Formal Analysis: Power change and war

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Week 4 Session 1

Application

A few comments on the application

What I heard too much of:

- intuition
- substantive knowledge
- applying the **conclusions** of models we examined rather than the **assumptions** and **techniques**

What I didn't hear enough of: "Let's look at each model in Gehlbach (Hotelling-Downs, Wittman, citizen-candidate with sincere voters, citizen-candidate with strategic voters, Wittman with uncertainty) and see how it would apply to this problem."

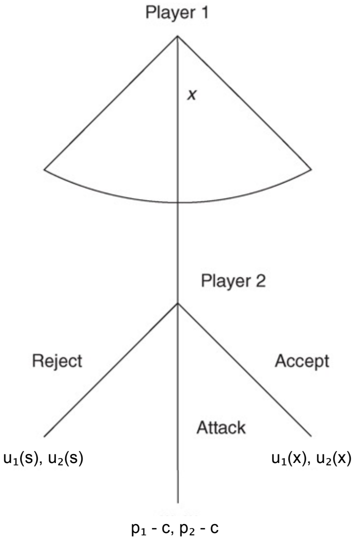
Bargaining and war: re-cap

Underlying puzzle

War is costly: it destroys some of the resource that states are fighting over. Why can't a peaceful allocation be found that makes everyone (weakly) better off?

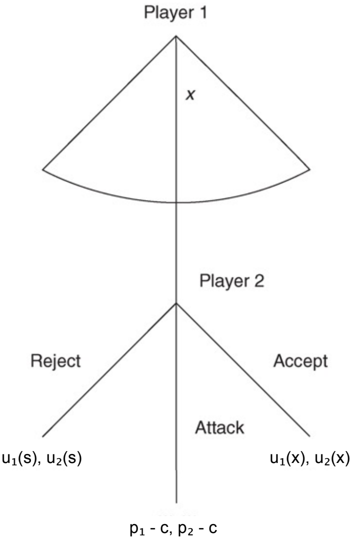
Answer in Kydd's Chapter 4: the resource at issue may be less valuable when divided, such that they prefer to fight rather than divide it.

War in Kydd's Chapter 4



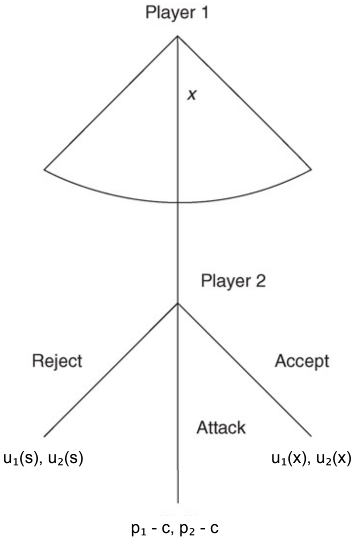
What are the conditions for war to take place?

Conditions for war (general)



War occurs when 2 chooses *Attack*.
So under what conditions does this happen?
(1) Player 2 prefers Attack to Reject.
(2) Player 2 prefers Attack to Accept.

Conditions for war (general, 2)



Define i's *bottom line* b_i such that $u_i(b_i) = p_i - c$.

(1) Player 2 prefers Attack to Reject.

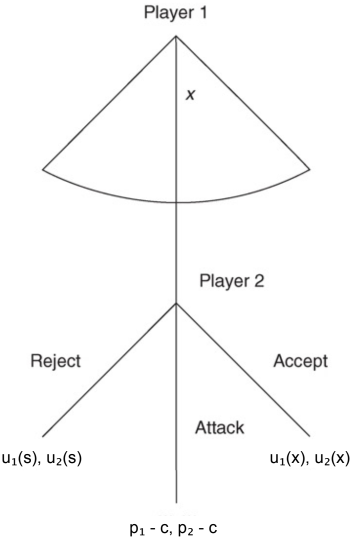
$$b_2 < s$$

(2) Player 2 prefers Attack to Accept.

$$b_2 < x$$

Combining, $b_2 < \min(s, x)$.

