Formal Analysis: Costly signaling (tying hands)

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Week 8 Session 1

Tying hands

Throwing out the steering wheel

Game of chicken

		Player 2: Swerve	Player 2: Straight
Player 1	Swerve	3, 3	2, 4
	Straight	4, 2	1, 1

Game of chicken after 1 removes steering wheel

		Player 2: Swerve	Player 2: Straight
Player 1	Straight	4, 2	1, 1

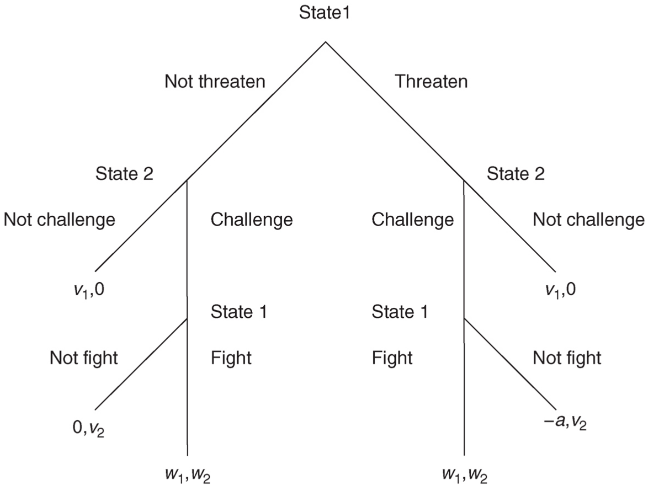
Burning ships/bridges

There are many accounts of military conquest in which the conqueror is said to have eliminated options of escape.

William the Conqueror (England, 1066) and Hernán Cortés (Mexico, 1519-1521) are said to have **burned their ships** on arrival to make escape impossible.

What could motivate this behavior? How could we use a model to explore the possible logic?

Actions that change future payoffs



An alternative approach

Assume probability of conflict is CSF, where e_i is i's effort:

$$\Pr(1 \text{ wins}) \equiv p_1 \equiv \frac{e_1}{e_1 + e_2}$$

The value of winning is 1. The value of losing is v_l. Expending effort e_1 costs γe_1.

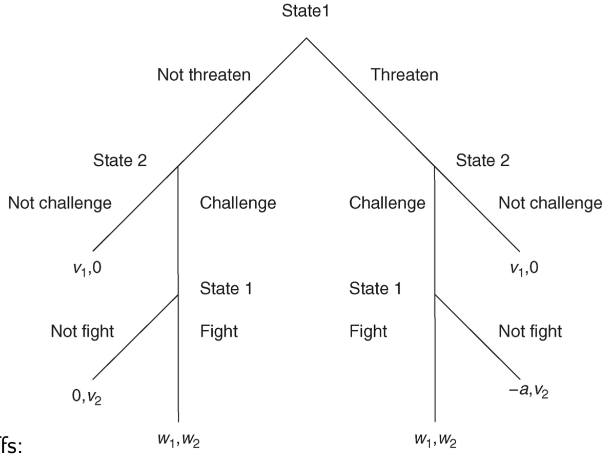
Then expected utility is:

$$\frac{e_1}{e_1 + e_2} + \left(1 - \frac{e_1}{e_1 + e_2}\right) v_l - e_1$$

What can player 1 accomplish by reducing v_l (e.g. by making escape impossible)?

Analysis of the costly signaling game in Kydd: complete information case

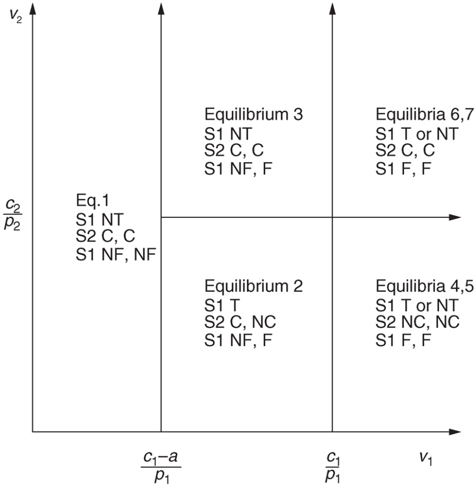
The complete information case



War payoffs:

