Causal inference

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Oxford DPIR

HT 2018

Getting started

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Mostly this means measuring the effect of a **treatment** on some **outcome**:

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- What is the effect of class size on test scores?
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Measuring causal relationships also important for answering **why** questions:

- explaining variation in outcomes (e.g. why are some countries so poor?)
- explaining relationships between variables (e.g. why do more nationalistic countries redistribute less?)

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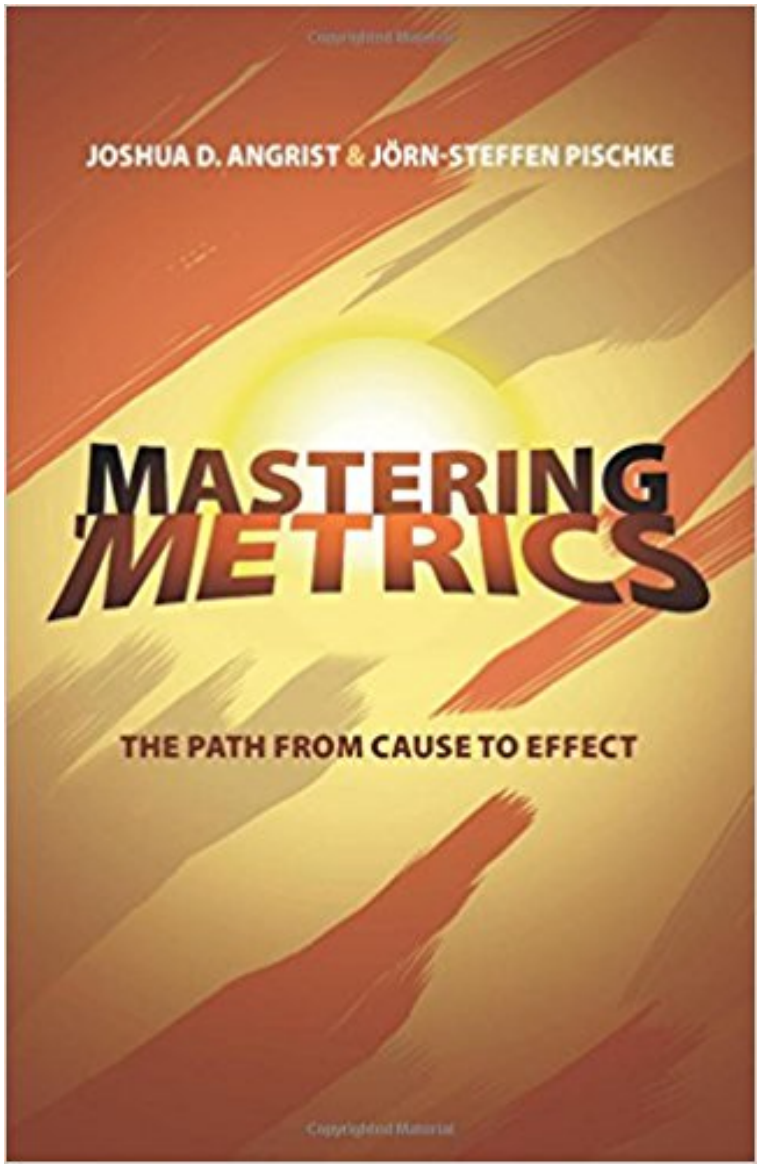
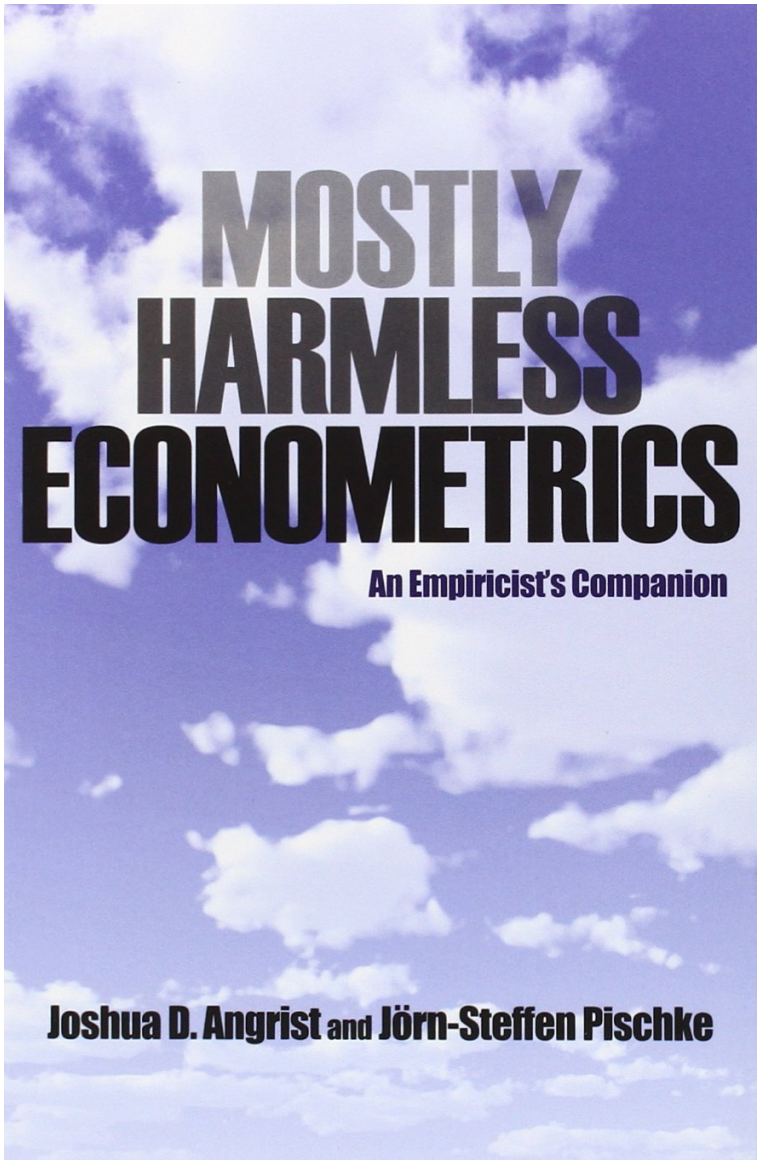
Plan by week:

