Formal Analysis: Bargaining

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Week 2 Session 1

Efficiency

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Definition 2.5 (Kydd p. 17): Given a set of actors with utility functions u_i defined over an outcome space X, an outcome $x' \in X$ is efficient if for any other outcome $x'' \in X$ that makes some player i better off, $u_i(x'') > u_i(x')$, there must be some other actor j that is worse off, $u_j(x') > u_j(x'')$.

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Related definitions:

- An outcome is **inefficient** (Pareto inefficient) if there *is* some way to make some players better off without making anyone worse off
- An efficient outcome is **Pareto optimal**
- An inefficient outcome is **Pareto inferior** to another outcome

Two questions about efficiency

1. Of the four possible strategy profiles in the prisoner's dilemma game, which are efficient, and which are inefficient?
2. If player 1 and player 2 agree on a peaceful division of the good in which player 1 gets $x \in [0, 1]$ and player 2 gets $1 - x$, their payoffs are x and $1 - x$ respectively. If they fight a war, their payoffs are w_1 and w_2, with $w_1 + w_2 < 1$. Show that given these assumptions *war is inefficient*.

Solutions

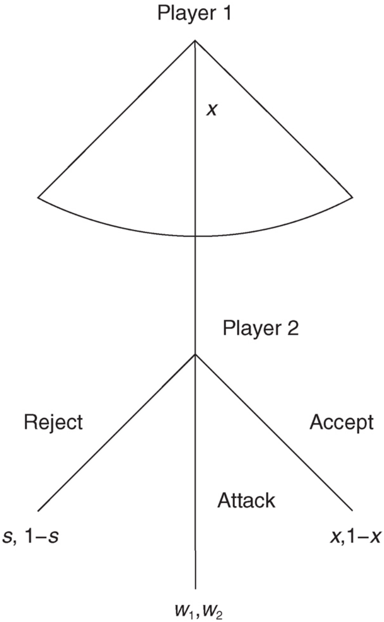
1. Three strategy profiles of the prisoner's dilemma yield efficient outcomes ($\{C, C\}; \{C, D\}; \{D, C\}$) and the remaining strategy profile $\{D, D\}$ yields an inefficient outcome. It is Pareto inferior to the outcome of $\{C, C\}$.
2. By definition, war is inefficient if there is at least one feasible peaceful allocation that is as good as war for both players and better than war for one player. Consider the peaceful allocation where $x = w_1$. w_1 is defined as player 1's war payoff, so this is as good as war for player 1. The peaceful allocation is better than war for the second player, because if $w_1 + w_2 < 1$ (as assumed), then $1 - w_1 > w_2$, i.e. 2's peaceful allocation is better than her war payoff. Thus war is inefficient: it is Pareto inferior to at least one peaceful outcome.

Bargaining

What is the point of these models?

The central puzzle about war, and also the main reason we study it, is that wars are costly but nonethelesss wars recur. Scholars have attempted to resolve the puzzle with three types of arguments. First, one can argue that people (and state leaders in particular) are sometimes or always irrational. They are subject to biases and pathologies that lead them to neglect the costs of war or to misunderstand how their actions will produce it. Second, one can argue that the leaders who order war enjoy its benefits but do not pay the costs, which are suffered by soldiers and citizens. Third, one can argue that even rational leaders who consider the risks and costs of war may end up fighting nonetheless. This article focuses on arguments of the third sort, which I will call rationalist explanations. (Fearon, "Rationalist explanations for war", 1995)

Bargaining with conflict



Backwards induction

"In games of complete information, . . . subgame perfection is equivalent to *backwards induction*. Backwards induction is solving the game from the terminal nodes, working backwards to the initial note. At each node, the optimal choice is made and the payoffs of the chosen successor node are implicitly substituted for the node. Then at the previous node the optimal choice is made, given the understanding of what would happen subsequently." (Kydd p. 59)