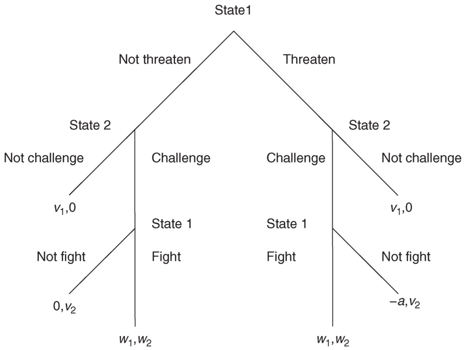
- **Player 2**: $w_2 = p_2 v_2 - c_2 \implies$ fight if $v_2 > \frac{c_2}{p_2}$
- **Player 1**: $w_1 = p_1 v_1 - c_1 \implies$ fight if
  - $v_1 > \frac{c_1}{p_1}$, assuming **did not** issue threat
  - $v_1 > \frac{c_1}{p_1} - a$, assuming **did** issue threat

The complete information case (2)



The interesting question

Q: Under what conditions would it be valuable to state 1 to be able to impose a cost a on backing down from a threat?

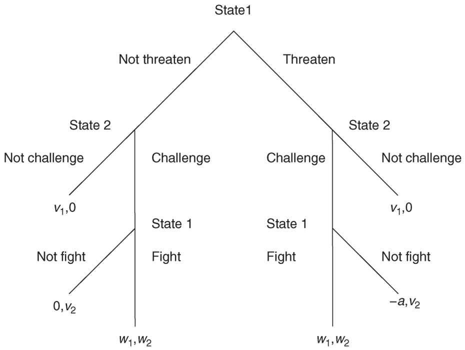


The interesting case

Q: Under what conditions would it be valuable to state be able to impose a cost *a* on backing down from a threat?

A: When state 1 is the type who would back down rather than fight.

Q: When is that true?



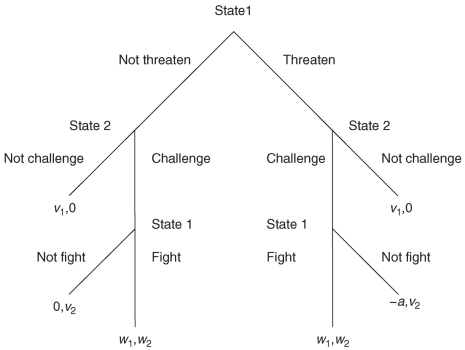
The interesting case

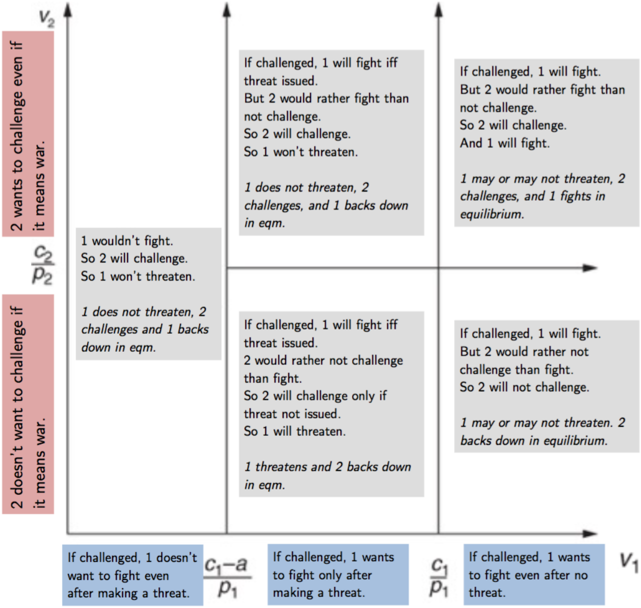
Q: Under what conditions would it be valuable to state be able to impose a cost a on backing down from a threat?

