Formal Analysis: Electoral competition under certainty

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Week 2 Session 2

Hotelling-Downs

Original article: setup

Harold Hotelling, "Stability in Competition", *The Economic Journal*, 1929.

Consider the following illustration. The buyers of a commodity will be supposed uniformly distributed along a line of

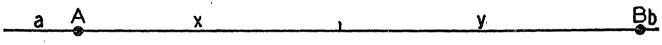


Fig. 1.

Market of length $l = 35$. In this example $a = 4, b = 1, x = 14, y = 16$.

length l, which may be Main Street in a town or a transcontinental railroad. At distances a and b respectively from the two ends of this line are the places of business of A and B (Fig. 1). Each buyer transports his purchases home at a cost c per unit distance. Without effect upon the generality of our conclusions we shall suppose that the cost of production to A and B is zero, and that unit quantity of the commodity is consumed in each unit of time in each unit of length of line. The demand is thus at the extreme of inelasticity. No customer has any preference for either seller except on the ground of price plus transportation cost. In general

Original article: varying location

Harold Hotelling, “Stability in Competition”, *The Economic Journal*, 1929.

As a further problem, suppose that A’s location has been fixed but that B is free to choose his place of business. Where will he set up shop? Evidently he will choose b so as to make

$$\pi_2 = \frac{c}{2}\left(l + \frac{b-a}{3}\right)^2$$

as large as possible. This value of b cannot be found by differentiation, as the value thus determined exceeds l and, besides, yields a minimum for π_2 instead of a maximum. But for all smaller values of b, and so for all values of b within the conditions of the problem, π_2 increases with b. Consequently B will seek to make b as large as possible. This means that he will come just as close to A as other conditions permit. Naturally, if A is not

Original article: application to politics

Harold Hotelling, "Stability in Competition", *The Economic Journal*, 1929.

fashion and imitation. But over and above these forces is the effect we have been discussing, the tendency to make only slight deviations in order to have for the new commodity as many buyers of the old as possible, to get, so to speak, *between* one's competitors and a mass of customers.

So general is this tendency that it appears in the most diverse fields of competitive activity, even quite apart from what is called economic life. In politics it is strikingly exemplified. The competition for votes between the Republican and Democratic parties does not lead to a clear drawing of issues, an adoption of two strongly contrasted positions between which the voter may choose. Instead, each party strives to make its platform as much like the other's as possible. Any radical departure would lose

One approach for finding Nash equilibria

In previous session, we solved for SPNE in an **sequential game** via backwards induction:

1. Figure out player 2's best response to each possible action by player 1
2. Figure out player 1's best action, given that player 2 will best-respond

Any best-response by 1 to a best-response by 2 is a Nash equilibrium.

But this approach may not find **all** Nash equilibria. **(Why not? Examples?)**

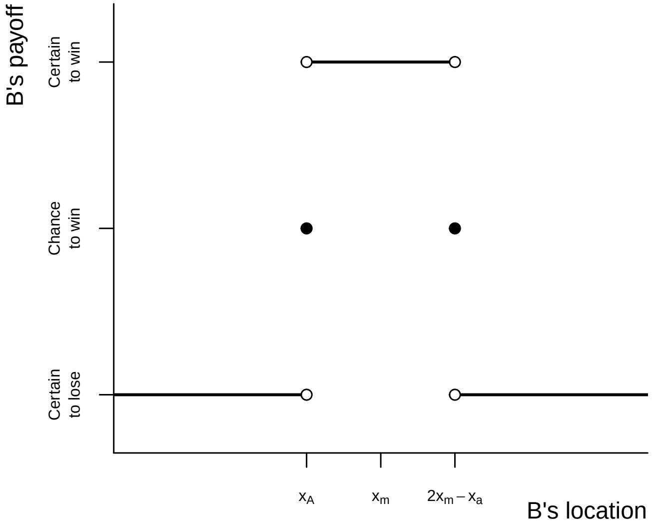
Hotelling-Downs

1. **Parties**: two parties, A and B, choosing position on $\mathbb{R}$. They want to win office.
2. **Voters**: Continuum of voters with ideal point $x_i \in \mathbb{R}$. Given policy x, voter i's utility is $u_i(x) = -|x - x_i|$. Voters vote sincerely and abstain if indifferent.
3. **Election rules**: plurality rule, with a fair lottery if election is tied.

