Formal Analysis: Electoral competition under certainty

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Week 2 Session 2

Hotelling-Downs

One approach for finding Nash equilibria

In previous session, we solved for SPNE in an **extensive form game** via backwards induction:

1. Figure out player 2's best response to each possible action by player 1
2. Figure out player 1's best action, given that player 2 will best-respond

Any best-response by 1 to a best-response by 2 is a Nash equilibrium.

But this approach may not find **all** Nash equilibria. **(Why not? Examples?)**

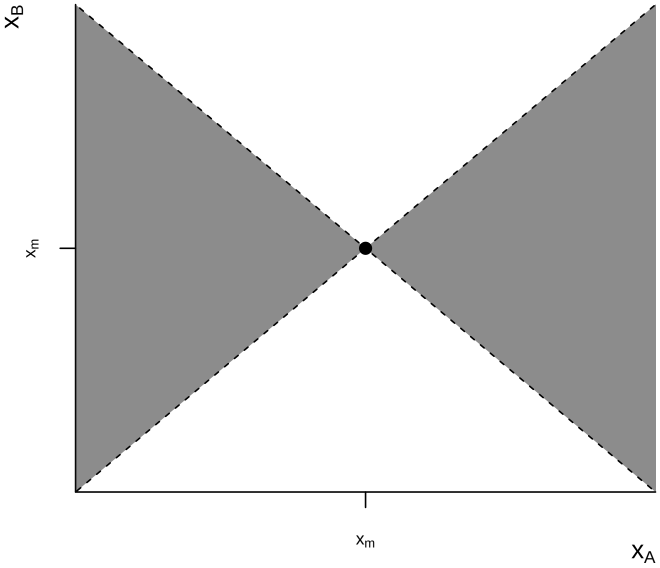
Hotelling-Downs

1. **Parties**: two parties, A and B, choosing position on $\mathbb{R}$. They want to win office.
2. **Voters**: Continuum of voters with ideal point $x_i \in \mathbb{R}$. Given policy x, voter i's utility is $u_i(x) = -|x - x_i|$. Voters vote sincerely and abstain if indifferent.
3. **Election rules**: plurality rule, with a fair lottery if election is tied.

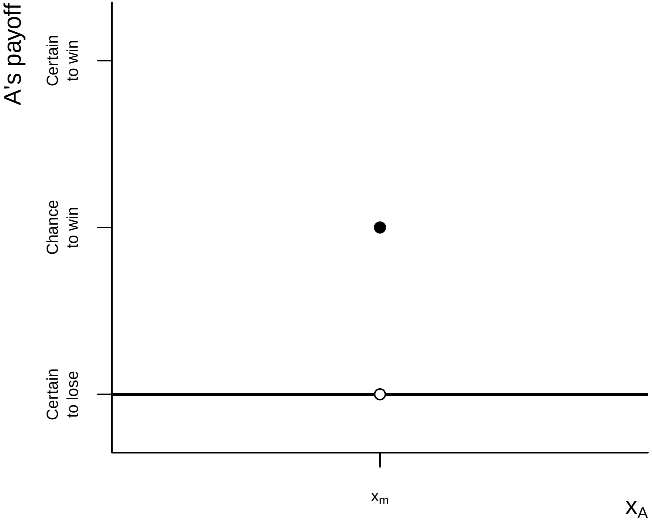
Method of finding equilibria:

1. Figure out party B's *best response correspondence* to each possible action by party A
2. Figure out party A's best action, given that party B will best-respond

Hotelling-Downs: optimal move by party *B*



Hotelling-Downs: payoff of party A (given B is best-responding)