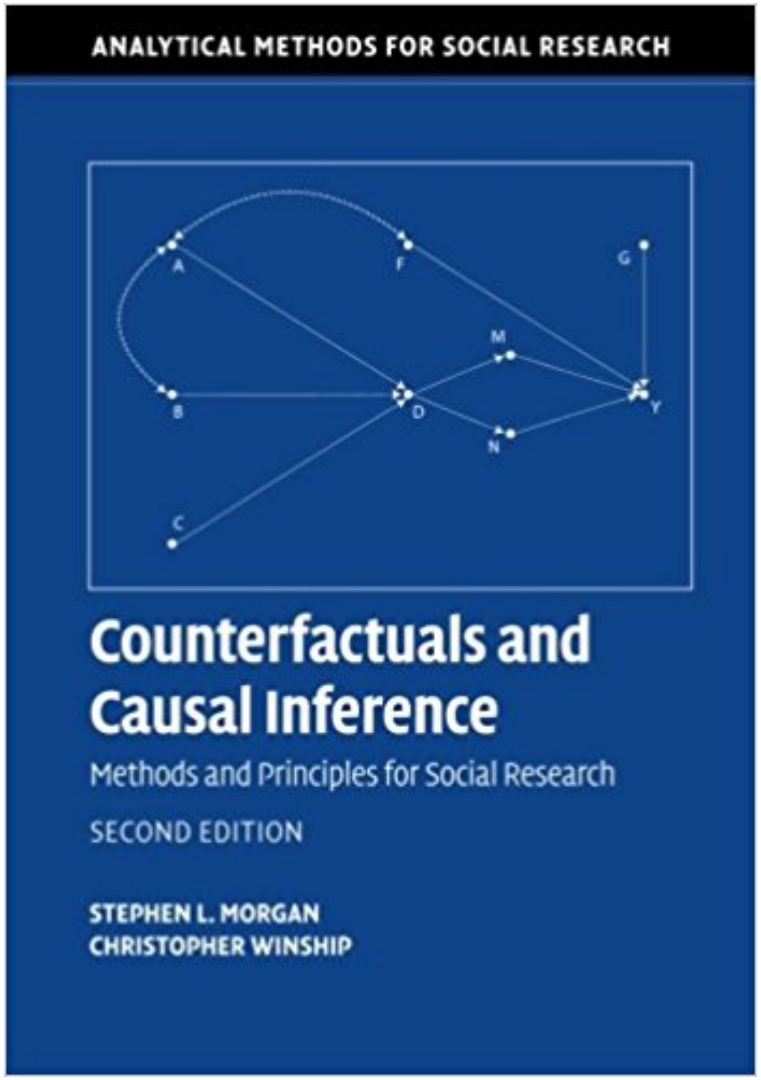
1. Potential outcomes framework and randomized experiments
2. Covariate adjustment 1 (Regression, matching, sub-classification)
3. Covariate adjustment 2 (Regression, matching, sub-classification)
4. Instrumental variables
5. Regression discontinuity design (RDD)
6. Differences-in-differences
7. Panel fixed-effects analysis
8. Treatment effect heterogeneity, mechanisms, and mediation

