Formal Analysis: Bargaining

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Week 2 Session 1

Efficiency

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Definition 2.5 (Kydd p. 17): Given a set of actors with utility functions u_i defined over an outcome space X, an outcome $x' \in X$ is efficient if for any other outcome $x'' \in X$ that makes some player i better off, $u_i(x'') > u_i(x')$, there must be some other actor j that is worse off, $u_j(x') > u_j(x'')$.

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Related definitions:

- An outcome is **inefficient** (Pareto inefficient) if there *is* some way to make some players better off without making anyone worse off
- An efficient outcome is **Pareto optimal** (but not necessarily good!)
- An inefficient outcome is **Pareto inferior** to another outcome

A question about efficiency

Of the four possible strategy profiles in the prisoner's dilemma game, which are efficient, and which are inefficient?

Solution

The strategy profile $\{D, D\}$ is inefficient: moving to $\{C, C\}$ would make both players better off.

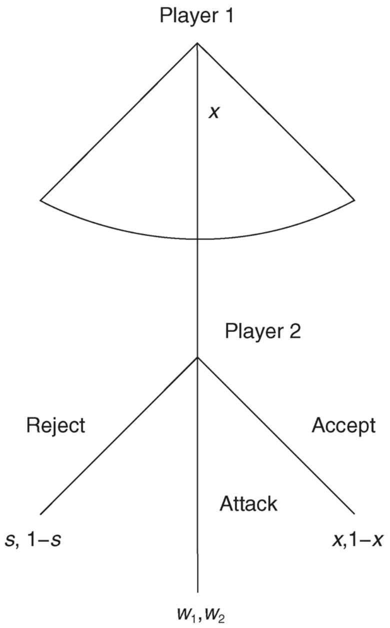
All three other strategy profiles are efficient. For example, $\{C, D\}$ yields a higher profile for player 2 than any other profile.

Bargaining

What is the point of these models?

The central puzzle about war, and also the main reason we study it, is that wars are costly but nonethelesss wars recur. Scholars have attempted to resolve the puzzle with three types of arguments. First, one can argue that people (and state leaders in particular) are sometimes or always irrational. They are subject to biases and pathologies that lead them to neglect the costs of war or to misunderstand how their actions will produce it. Second, one can argue that the leaders who order war enjoy its benefits but do not pay the costs, which are suffered by soldiers and citizens. Third, one can argue that even rational leaders who consider the risks and costs of war may end up fighting nonetheless. This article focuses on arguments of the third sort, which I will call rationalist explanations. (Fearon, "Rationalist explanations for war", 1995)

Bargaining with conflict



Backwards induction

"In games of complete information, . . . subgame perfection is equivalent to *backwards induction*. Backwards induction is solving the game from the terminal nodes, working backwards to the initial node. At each node, the optimal choice is made and the payoffs of the chosen successor node are implicitly substituted for the node. Then at the previous node the optimal choice is made, given the understanding of what would happen subsequently." (Kydd p. 59)

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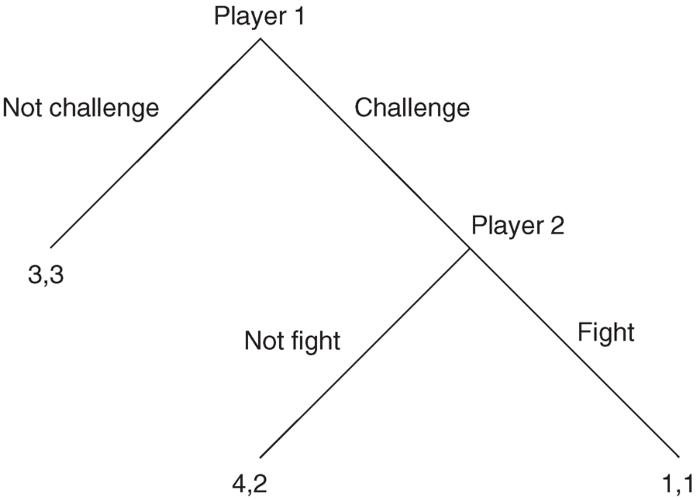
We find **subgame perfect Nash equilibria** (SPNE) with backwards induction.