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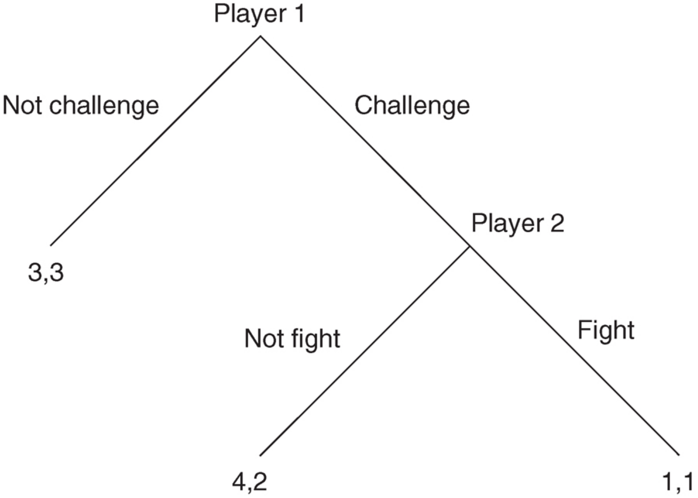
We find **subgame perfect Nash equilibria** (SPNE) with backwards induction.

Backwards induction in practice: player 1 choosing between two actions



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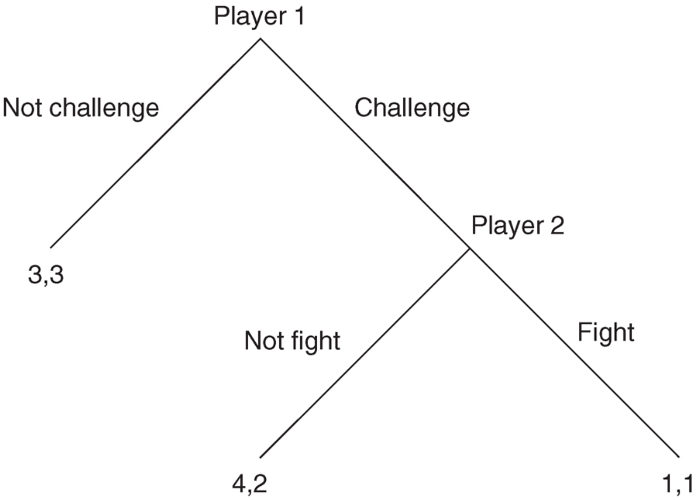
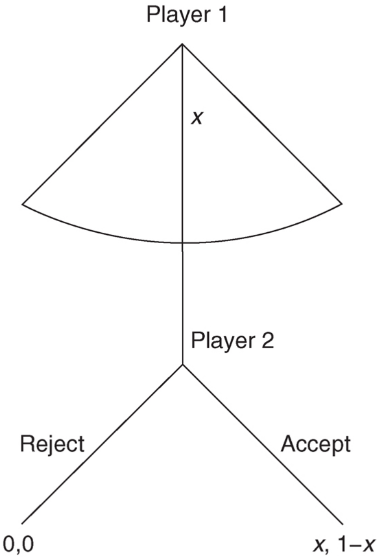
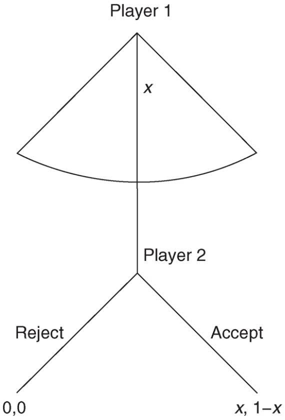


SPNE: {Challenge, Not fight}

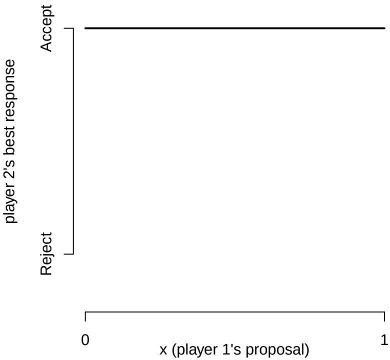
Backwards induction in practice: ultimatum game (continuous proposal)



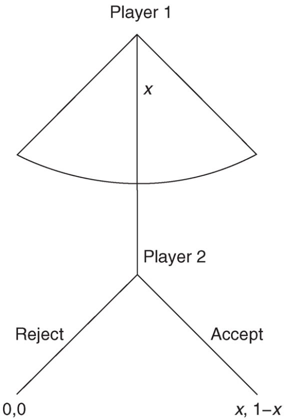
Backwards induction in practice: ultimatum game (2)