Conditions for war (general, 3)



War takes place when $b_2 < \min(s, x)$.

But what is x?

We only need to consider $x = b_1$: if player 2 accepts b_1, then war cannot take place because 1 would prefer to propose $b_1 + \epsilon$ rather than fight a war. So in equilibrium war occurs if and only if player 2 rejects $x = b_1$.

Restating, war takes place if and only if $b_2 < \min(s, b_1)$.

Conditions for war (specific)

Assumptions:

1. $p_2 = 1 - p_1$
2. $u_1(x) = x^a$, $u_2(x) = (1 - x)^a$, with $a > 0$

Let's calculate each player's **bottom line —** the allocation x that is just as good as a war.

Expected utility of war for player 1: $p_1 - c$

Expected utility of war for player 2: $p_2 - c = 1 - p_1 - c$

By definition, $u_1(b_1) = p_1 - c$, so $b_1 = (p_1 - c)^{1/a}$.

By definition, $u_2(b_2) = 1 - p_1 - c$, so $b_2 = 1 - (1 - p_1 - c)^{1/a}$.

Conditions for war (cont'd)

So new condition for war is:

$$1 - (1 - p_1 - c)^{1/a} < \min\left(s, (p_1 - c)^{1/a}\right).$$

It would be nice to rearrange and isolate a, but I can't.

Instead, we can try out different values of a, p_1, and c using this `shiny` app I made with `R`:

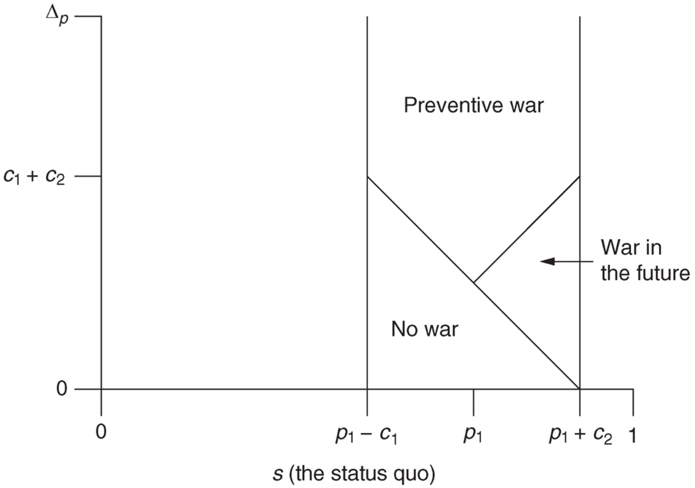
`https://andyeggers.shinyapps.io/intermediate_values/`

Takeaways

1. **Substance:**
   - no war occurs when war is costly and intermediate outcomes sufficiently valued (e.g. linear payoffs)
   - war may occur if the resource at issue is much less valuable when divided.
2. **Procedure:** tricks for solving the problem
   - backwards induction
   - working with *bottom lines* (allocations) rather than utilities
   - numerical examples (perhaps using `shiny` app)
3. **Setup:** Proposal x followed by decision to accept, reject, or fight — what is captured, what is missing?

War from changing power: no bargaining

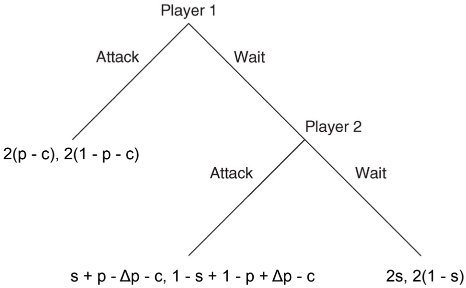
Describing equilibria in terms of model parameters



Changing power with no bargaining

Consider simplest game in chapter 5, and simplify further:

- $p_1 = p_2 = p$
- $c_1 = c_2 = c$
- linear payoffs: $u_1(x) = x$; $u_2(x) = 1 - x$



Under what conditions can we expect war in this game?