Competition when policy is multidimensional

Condorcet winner and Condorcet's paradox

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2. $y \succ z \succ x$
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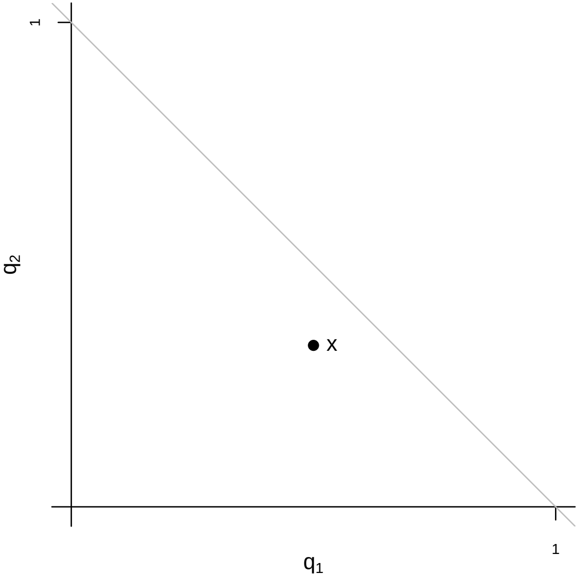
Then no Condorcet winner.

No Condorcet winner when dividing a pie



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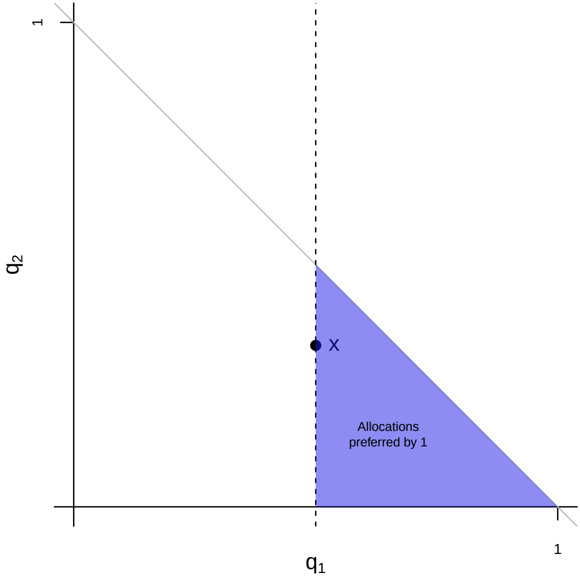

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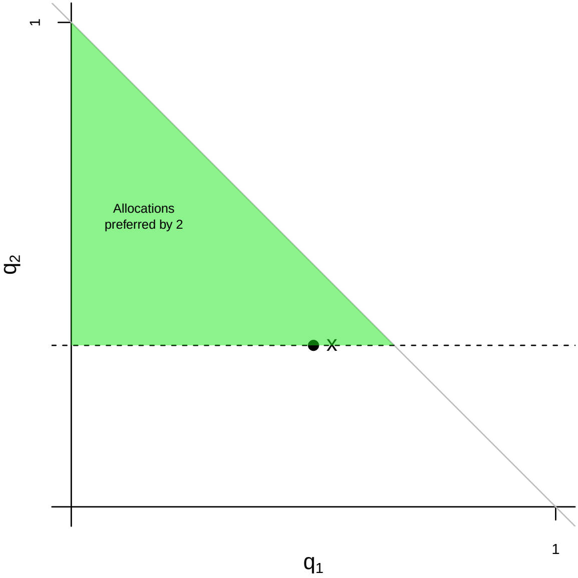
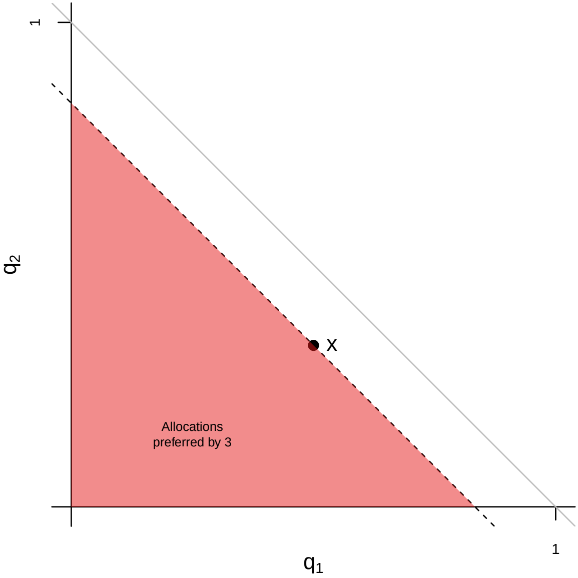