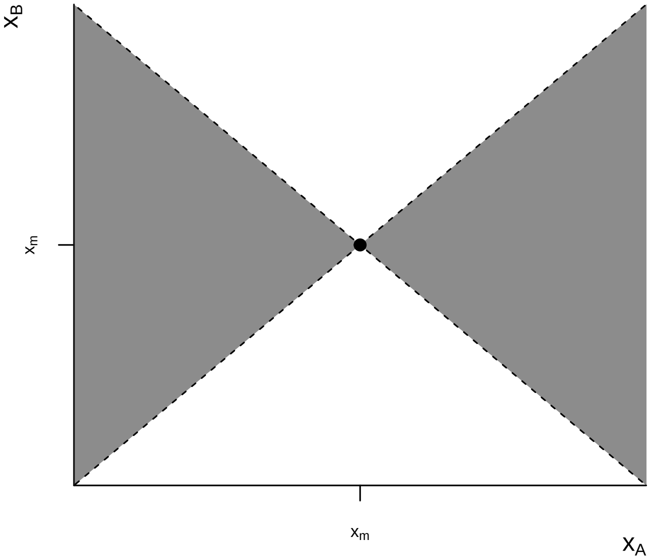
Method of finding equilibria:

1. Figure out party B's *best response correspondence* to each possible action by party A
2. Figure out party A's best action, given that party B will best-respond

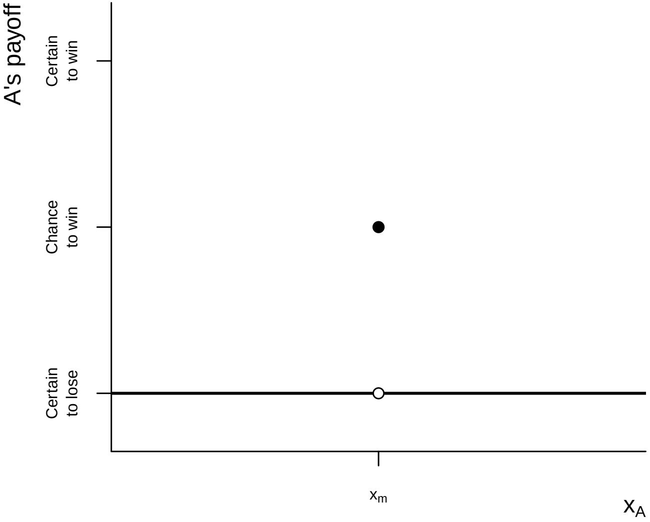
Hotelling-Downs: payoff of party B (given A has chosen x_A)



Hotelling-Downs: optimal move by party B as function of x_A



Hotelling-Downs: payoff of party A (given B is best-responding)



Competition when policy is multidimensional

Condorcet winner and Condorcet's paradox

A Condorcet winner defeats any other alternative in a pairwise majority vote.

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1. $x \succ y \succ z$
2. $y \succ z \succ x$
3. $z \succ x \succ y$