Changing power with no bargaining (3)

Player 2 attacks if

$$1 - s + 1 - p + \Delta p - c > 2(1 - s)$$
$$\text{Cond A:} \quad \Delta p > p + c - s$$

Player 1 attacks if

Given $\Delta p > p + c - s$ (2 attacks):

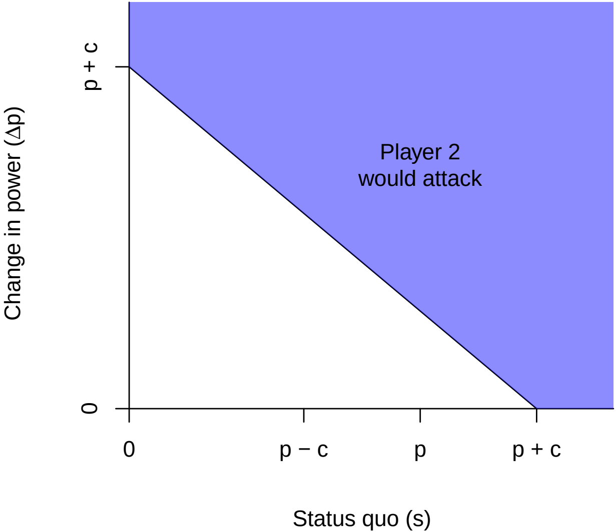
$$2(p - c) > s + p - \Delta p - c$$
$$\text{Cond B.1:} \quad \Delta p > -p + c + s$$

Given $\Delta p \leq p + c - s$ (2 waits):

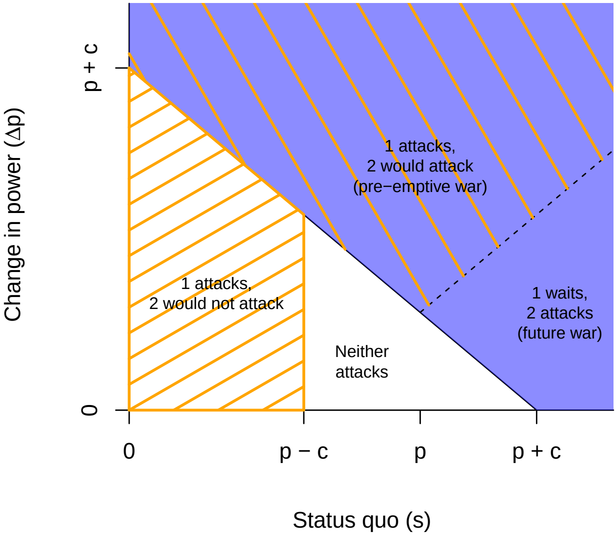
$$2(p - c) > 2s$$
$$\text{Cond B.2:} \quad s < p - c$$

Making a diagram: condition A

Condition A: player 2 attacks if $\Delta p > p + c - s$



Making a diagram: adding conditions B.1 and B.2

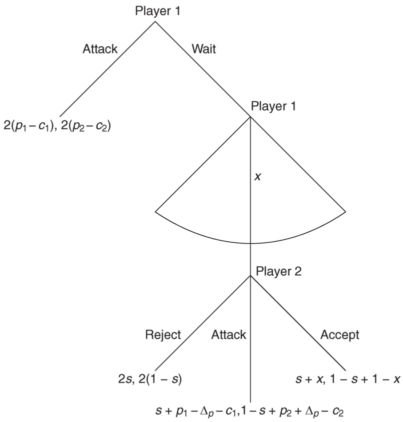


Another shiny app

https://andyeggers.shinyapps.io/preventive_war/

War from changing power with bargaining

Key points from power-change-with-bargaining model

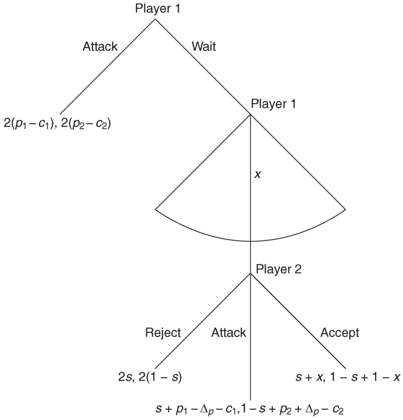


Key question for 1: war now, while strong, or crisis bargaining in future when weak.