No Condorcet winner when dividing a pie



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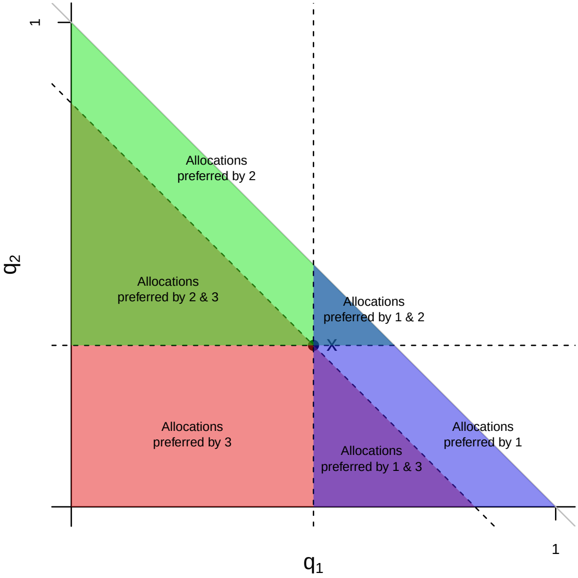
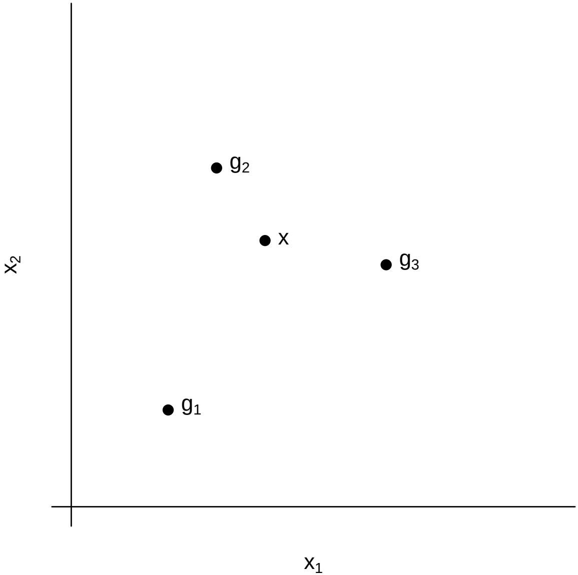


Two policy dimensions, three groups w. Euclidean preferences



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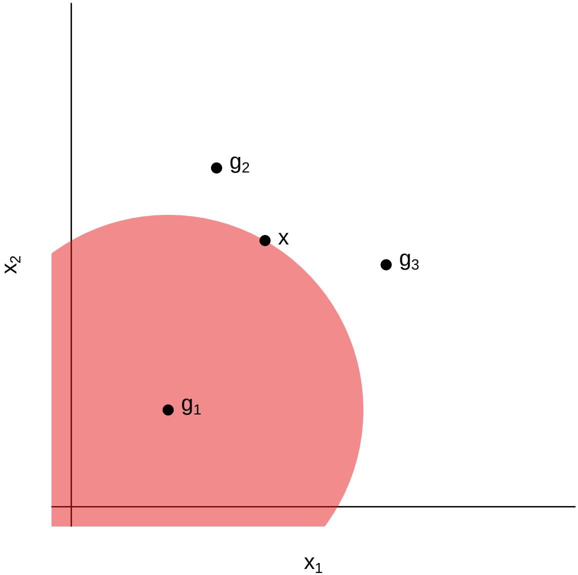