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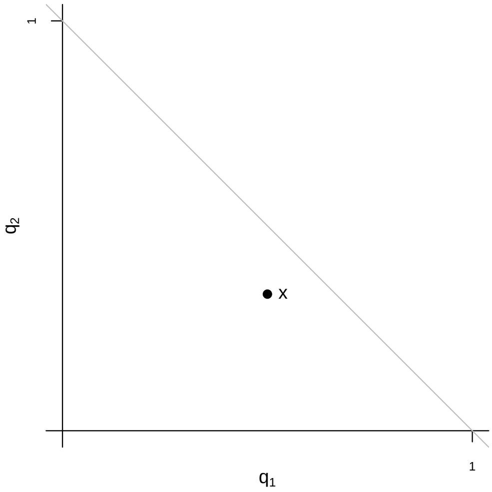
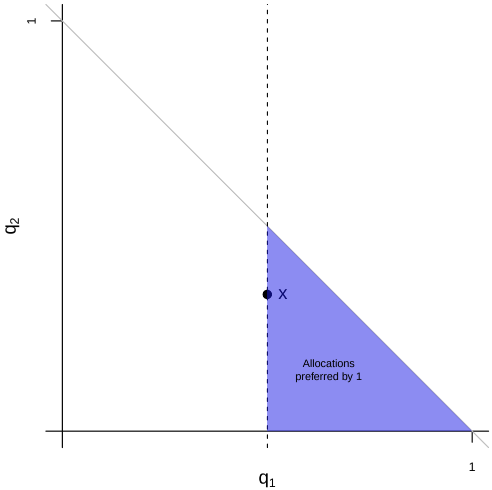
Then no Condorcet winner.

No Condorcet winner when dividing a pie



No Condorcet winner when dividing a pie



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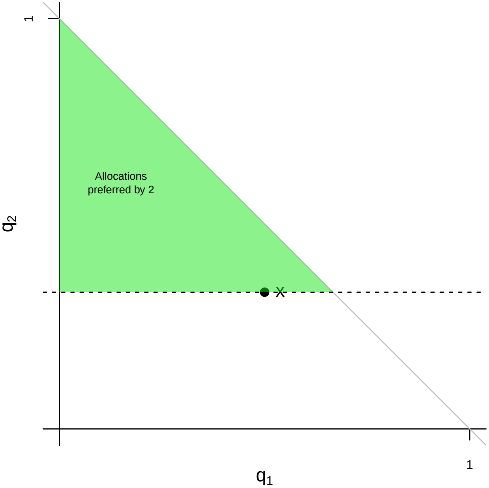

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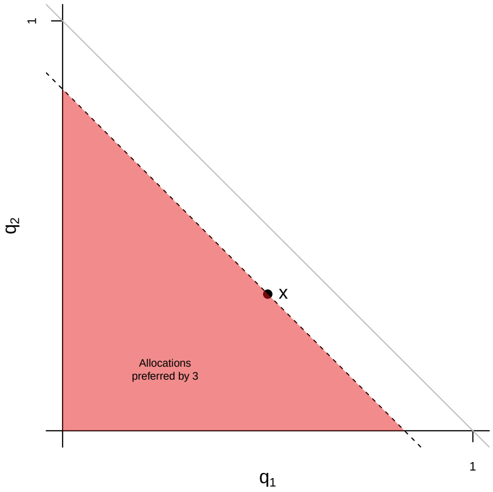
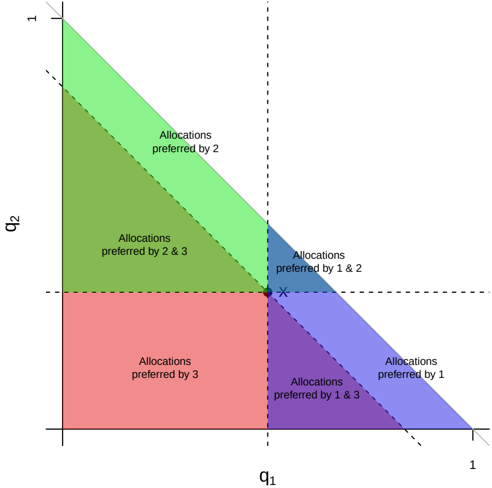
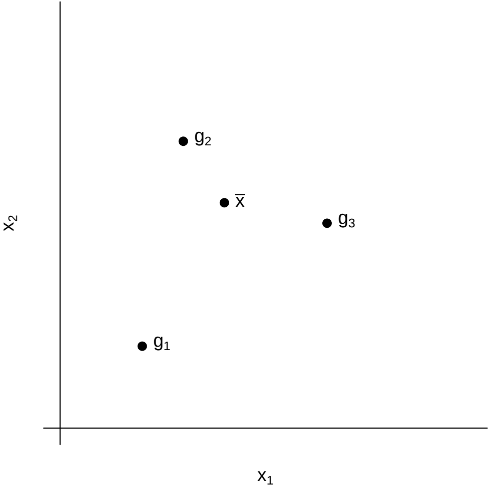