Backwards induction in practice: player 1 choosing between two actions



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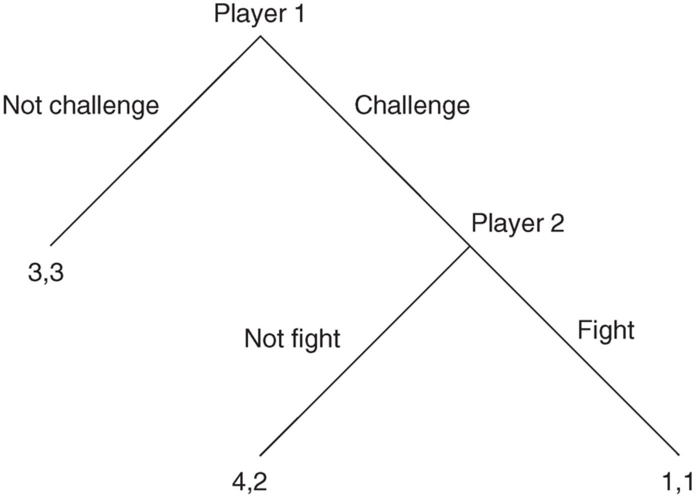
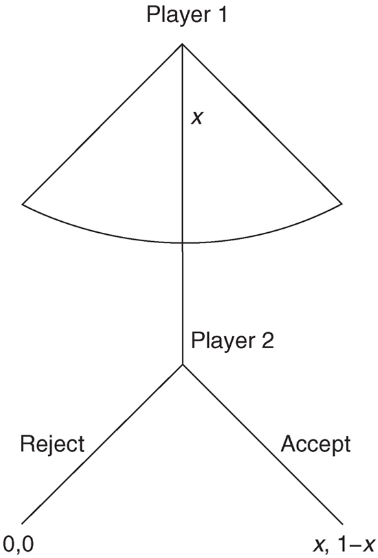
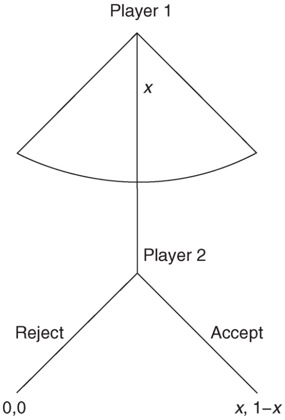


SPNE: {Challenge, Not fight}

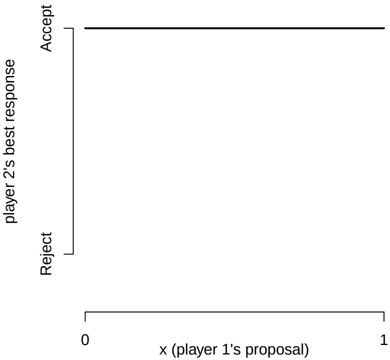
Backwards induction in practice: ultimatum game (continuous proposal)



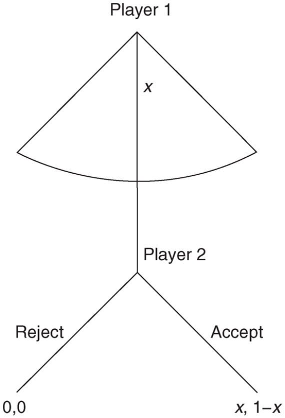
Backwards induction in practice: ultimatum game (2)



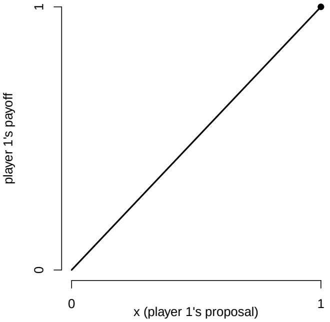
First, **player 2's best response** to all possible proposals from player 1:



Backwards induction in practice: ultimatum game (3)



Now, **player 1's payoff** as a function of player 1's proposal x (given 2's best response):



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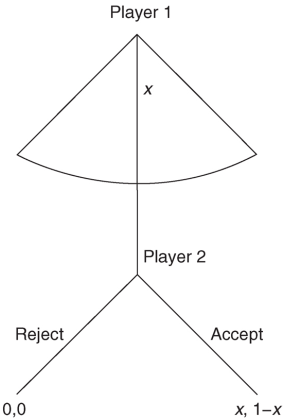