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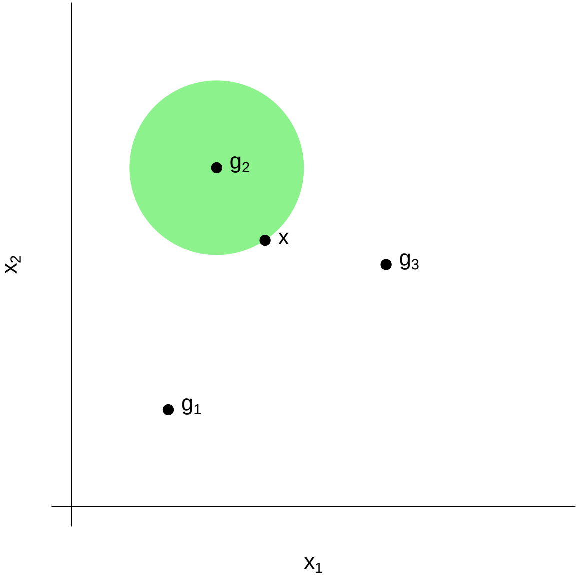

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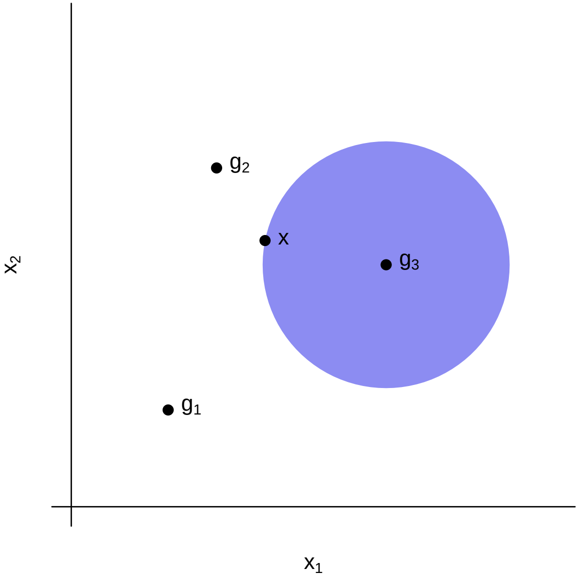

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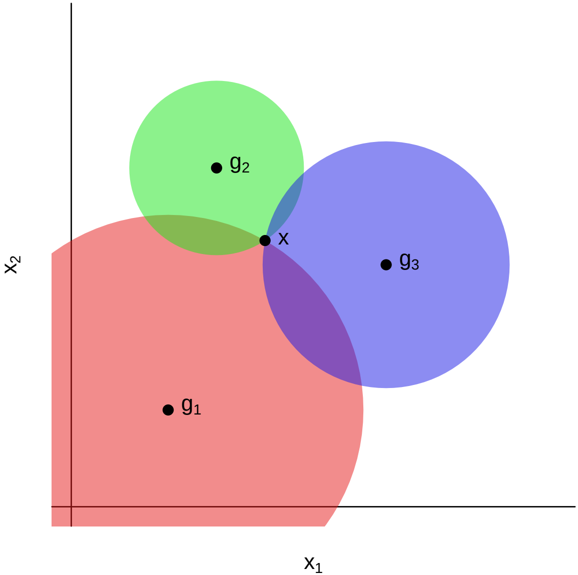
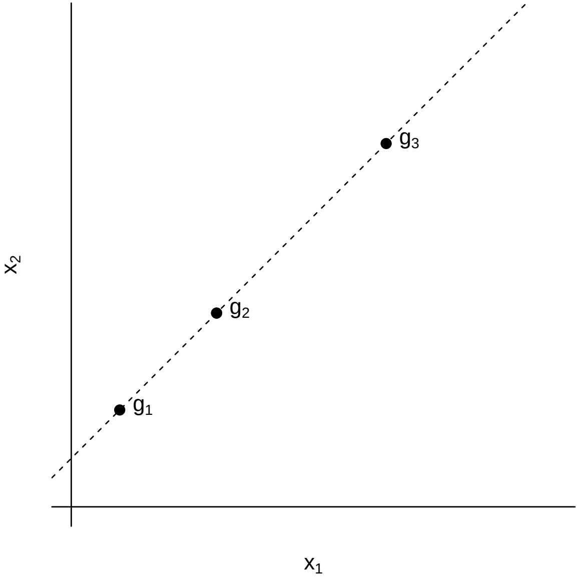