No Condorcet winner when dividing a pie



Two policy dimensions, three groups w. Euclidean preferences



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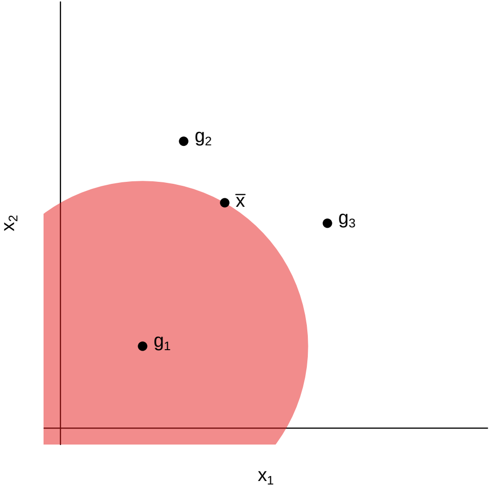

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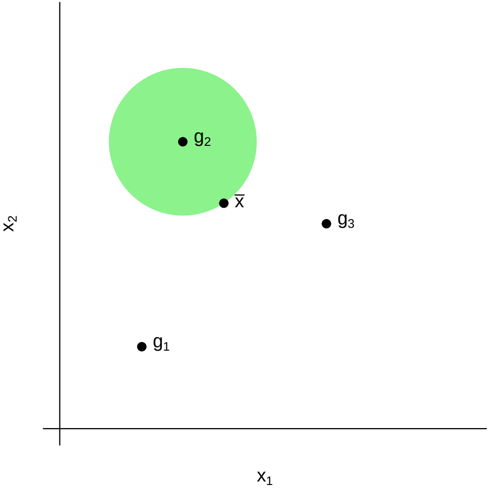

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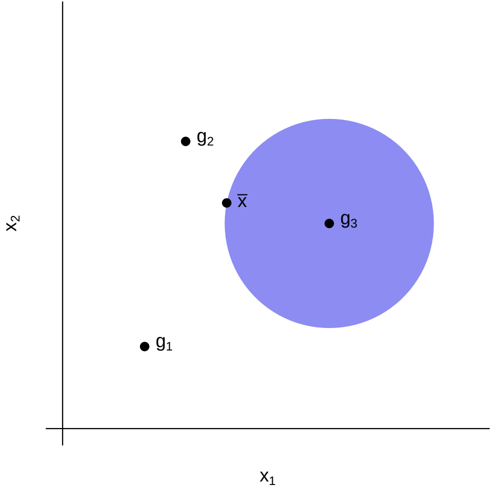

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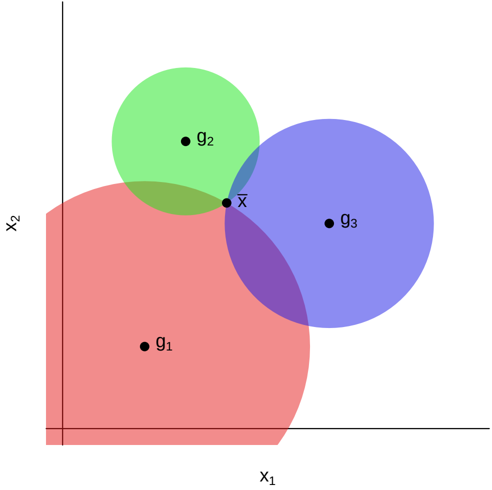
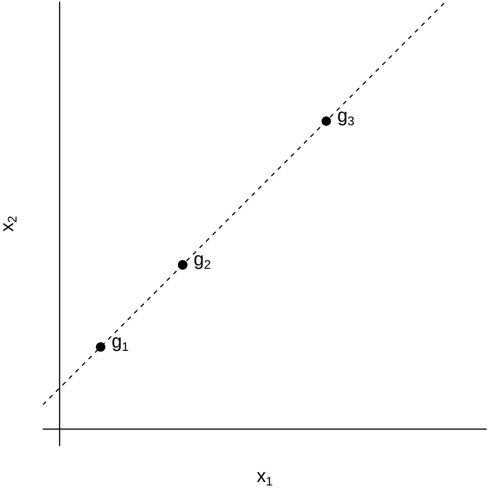


