eqm instead of no Nash eqm.

Divergence in the Wittman model

Divergence: one reason to consider uncertainty

Hotelling-Downs predicts platform convergence, but generally parties are not identical (nor would voters bother to vote if they were).

What is missing from the model?

Grofman (2004) "Downs and Two-Party Convergence" shows **17** ways to relax one assumption in Hotelling-Downs and produce divergence.

> *Recent work allows us to turn what is taken to be the Downsian view on its head: Although there are pressures in two-party competition for the two parties to converge, in general we should expect nonconvergence. (Grofman 2004)*

Explaining divergence without uncertainty

In Chapter 1 we already saw some ways to explain divergence without uncertainty, and you can probably think of others. **Name some.**

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- Multiple parties competing: e.g. equilibrium with two parties on each side of the median
- Citizen-candidate model: equilibria with divergent candidates competing, in which
  - Centrist would win but doesn't want to enter
  - Centrist would lose because of strategic voting

Wittman model with uncertainty

Setup:

- Parties L and R have Euclidean policy preferences, with ideal points at 0 and 1 respectively
- Position of median voter x_m uncertain; distributed uniformly on $[\mu - a, \mu + a]$

Party L chooses policy x_L to maximize

$$\pi(x_L, x_R)(-|x_L|) + [1 - \pi(x_L, x_R)](-|x_R|)$$

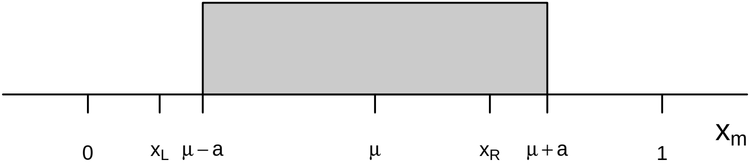
If $0 < x_L < x_R < 1$, then we can drop absolute values and this simplifies to

$$-x_R + \pi(x_L, x_R)(x_R - x_L).$$

Could proceed with $\pi(x_L, x_R)$, but given assumptions we can unpack it and find optimal x_L.

Wittman model with uncertainty (2)

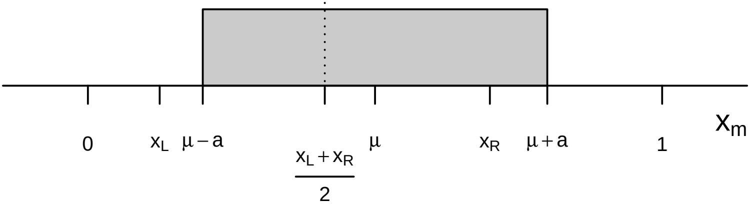
So what is the probability of L winning, given some arbitrary x_L and x_R?



Wittman model with uncertainty (3)

The voter who is indifferent between L and R is located at $\frac{x_L+x_R}{2}$.

The probability of L winning is the probability that $x_m < \frac{x_L+x_R}{2}$.



So:

$$\pi(x_L, x_R) = \frac{\frac{x_L+x_R}{2} - (\mu - a)}{(\mu + a) - (\mu - a)}$$
$$= \frac{x_L + x_R}{4a} - \frac{\mu}{2a} + \frac{1}{2}$$

Wittman model with uncertainty (4)

Then L's problem becomes

$$\max_{x_L} \left(\frac{x_L + x_R}{4a} - \frac{\mu}{2a} + \frac{1}{2} \right) (x_R - x_L)$$

yielding solutions of

$$x_L^* = \mu - a$$

and

$$x_R^* = \mu + a.$$

- Does this make sense?
- What changes to the model would lead to more or less divergence?
- How might the parties be assumed to differ?

“Excessive electoral manipulation” in the Simpser model

Simpser's puzzle of "excessive electoral manipulation"

Electoral manipulation "is frequently perpetrated far beyond the victory threshold and in excess of any plausible safety margin" (Simpser 2012, pg. 1)

Why? Several answers, including incumbent's desire to send a signal of strength.

We focus on this explanation:

1. If incumbent wins, he punishes opposition supporters who turned out to vote.
2. Therefore, many opposition supporters refuse to vote when they expect the incumbent to win.

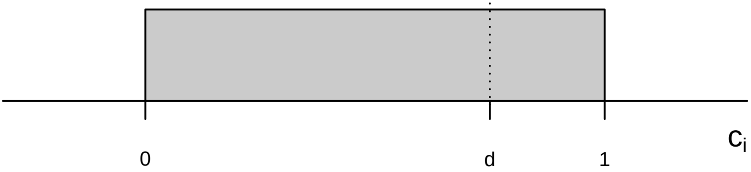
Modeling Simpser's theory (0)

What is the simplest model we could produce to capture this logic?