Two dimensions that are really just one



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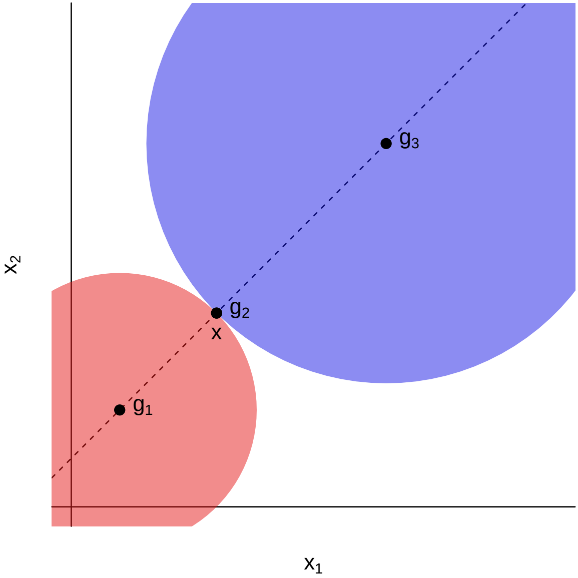


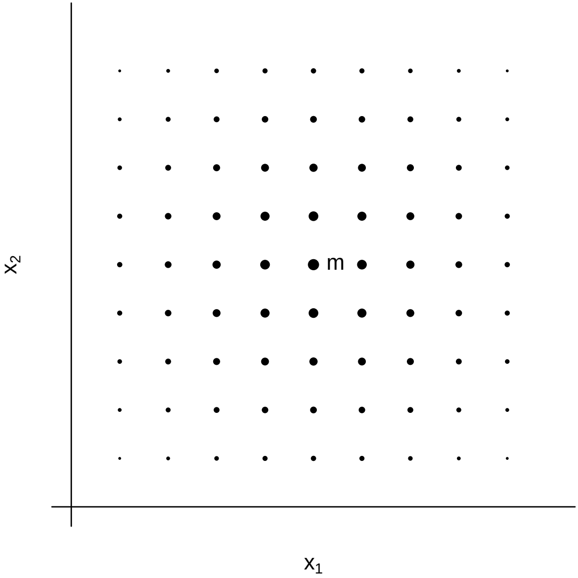
When is there a Condorcet winner in 2 dimensions?

Definition: A *median line* is a line such that at least half the voter ideal points lie either on it or to the right of it and at least half the voter ideal points lie either on it or to the left of it.

Theorem (Davis, DeGroot, and Hinich, 1972): There exists a Condorcet winner if and only if there exists a voter's ideal point, M, such that every line passing through it is a median line. If so, the alternative, M, corresponding to that point will be a Condocet winner.

Feld and Grofman, *AJPS* 1987, "Necessary and sufficient conditions for a majority winner in n-dimensional spatial voting games: an intuitive geometric approach"

Graphical illustration



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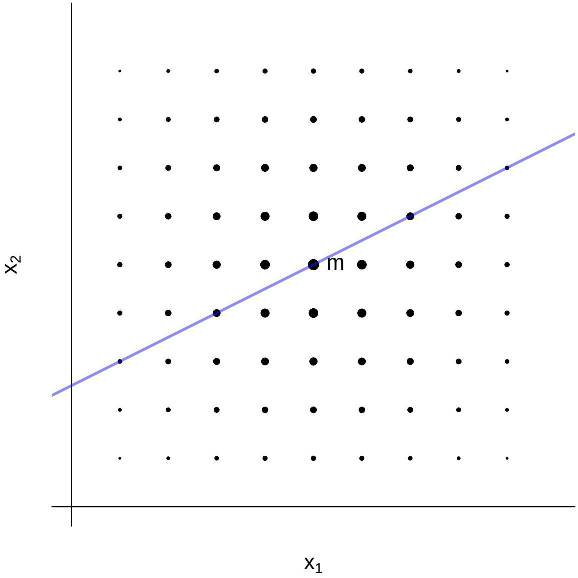