Two dimensions that are really just one



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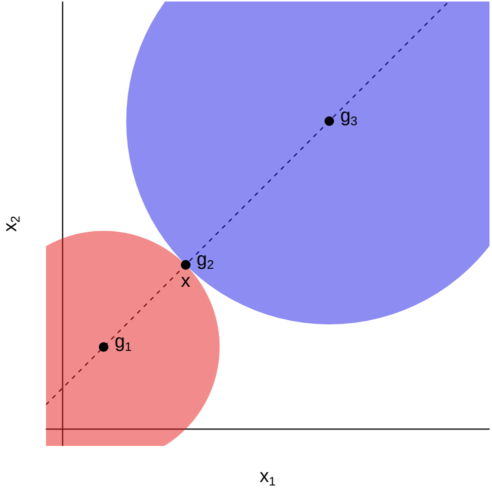


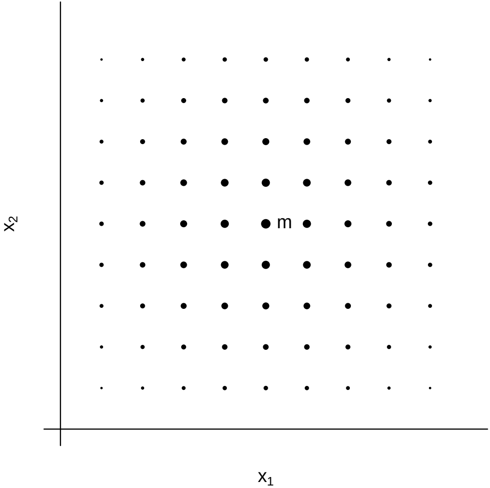
When is there a Condorcet winner in 2 dimensions?

Definition: A *median line* is a line such that at least half the voter ideal points lie either on it or to the right of it and at least half the voter ideal points lie either on it or to the left of it.

Theorem (Davis, DeGroot, and Hinich, 1972): There exists a Condorcet winner if and only if there exists a voter's ideal point, M, such that every line passing through it is a median line. If so, the alternative, M, corresponding to that point will be a Condorcet winner.

Feld and Grofman, *AJPS* 1987, "Necessary and sufficient conditions for a majority winner in *n*-dimensional spatial voting games: an intuitive geometric approach"

Graphical illustration



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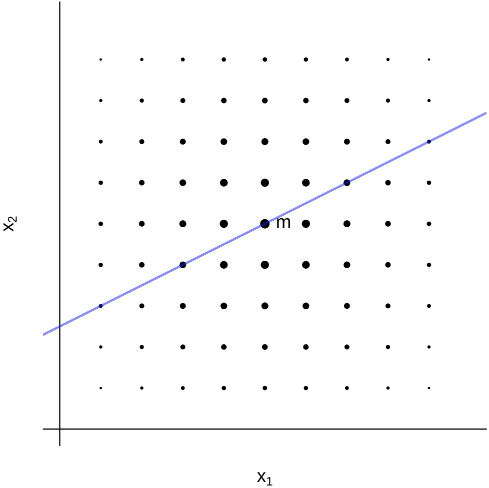
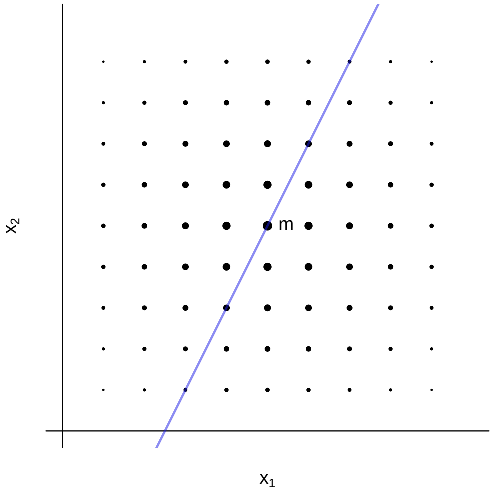