1. If incumbent wins, he punishes opposition supporters who turned out to vote.
2. Therefore, many opposition supporters refuse to vote when they expect the incumbent to win.

Simpser's approach (via Gehlbach):

Citizens all receive benefit $d \in (0,1)$ from voting, but they vary in the cost of voting, with c_i uniformly distributed on $[0,1]$.



Share α_O support opposition, share α_I support incumbent, with $\alpha_O > \alpha_I$.

If incumbent wins, he imposes cost s on opposition supporters who voted.

Then opposition supporter should vote if

$$d - c_i - \pi(\sigma)s > 0,$$

where $\pi(\sigma)$ denotes the probability of incumbent victory, given profile of voting strategies σ.

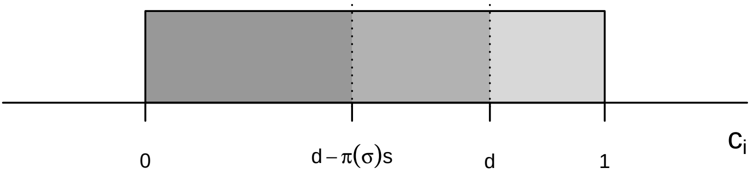
Simpser's model (2)

Rearranging, opposition supporters who vote are those with

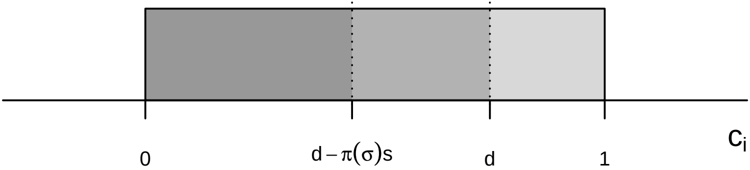
$$c_i < d - \pi(\sigma)s,$$

meaning that the opposition's *turnout rate* is $d - \pi(\sigma)s$.

The more likely the incumbent is to win, the fewer opposition supporters turn out.



Simpser's model (3)



Two equilibria:

- **Opposition victory** Suppose $\pi(\sigma) = 0$ (incumbent sure to lose). Then every opposition member with $c_i < d$ votes, in which case $\pi(\sigma) = 0$, because we assumed $\alpha_O > \alpha_I$.
- **Incumbent victory** Suppose $\pi(\sigma) = 1$ (incumbent sure to win). Then only opposition members with $c_i < d - s$ vote. If $(d - s)\alpha_O < d\alpha_I$, then $\pi(\sigma) = 1$.

When we see "excessive electoral manipulation", it may be because we are in the second equilibrium. Was it really "excessive"?

Another look at Simpser's model (4a)

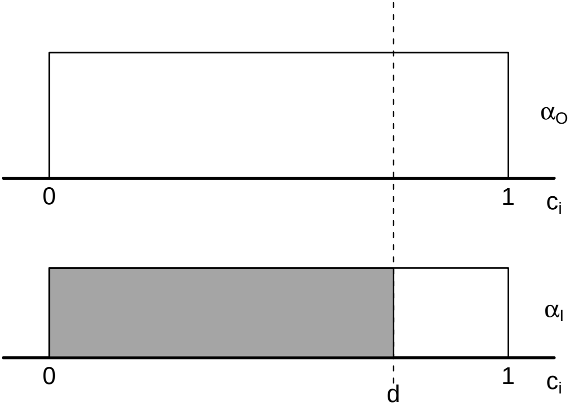
Basic setup: $\alpha_O > \alpha_I$, $c_i \sim \mathrm{Unif}(0,1)$ in both groups

Another look at Simpser's model (4b)

Basic setup: intrinsic benefit $d \in (0, 1)$ for all voters

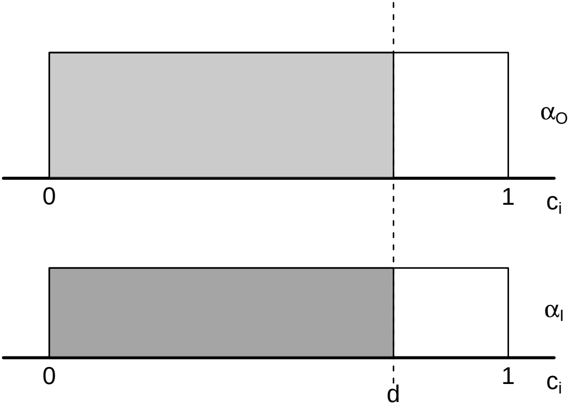
Another look at Simpser's model (4c)

Incumbent supporters with $c_i < d$ vote



Another look at Simpser's model (4d)

Eqm 1: Opposition wins, no extra cost for opposition voters



Another look at Simpser's model (4e)

Eqm 2: Opposition loses, extra cost *s* for opposition voters