Kydd assumes linear payoffs so we know what happens in the crisis bargaining sub-game:

- it is always optimal for player 1 to make a proposal that makes 2 indifferent between attacking and accepting (i.e. b_2'), so 2 never attacks
- if $s < b_2'$, then player 2 rejects and wasn't going to fight anyway (no "credible threat to fight")

Key points from the power-change-with-bargaining model (2)

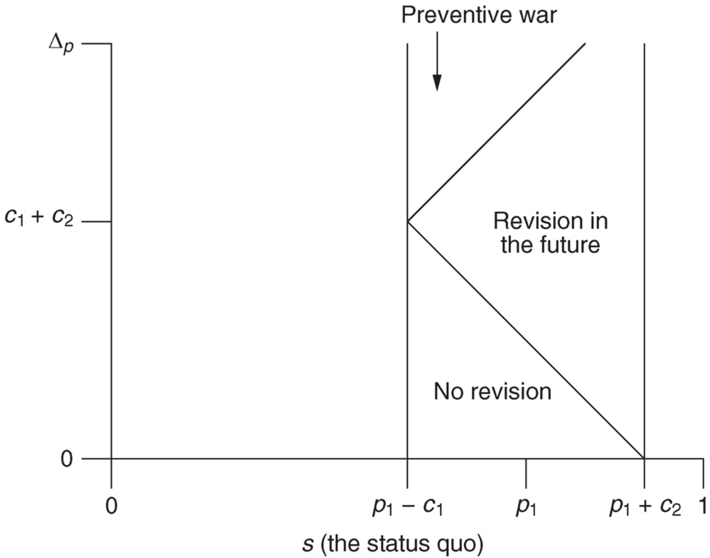


So war does not happen in the future. And Kydd assumes $s > p_1 - c_1$, i.e. player 1 wouldn't attack in the absence of power change.

So the whole question is whether 1 prefers attacking now to making concessions in the future.

- the proposal 1 must make to appease 2 is $x = b_2' = p_1 + c_2 - \Delta p$
- 1 prefers attacking to making concessions if $2(p_1 - c_1) > s + b_2'$ i.e. if $\Delta p > s - (p_1 - c_1) + c_1 + c_2$

Equilibria with bargaining in period 2



Recap

Big picture

We are looking at rationalist explanations for war one by one.

Each toy model aims to isolate a single mechanism (cf one model with all mechanisms) while **shutting down** other channels so we know what is producing the war.

Big picture: a table

	Bargaining?	Power change?	Credible threat w/o change?	Credible threat post-change?	Linear payoffs?	Outcome
	None	No	No	-	-	No war
	None	No	Yes	-	-	War
4.4	Yes	No	No	-	-	No concession, no war
4.4	Yes	No	Yes	-	Yes	Concession, no war
4.6	Yes	No	Yes	-	No	War if intermediate outcomes undervalued
5.1	None	Yes	No	No	-	No war
5.1	None	Yes	No	Yes	-	Preventive or future war
5.2	2nd rd.	Yes	No	No	Yes	No concession, no war
5.2	2nd rd.	Yes	No	Yes	Yes	Concession or preventive war
5.3	Both rd.	Yes	No	No	Yes	No concession, no war
5.3	Both rd.	Yes	No	Yes	Yes	Concession (once/twice) or preventive war if 2 can't buy off 1