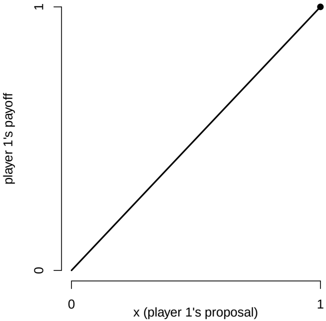
First, **player 2's best response** to all possible proposals from player 1:



Backwards induction in practice: ultimatum game (3)



Now, **player 1's payoff** as a function of player 1's proposal x (given 2's best response):



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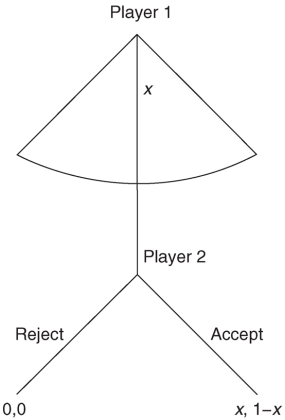

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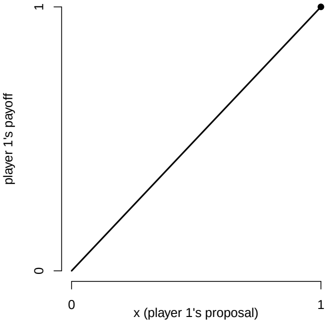
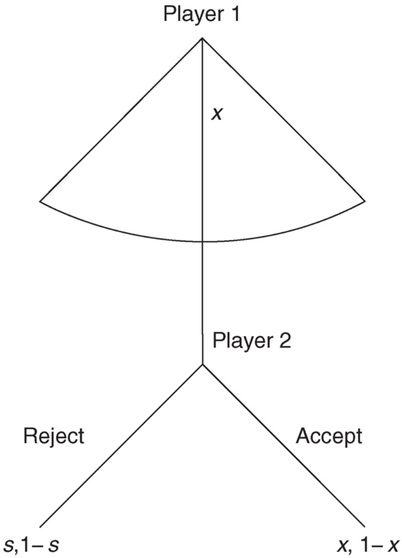
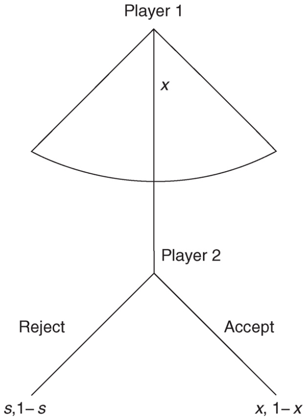


SPNE is: $\{$Propose $x = 1$, accept any proposal$\}$

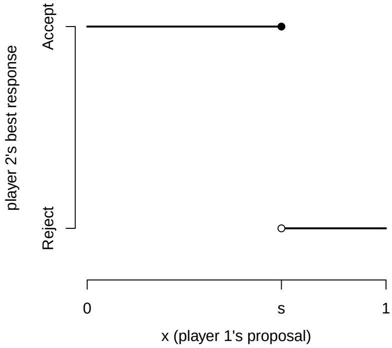
Backwards induction in practice: ultimatum game with status quo



Backwards induction in practice: ultimatum game w. SQ (2)



First, **player 2's best response** to any possible proposal x:



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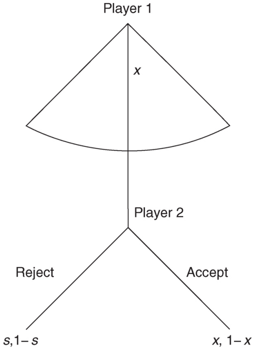

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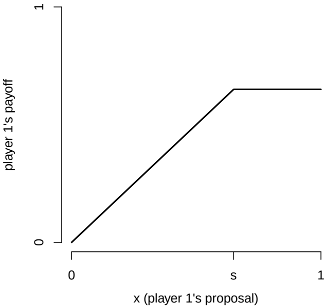