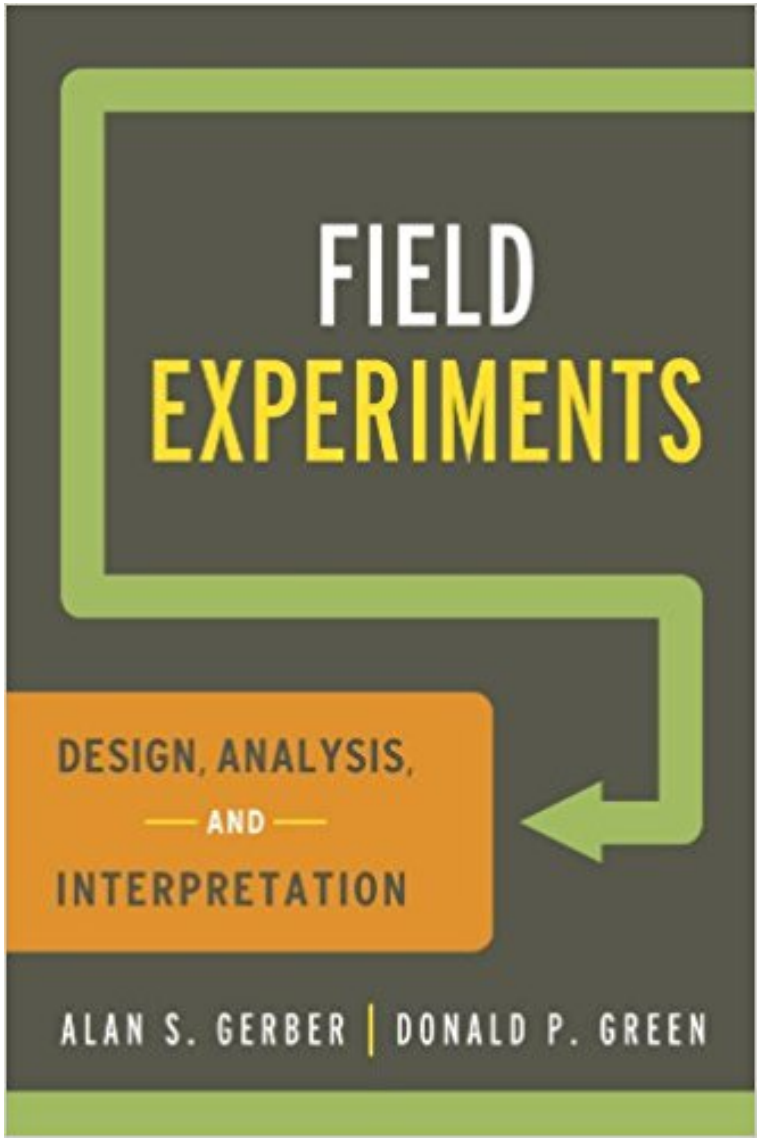
Textbooks


JOSHUA D. ANGRIST & JÖRN-STEFFEN PISCHKE
MASTERING 'METRICS
THE PATH FROM CAUSE TO EFFECT


MOSTLY HARMLESS ECONOMETRICS
An Empiricist's Companion
Joshua D. Angrist and Jörn-Steffen Pischke

Textbooks (2)


ANALYTICAL METHODS FOR SOCIAL RESEARCH
A
F
G
M
B
D
Y
N
C
Counterfactuals and Causal Inference
Methods and Principles for Social Research
SECOND EDITION
STEPHEN L. MORGAN
CHRISTOPHER WINSHIP


FIELD EXPERIMENTS
DESIGN, ANALYSIS, AND INTERPRETATION
ALAN S. GERBER | DONALD P. GREEN

Assessment

Problem sets (each 20% of final mark, total of 60%):

- PS 1: Distributed week 1, due noon on Thursday of Week 3.
- PS 2: Distributed week 3, due noon on Thursday of Week 6.