A: When state 1 is the type who would back down rather than fight.

Q: When is that true?

A: When $w_2 < 0$, i.e. when $v_2 < \frac{c_2}{p_2}$.





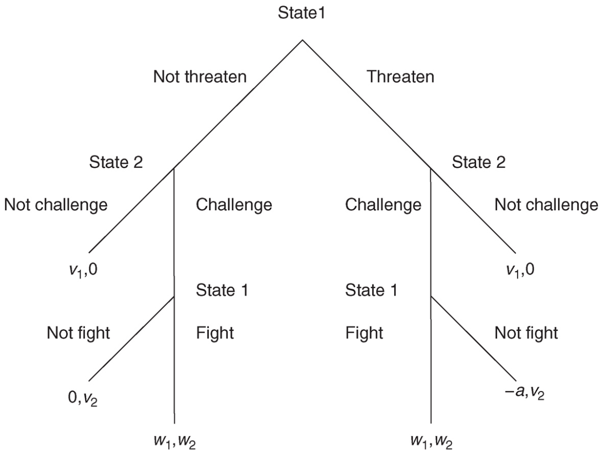
V_2
2 wants to challenge even if it means war.
$\frac{c_2}{p_2}$
2 doesn't want to challenge if it means war.
1 wouldn't fight.
So 2 will challenge.
So 1 won't threaten.
1 does not threaten, 2 challenges and 1 backs down in eqm.
If challenged, 1 will fight iff threat issued.
But 2 would rather fight than not challenge.
So 2 will challenge.
So 1 won't threaten.
1 does not threaten, 2 challenges, and 1 backs down in eqm.
If challenged, 1 will fight.
But 2 would rather fight than not challenge.
So 2 will challenge.
And 1 will fight.
1 may or may not threaten, 2 challenges, and 1 fights in equilibrium.
If challenged, 1 will fight iff threat issued.
2 would rather not challenge than fight.
So 2 will challenge only if threat not issued.
So 1 will threaten.
1 threatens and 2 backs down in eqm.
If challenged, 1 will fight.
But 2 would rather not challenge than fight.
So 2 will not challenge.
1 may or may not threaten. 2 backs down in equilibrium.
If challenged, 1 doesn't want to fight even after making a threat.
$\frac{c_1-a}{p_1}$
If challenged, 1 wants to fight only after making a threat.
$\frac{c_1}{p_1}$
If challenged, 1 wants to fight even after no threat.
V_1

Analysis of the costly signaling game in Kydd: incomplete information case

What does incomplete information mean here?

The values v_1 & v_2 are distributed according to f_1 & f_2 (F_1 & F_2).

Backward induction harder when you don't know the other player's type.



Approach to solving the incomplete information game

Types of state 1 with v_1 above a certain value will threaten.

Two cases:

- **no-bluffing equilibrium**: cutoff is between $\frac{c_1 - a}{p_1}$ and $\frac{c_1}{p_1}$, so the type at the threshold is one who would fight if challenged
- **no-bluffing equilibrium**: cutoff is below $\frac{c_1 - a}{p_1}$, so the type at the threshold is one who would not fight if challenged

Approach to solving the incomplete information game: no-bluff equilibrium

No-bluffing equilibrium: threat cutoff is between $\frac{c_1-a}{p_1}$ and $\frac{c_1}{p_1}$, so the type at the threshold is one who would fight if challenged

State 2 knows it will have a war if it challenges a state who has threatened $\implies$ state 2 challenges only if $v_2 > \frac{c_2}{p_2} \equiv v_2^*$.

Where should the threat cutoff be?

Approach to solving the incomplete information game: no-bluff equilibrium (2)

Where should state 1's threat cutoff v_1^* be?

If threaten, then two possibilities:

- $v_2 < v_2^*$: state 2 does not challenge, state 1 gets v_1
- $v_2 > v_2^*$: state 2 challenges, state 1 gets $p_1 v_1 - c_1$

If not threaten, then state 2 will challenge and state 1 will get 0.

How do we solve for the optimal v_1^*?