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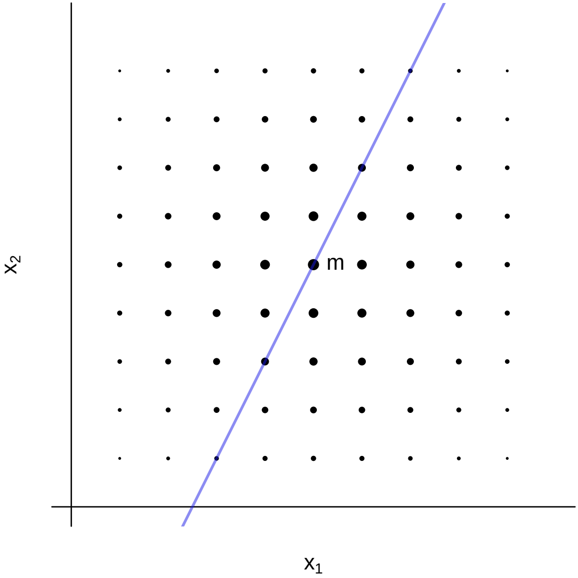

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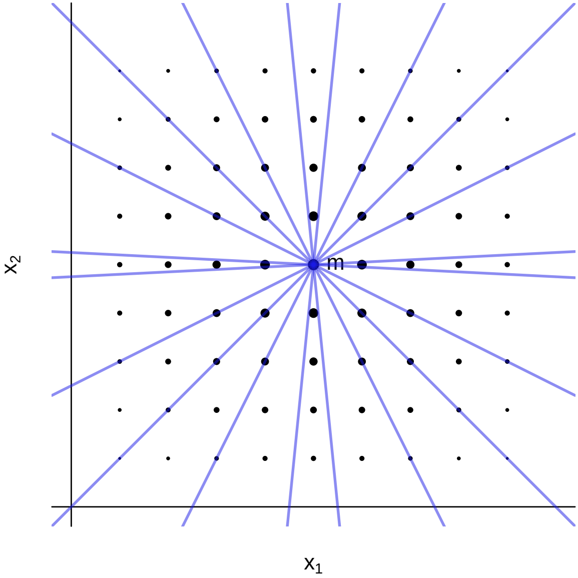


Multiparty competition

Cox's Lemma 1

Suppose $M > 2$ vote-maximizing parties, sincere voters, and unidimensional policy space.

Cox (1987) proves that in equilibrium:

1. No more than two parties occupy any one position.
2. Each extremist position (i.e. left-most or right-most among all parties) is occupied by exactly two parties.
3. If two parties occupy the same position x, then the share of voters to the left of x who most prefer the parties at x must be equal to the share of voters to the right of x who most prefer the parties at x.

These conditions are stated without proof in Gehlbach. Let's prove them.

Proving Cox's Lemma 1 (1)

1. No more than two parties occupy any one position.

Suppose $n > 2$ parties are at a position x, with L voters to the left of x and R voters to the right of x preferring the parties at x to any other. Without loss of generality, let $L \geq R$. The parties at x each win $\frac{L+R}{n}$ in vote share. If party j, located at x, deviates slightly to the left, she wins a vote share of L, which is larger than $\frac{L+R}{n}$ for any $n > 2$.

Proving Cox's Lemma 1 (2)

2. Each extremist position (i.e. left-most or right-most among all candidate positions) is occupied by exactly two parties.

We have already shown that no position can be occupied by more than three parties. Suppose a single party, j, is at an extremist position x. Without loss of generality we shall say that this is the left-most position. By moving to the right j can win more votes.