- PS 3: Distributed week 6, due noon on Thursday of Week 9.

Take-home exam (40% of final mark): Distributed by Friday of Week 8, due noon on Friday of week 0 of Trinity Term.

Weekly lab sessions

Spyros Kosmidis

Vuk Vukovic

Schedule:

- Group 1: Thursday 1500-1630
- Group 2: Thursday 1630-1800

Basics (in plain-ish language)

What does it mean for some **causal factor** (e.g. time studying) to **affect** some **outcome** (e.g. final mark) for an **individual** (e.g. you)?

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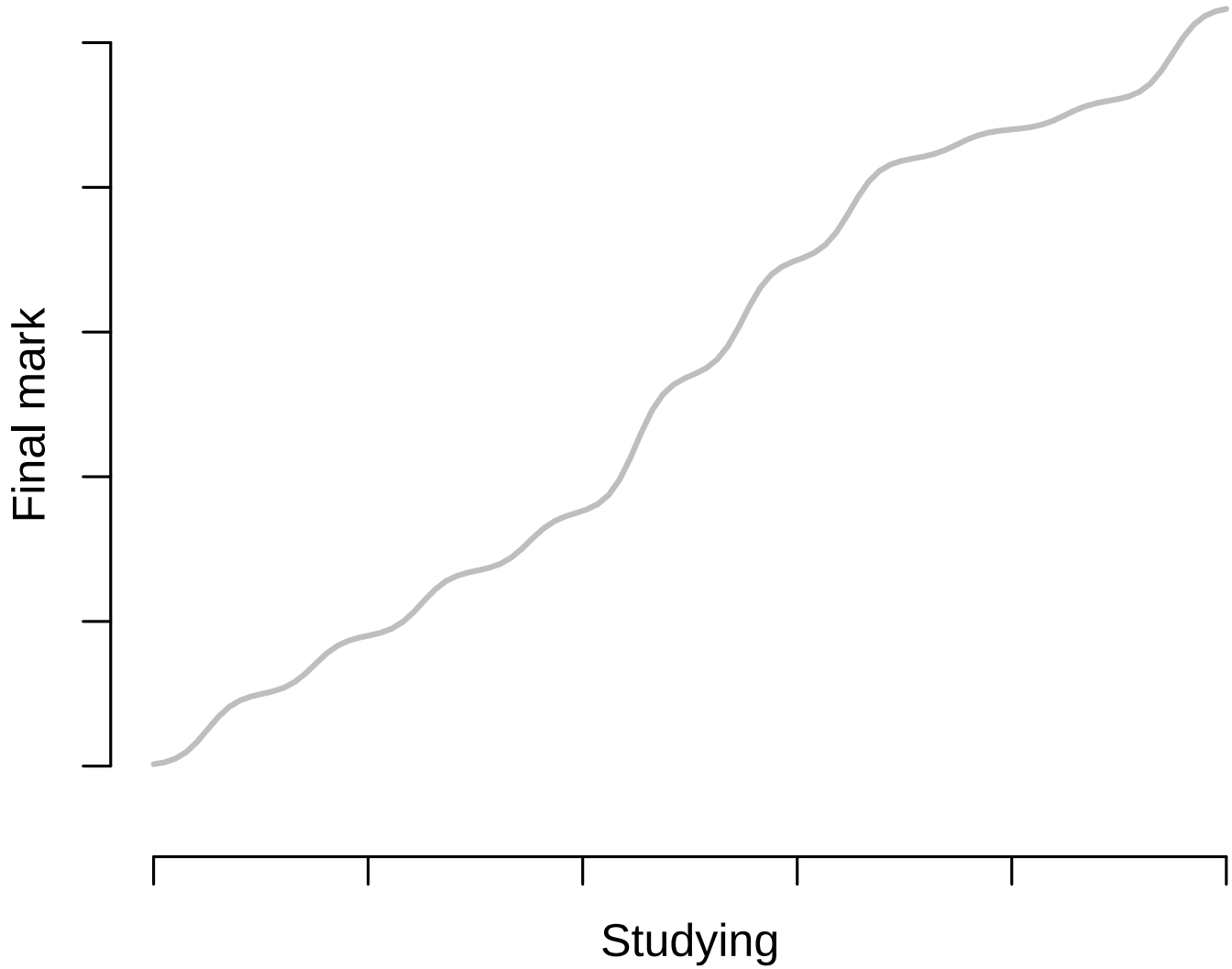
We call these hypothetical outcomes (one for each hypothetical value of the causal factor) **potential outcomes**.