Graphical illustration



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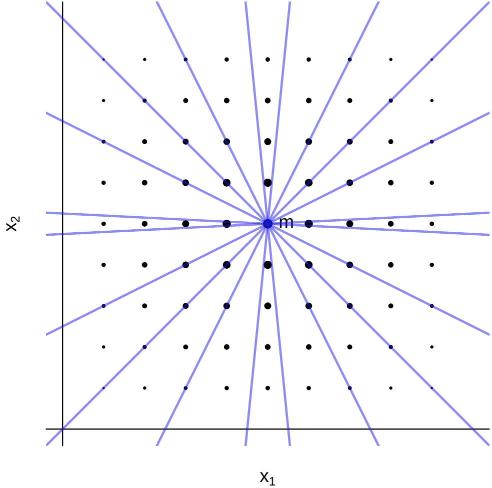


Multiparty competition

Cox's Lemma 1

Suppose $M > 2$ vote-maximizing parties, sincere voters, and unidimensional policy space.

Cox (1987) proves that in equilibrium:

1. No more than two parties occupy any one position.
2. Each extremist position (i.e. left-most or right-most among all parties) is occupied by exactly two parties.
3. If two parties occupy the same position x, then the share of voters to the left of x who most prefer the parties at x must be equal to the share of voters to the right of x who most prefer the parties at x.

These conditions are stated without proof in Gehlbach. Let's prove them.

Proving Cox's Lemma 1 (1)

1. No more than two parties occupy any one position.

Suppose $n \geq 2$ parties are at a position x, with L voters to the left of x and R voters to the right of x preferring the parties at x to any other. Without loss of generality, let $L \geq R$. The parties at x each win $\frac{L+R}{n}$ in vote share. If party j, located at x, deviates slightly to the left, she wins a vote share of L. For $n \geq 2$ parties to be located at position x in equilibrium, then, it must be the case that $L \leq \frac{L+R}{n}$, which can be rearranged to $L \leq \frac{R}{n-1}$. For $n = 2$ this could be true if $L = R$, but given the assumption that $L \geq R$ it cannot be true for $n > 2$.

Proving Cox's Lemma 1 (2)

2. Each extremist position (i.e. left-most or right-most among all candidate positions) is occupied by exactly two parties.

We have already shown that no position can be occupied by more than two parties. Suppose a single party, j, is at an extremist position x. Without loss of generality we shall say that this is the left-most position. By moving to the right j can win more votes.

Proving Cox's Lemma 1 (3)

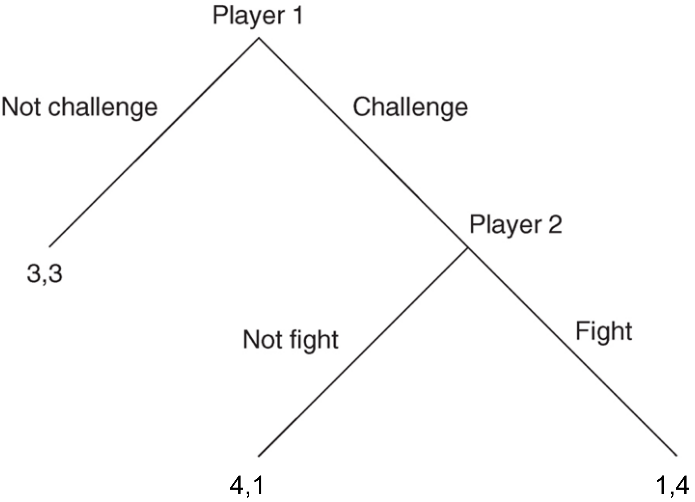
3. If two parties occupy the same position x, then the share of voters to the left of x who most prefer the parties at x must be equal to the share of voters to the right of x who most prefer the parties at x.