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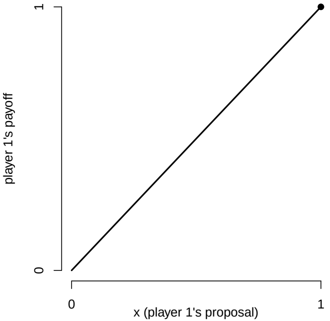
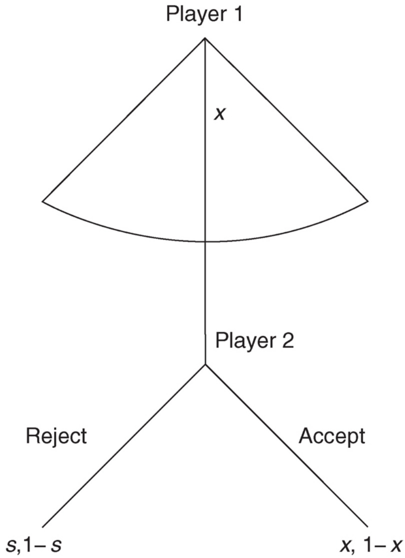
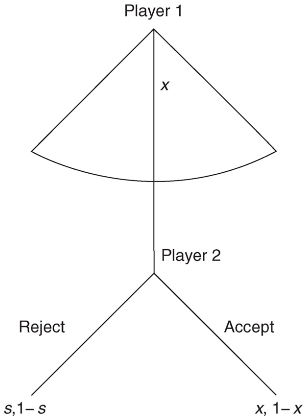


SPNE is: $\{$Propose $x = 1$, accept any proposal$\}$

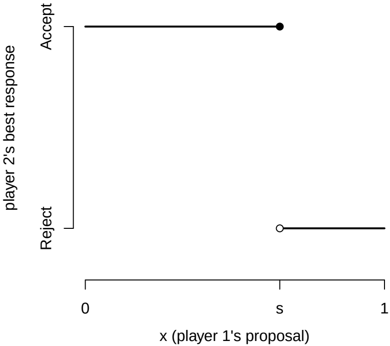
Backwards induction in practice: ultimatum game with status quo



Backwards induction in practice: ultimatum game w. SQ (2)



First, **player 2's best response** to any possible proposal x:



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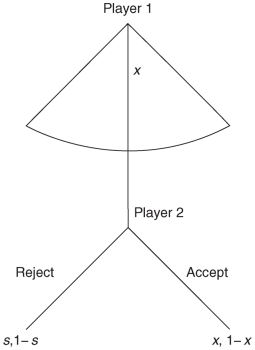

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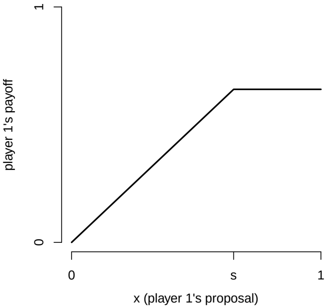

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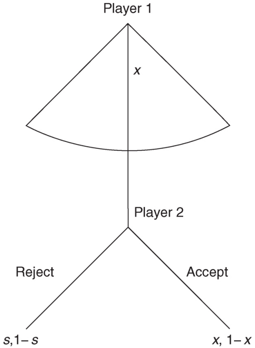

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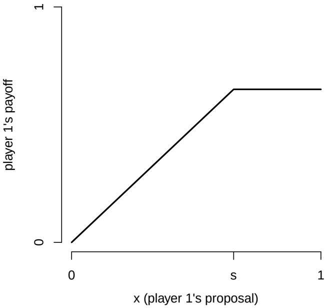

SPNE: $\{$Propose $x \geq s$, accept $x \leq s\}$

Bargaining and war

Underlying puzzle

War is costly: it destroys some of the resource that states are fighting over. Why can't a peaceful allocation be found that makes everyone (weakly) better off?

Answer in Kydd's Chapter 4: the resource at issue may be less valuable when divided, such that they prefer to fight rather than divide it.

Key concept: bargaining range

The **bargaining range** is "the set of agreements that both sides prefer to conflict" (Kydd, 64).

In rest of book, assuming bargaining range exists, but cannot be *identified* or *implemented*.