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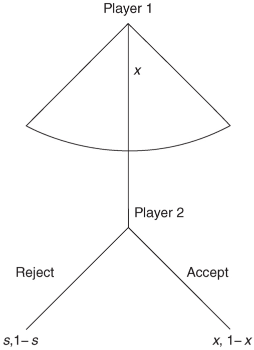

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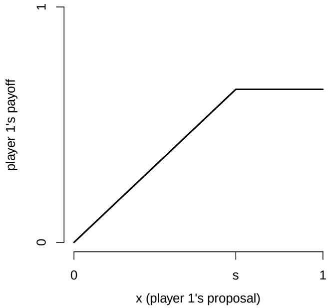
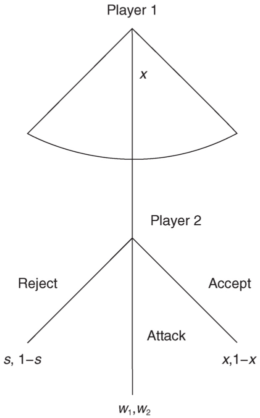


SPNE: $\{$Propose $x \geq s$, accept $x \leq s\}$

Backwards induction in practice: bargaining with conflict



Game features:

- **Timing and actions:**
  - Player 1 proposes division $\{x, 1-x\}$
  - Player 2 can then
    - accept $\rightarrow$ they get $\{x, 1-x\}$
    - reject $\rightarrow$ they get $\{s, 1-s\}$ $(s \in [0,1])$
    - attack $\rightarrow$ they fight.
- **Utility functions**: $u_1'(x) > 0$ and $u_2'(x) < 0$; both are continuous. **Not** assuming linear utility function as in figure at left..

Can war be averted?

Bargaining with conflict: solution

As before, proceed by backwards induction: what will player 2 do, given proposal of x from player 1?

- **Case 0**: war better than best possible deal for 2 (so player 2 attacks no matter what player 1 does)
- **Case 1**: status quo better for 2 than fighting (so if player 2 doesn't like the proposal, she rejects without attacking)
- **Case 2**: status quo worse for 2 than fighting (so if player 2 doesn't like the proposal, she attacks)

As in Kydd:

- Normalize utility such that $u_2(1) = 0$, $u_2(0) = 1$
- w_i denotes player i's expected payoff from a war
- b_i denotes allocation x such that player i indifferent between x and war:

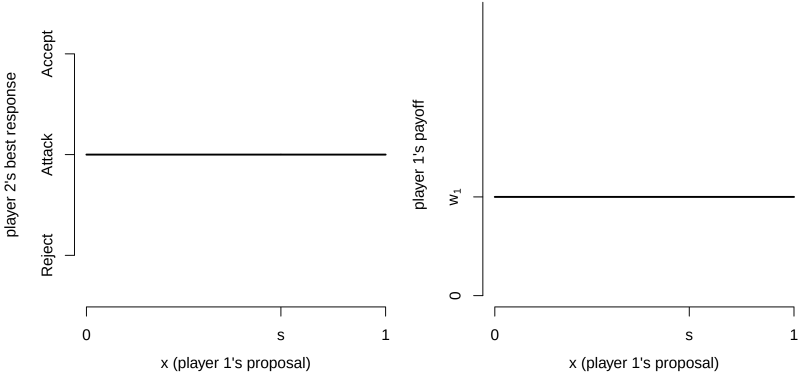
$$b_i = \{x | u_i(x) = w_i\}$$

Then:

- **Case 0**: $w_2 > 1$
- **Case 1**: $s < b_2$
- **Case 2**: $s > b_2$

Bargaining with conflict: solution in case 0

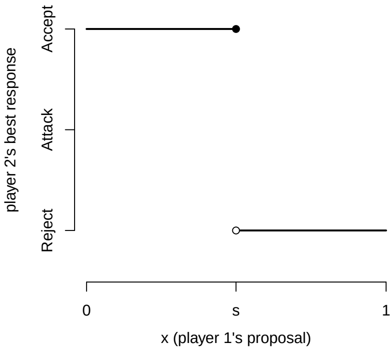
No matter what proposal 1 makes, 2 attacks and they each get their war payoff.

