Formal Analysis: Cooperation theory

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Week 5 Session 2

Cooperation theory

Big picture

Cooperation theory studies situations where players face "social dilemmas" but can overcome them through voluntary arrangements, i.e. cooperation (cf outside enforcement).

Broad goals:

- explain variation in cooperation
- explain role of "institutions" in achieving cooperation
- design institutions for cooperation

Going beyond IR

In what situations must arrangements be voluntary, i.e. not rely on outside enforcement?

Conditions of anarchy:

- international relations
- state's interactions with its citizens
- historical long-distance trade (Greif)
- state formation and state failure
- illegal activites (mafia, pirates)
- quasi-illegal activities (lobbying)
- incomplete contracts

Escaping social dilemmas (e.g. PD)

Why would some groups be able to cooperate when others can't, given the same raw payoffs (i.e. technology)?

- internalized values
- government
- repetition (& monitoring)

Repeated games: theory and evidence

Reminder about the value of discounted future payoffs

Let δ be the discount rate/probability of continuation, and let x be the payoff in each time period.

Define $V \equiv x + \delta x + \delta^2 x + \ldots$.

To restate V more compactly, note

$$V - \delta V = x$$

So $V = \frac{x}{1-\delta}$.

Stationary bandit

The citizen have an endowment y from which they can choose to invest $x \in [0, y]$ in some productive activity. The ruler then takes proportion τ. The citizens' payoffs τx. The game repeats.

The ruler has discount rate δ.

A prisoner's dilemma game

Let δ be the discount rate/probability of continuation.

Under what condition can C, C be sustained by "grim trigger" strategies given this stage game?

		Blue player	
		C	D
Red player	C	75, 75	10, 100
	D	100, 10	45, 45

An experiment (Dal Bo 2005)

Subjects in a lab paired and asked to play PD game below (or another very similar).

Treatments:

- **Infinite game**: play game; play again with same partner with probability $\delta \in \{0, \frac{1}{2}, \frac{3}{4}\}$; repeat
- **Finite game**: play game $H \in \{1, 2, 4\}$ times with same partner

		Blue player	
		C	D
Red player	C	75, 75	10, 100
	D	100, 10	45, 45

Results (Dal Bo 2005)

Treatments:

- **Infinite game**: play game; play again with same partner with probability $\delta \in \{0, \frac{1}{2}, \frac{3}{4}\}$; repeat
- **Finite game**: play game $H \in \{1, 2, 4\}$ times with same partner

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TABLE 6—PERCENTAGE OF COOPERATION BY ROUND AND TREATMENT*

		Round											
		1	2	3	4	5	6	7	8	9	10	11	12
Dice	$\delta = 0$	9.17											
	$\delta = \frac{1}{2}$	30.93	26.10	19.87	12.50	12.96							
	$\delta = \frac{3}{4}$	46.20	40.76	38.76	34.58	33.04	27.27	24.75	26.28	29.17	26.04	32.29	31.25
Finite	$H = 1$	10.34											
	$H = 2$	13.31	6.90										
	$H = 4$	34.58	21.55	18.97	10.63								

* *All sessions, matches four through ten.*

Question: How do the results compare with theory?

Problem set 1

Utilities in the strategic voting question

An election takes place with K candidates under first-past-the-post (plurality) rules.

You survey a voter and ask her to indicate how much she likes each candidate on a 0-100 scale. Denote by u_i the voter's response with respect to candidate i, and denote by $\mathbf{u}$ the vector of responses $\{u_1, u_2, \ldots, u_K\}$.

(1.1) If you could ask the voter additional questions, how could you determine whether the elements of $\mathbf{u}$ are ordinal utilities corresponding to election outcomes?

(1.2) If you could ask the voter additional questions, how could you determine whether the elements of $\mathbf{u}$ are VNM (cardinal) utilities corresponding to election outcomes?

Confusion about ordinal utilities

(1.1) If you could ask the voter additional questions, how could you determine whether the elements of $\mathbf{u}$ are ordinal utilities corresponding to election outcomes?

I was not asking, "Are the voter's preferences rational (complete, transitive)".

I was asking whether $\mathbf{u}$ is an **ordinal representation of those preferences**.

This is true if, for any pair of candidates, a higher score identifies the voter's preferred candidate, i.e. $u_i \geq u_j \iff$ voter weakly prefers candidate i to candidate j.

Confusion about cardinal utilities

(1.2) If you could ask the voter additional questions, how could you determine whether the elements of $\mathbf{u}$ are VNM (cardinal) utilities corresponding to election outcomes?

Common but incorrect response: "If the voter gives a score of 10 to a and 5 to b, ask voter if she gets twice as much utility from a as from b." But how can the voter answer that question?

Evaluating VNM/cardinal utilities requires talking about a **lottery**.

Checking equilibria

When asked, "Is strategy profile σ an equilibrium?" or asked to "Show that strategy profile σ is not an equilibrium":

just check whether either player has a profitable deviation

if σ is not an equilibrium, you do **not** need to find the equilibrium.

For example, "In the Wittman model, is it an equilibrium for L and R to be located at $x_m - \Delta$ and $x_m + \Delta$ (assuming a uniform distribution of voters)?"

- **Good:** No. Either candidate could win with certainty by moving slightly closer to the median.
- **Correct but contains extraneous elements:** No. Either candidate could win with certainty by moving slightly closer to the median. But then the other one would do the same thing. And then the other would respond in a similar fashion, and might be wearing a silly hat. And so they would end up at the median, possibly both wearing silly hats.

Analogy to criminal defense: you don't need to identify the killer, just show your guy didn't do it.

Mixed strategy equilibrium intuition

Almost everyone got the correct mixing probabilities.

But almost everyone got this wrong: In nuclear crisis game (chicken), 2's payoff for "back down" (swerve) goes up. Why does 1's probability of playing "back down" (swerve) go up?

Mixed strategy equilibrium intuition (2)

But almost everyone got this wrong: In nuclear crisis game (chicken), 2's payoff for "back down" (swerve) goes up. Why does 1's probability of playing "back down" (swerve) go up?

Logic: "Back down" has become more attractive to 2. In response, 1 needs to make "Back down" less atttractive. 1 accomplishes this by playing "Back down" more.

Marking

My notation in the margins:

✓	"correct"	70s and 80s
(✓)	"not quite correct" or "close"	60s
(X)	"mostly incorrect"	50s
X	"incorrect"	40s

Overall mark roughly averages the marginal marks: (✓), (✓)$-$ or (X)$+$.

Distribution of marks