Chapter 4 considers possibility it does not exist.

Question: Did a bargaining range exist in a historical conflict you know about?

Baseline case

Let x be player 1's share of the good and $1 - x$ player 2's share.

Suppose each player gets (cardinal) utility 1 from possessing the whole good and utility 0 from possessing none of the good. (We have normalized.)

If there is a war,

- player 1 wins with probability p and player 2 wins with probability $1 - p$;
- the winner obtains the whole good; and
- each side loses utility $c \geq 0$ from fighting.

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Question: Does a bargaining range exist?

Baseline case with linear preferences

Same as previous slide:

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Question: Does a bargaining range exist? What is it?

Baseline case w. linear prefs: bargaining range

1 prefers x to war if

$$x > p - c$$

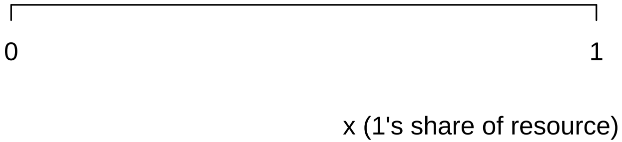
2 prefers x to war if

$$1 - x > 1 - p - c$$

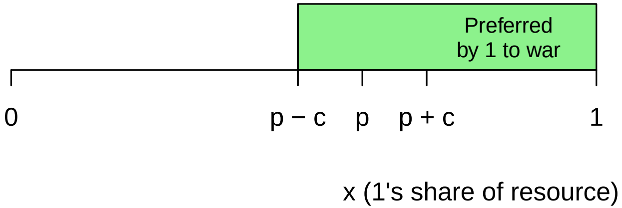
i.e.

$$x < p + c$$

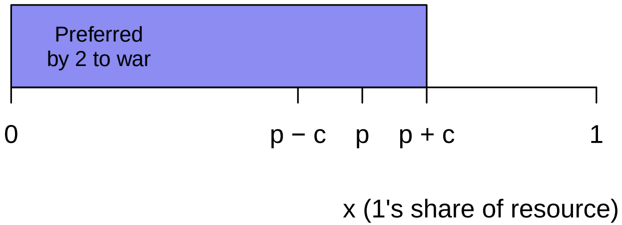
Bargaining range with linear prefs (1)



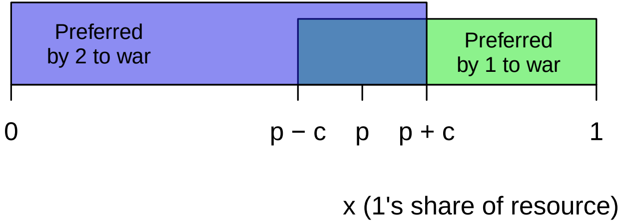
Bargaining range with linear prefs (2)



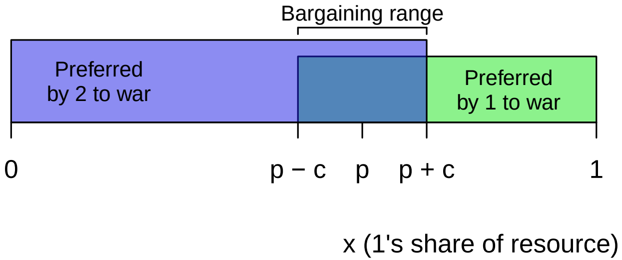
Bargaining range with linear prefs (3)



Bargaining range with linear prefs (4)



Bargaining range with linear prefs (5)



More general preferences

Suppose $u_1(x) = x^a$ and $u_2(x) = (1 - x)^a$, for $a > 0$.

Otherwise same setup.

Does a bargaining range exist?

More general preferences: condition for bargaining range

1 prefers x to war if	2 prefers x to war if
$x^a > p - c$	$(1-x)^a > 1 - p - c$
i.e.	i.e.
$x > (p-c)^{\frac{1}{a}}$	$1 - (1-p-c)^{\frac{1}{a}} > x$

Whether bargaining range exists depends on a, p, c.

Let's try different values (and also consider status quo) at https://andyeggers.shinyapps.io/intermediate_values/

Discussion

- What aspects of bargaining and conflict does this capture? What is missing?
- What political phenomena other than inter-state war might this model describe?