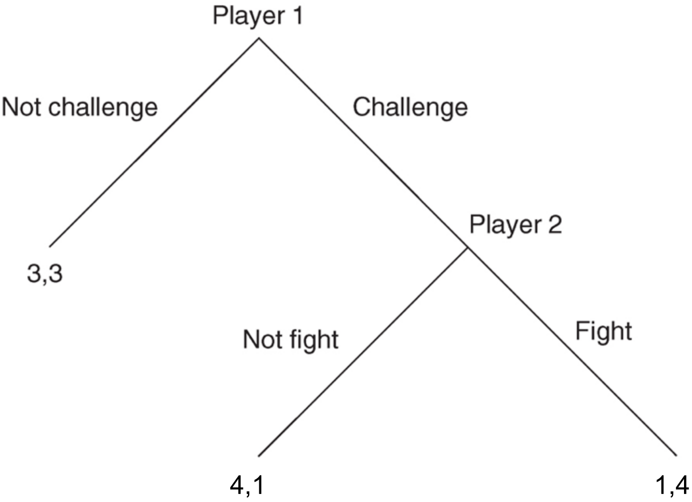
Proving Cox's Lemma 1 (3)

3. If two parties occupy the same position x, then the share of voters to the left of x who most prefer the parties at x must be equal to the share of voters to the right of x who most prefer the parties at x.

Suppose 2 parties, i and j, are located at a position x, with L voters to the left of x and R voters to the right of x preferring the parties at x to any other. The parties at x each win $\frac{L+R}{2}$ in vote share. If $L > R$, then a party j located at x would benefit from moving slightly to the left, thus winning $L > \frac{L+R}{2}$.

“Off the equilibrium path”

Easy example



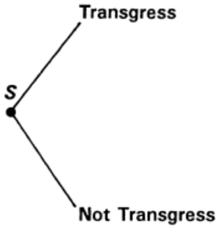
SPNE is: {Not challenge; Fight} – but "Fight" is **off the equilibrium path**.

Weingast (1997) "The political foundations of democracy and the rule of law"

FIGURE 2. Payoffs for the Sovereign-Constituency Coordination Game

S Moves first

Induced subgame between *A* and *B* (payoffs: *S*,*A*,*B*)



Transgress:

		B: Acquiesce	B: Challenge
A	Acquiesce	8, 2, 2	8, 2, 1
A	Challenge	8, 1, 2	0, 7, 7

Not Transgress:

		B: Acquiesce	B: Challenge
A	Acquiesce	2, 8, 8	2, 8, 7
A	Challenge	2, 7, 8	0, 7, 7

Weingast (1997) (2)

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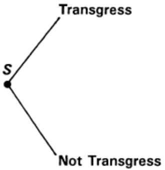

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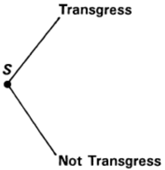

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Acquiesce	2, 8, 8	2, 8, 7
Challenge	2, 7, 8	0, 7, 7

Two SPNEs:

- {Transgress; Acquiesce if transgress, acquiesce if not transgress; Acquiesce if transgress, acquiesce if not transgress}
- {Not transgress; Challenge if transgress, acquiesce if not transgress; Challenge if transgress, acquiesce if not transgress}

Use the phrase "off the equilibrium path" to describe these SPNEs.

"Off the equilibrium path" in Gehlbach chapter 1

Feddersen-Sened-Wright model: Like Hotelling-Downs, but

- M candidates compete, with endogenous entry (benefit of winning v and cost of entry δ)
- voters vote strategically.

There exist equilibria in which 3 or more candidates locate at the median, x_m.

> *Such equilibria are supported by the following voting stratgies off the equilibrium path: if any potential candidate . . . deviates to a position other than x_m, then all voters with ideal points equal to or to the other side of x_m vote for one of the candidates who remains at x_m.*

"Off the equilibrium path" in Gehlbach chapter 1 (2)

Citizen-candidate model: Candidates cannot commit to implement policies other than their (common-knowledge) ideal point; endogenous entry.

With strategic voting, can have equilibria with one candidate on the extreme left and one on the extreme right. This is sustained by voting strategies **off the equilibrium path** in which, if a centrist entered, voters would continue to vote for their preferred extremist.