The proof of the first point above showed that 2 parties locating at a single position can be an equilibrium only if $L = R$.

To prove the specific point here: Suppose 2 parties are located at x, with L voters to the left of x and R voters to the right of x preferring the parties at x to any other. The parties at x each win $\frac{L+R}{2}$ in vote share. If $L > R$, then one of the parties would benefit from moving slightly to the left, thus winning $L > \frac{L+R}{2}$.

“Off the equilibrium path”

Easy example

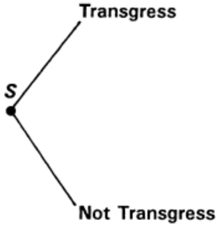


SPNE is: {Not challenge; Fight} – but "Fight" is **off the equilibrium path**.

Weingast (1997) "The political foundations of democracy and the rule of law"

FIGURE 2. Payoffs for the Sovereign-Constituency Coordination Game

S Moves first



Induced subgame between *A* and *B* (payoffs: *S,A,B*)

Transgress:

A \ *B*	Acquiesce	Challenge
Acquiesce	8, 2, 2	8, 2, 1
Challenge	8, 1, 2	0, 7, 7

Not Transgress:

A \ *B*	Acquiesce	Challenge
Acquiesce	2, 8, 8	2, 8, 7
Challenge	2, 7, 8	0, 7, 7

Weingast (1997) (2)

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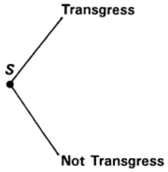

Transgress:

		B: Acquiesce	*B*: Challenge
A	Acquiesce	8, 2, 2	8, 2, 1
	Challenge	8, 1, 2	0, 7, 7

Not Transgress:

		B: Acquiesce	*B*: Challenge
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Weingast (1997) (2)

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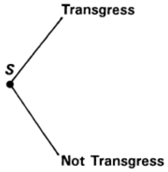

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Not Transgress:

		B: Acquiesce	*B*: Challenge
A	Acquiesce	2, 8, 8	2, 8, 7
	Challenge	2, 7, 8	0, 7, 7

Two SPNEs:

- {Transgress; Acquiesce if transgress, acquiesce if not transgress; Acquiesce if transgress, acquiesce if not transgress}
- {Not transgress; Challenge if transgress, acquiesce if not transgress; Challenge if transgress, acquiesce if not transgress}

Use the phrase “off the equilibrium path” to describe these SPNEs.

"Off the equilibrium path" in Gehlbach chapter 1

Feddersen-Sened-Wright model: Like Hotelling-Downs, but

- M candidates compete, with endogenous entry (benefit of winning v and cost of entry δ)
- voters vote strategically.

There exist equilibria in which 3 or more candidates locate at the median, x_m.

> *Such equilibria are supported by the following voting strategies off the equilibrium path: if any potential candidate . . . deviates to a position other than x_m, then all voters with ideal points equal to or to the other side of x_m vote for one of the candidates who remains at x_m.*

"Off the equilibrium path" in Gehlbach chapter 1 (2)

Citizen-candidate model: Candidates cannot commit to implement policies other than their (common-knowledge) ideal point; endogenous entry.

With strategic voting, can have equilibria with one candidate on the extreme left and one on the extreme right. This is sustained by voting strategies **off the equilibrium path** in which, if a centrist entered, voters would continue to vote for their preferred extremist.