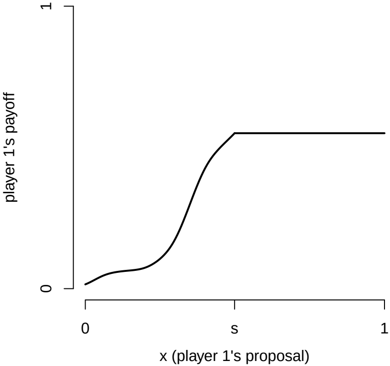


What assumption in Kydd rules out this case?

Bargaining with conflict: solution in case 1

If player 2 prefers status quo to fighting, she will accept a proposal that is better for her than the status quo, and reject (not attack) if the proposal is worse than the status quo. No fighting.

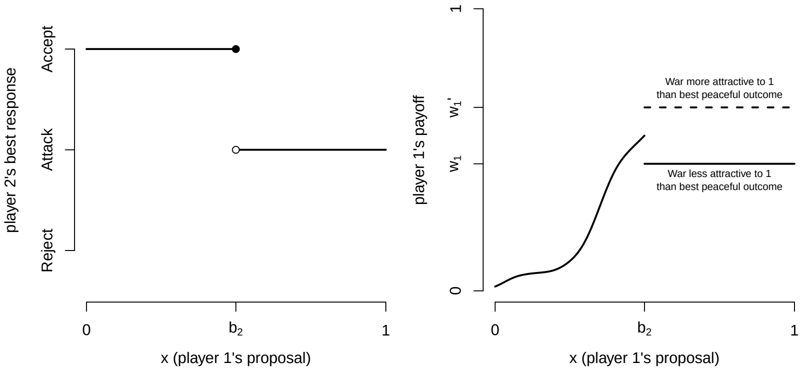




Have not specified $u(x)$ – generic curve used here.

Bargaining with conflict: solution in case 2

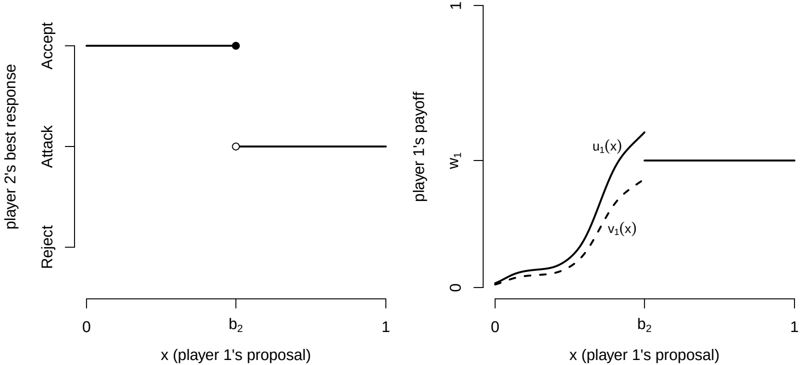
If player 2 prefers fighting to status quo, she will accept a proposal that is better than fighting, and attack (not reject) if the proposal is worse than fighting.



War happens if 1's war payoff is w_1', but not if it is w_1.

Bargaining with conflict: solution in case 2 (2)

Same thing, but focus on utility of intermediate outcomes rather than war payoff:



Given value of war, war happens if 1's utility function is $u_1(x)$, but not if it is $v_1(x)$.

Ordinal and cardinal utility again

The fundamental point is that in some circumstances there might not be peaceful divisions that both prefer to war, i.e. the bargaining range might be empty ($b_1 > b_2$).

Two ways of showing this in a model:

1. $u_i(x)$ is **ordinal** (monotonically increasing for 1, decreasing for 2); value of war given as w_1, w_2; war happens when w_1, w_2 chosen such that $b_1 > b_2$
2. $u_i(x)$ is **cardinal**, value of war stated as lottery over $x \in [0, 1]$ (binary or continuous) minus costs; war happens when $u_i(x)$ and costs chosen such that $b_1 > b_2$

Discussion

- What aspects of bargaining and conflict does this capture? What is missing?
- What political phenomena other than inter-state war might this model describe?