Potential outcomes (continuous case)



Effects from potential outcomes

In the potential outcomes framework,
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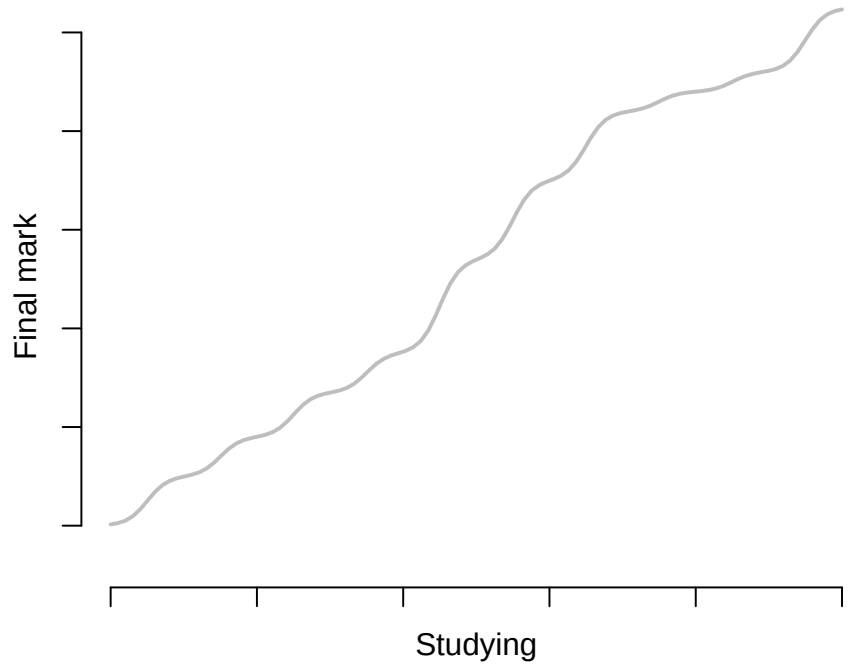
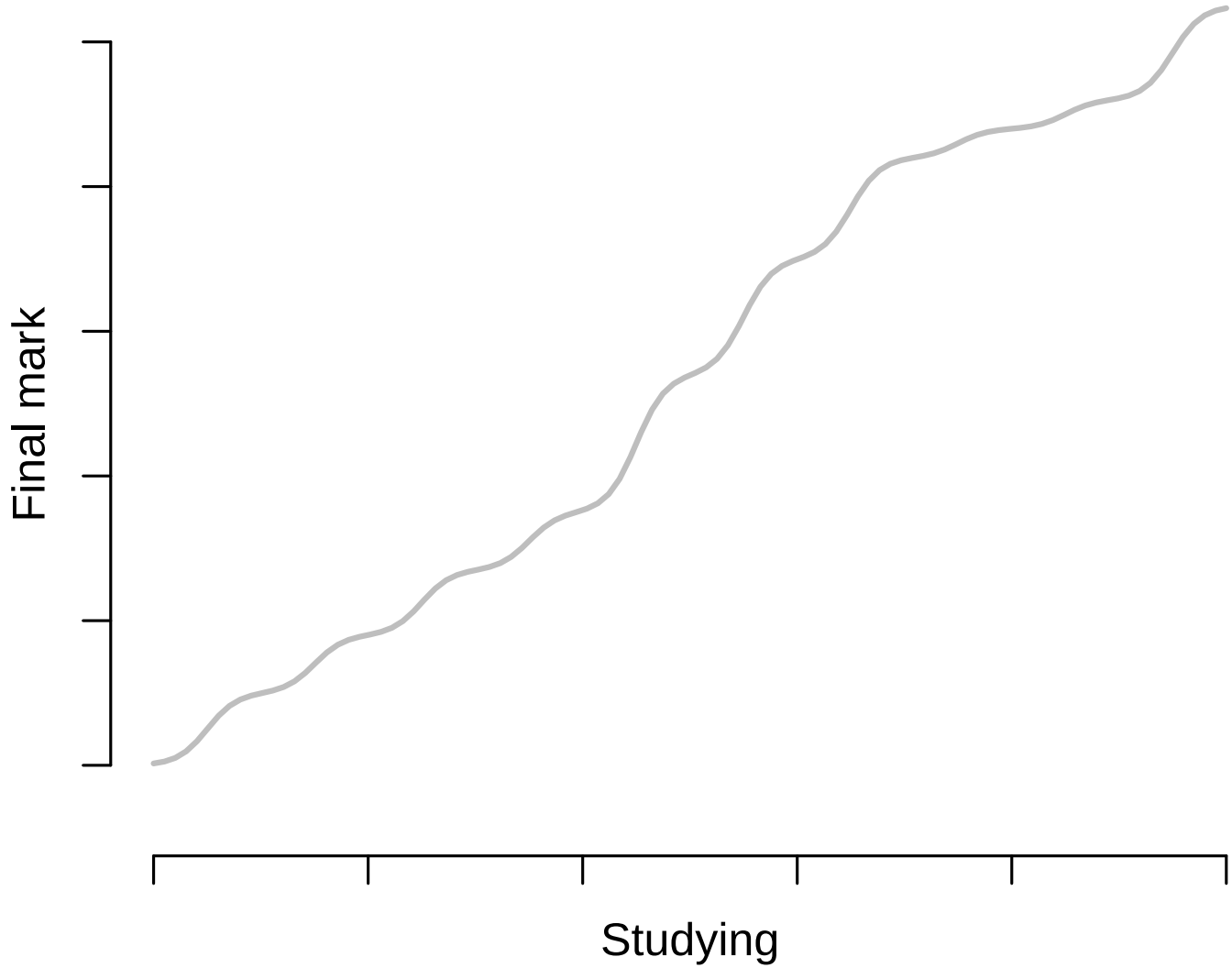
If we knew all potential outcomes for an individual, we could also know

- the effect of studying 100 hours vs. 1 hour on final mark
- the average effect of studying an additional hour on final mark
- etc.



Fundamental problem of causal inference

We only observe **one** potential outcome per unit.



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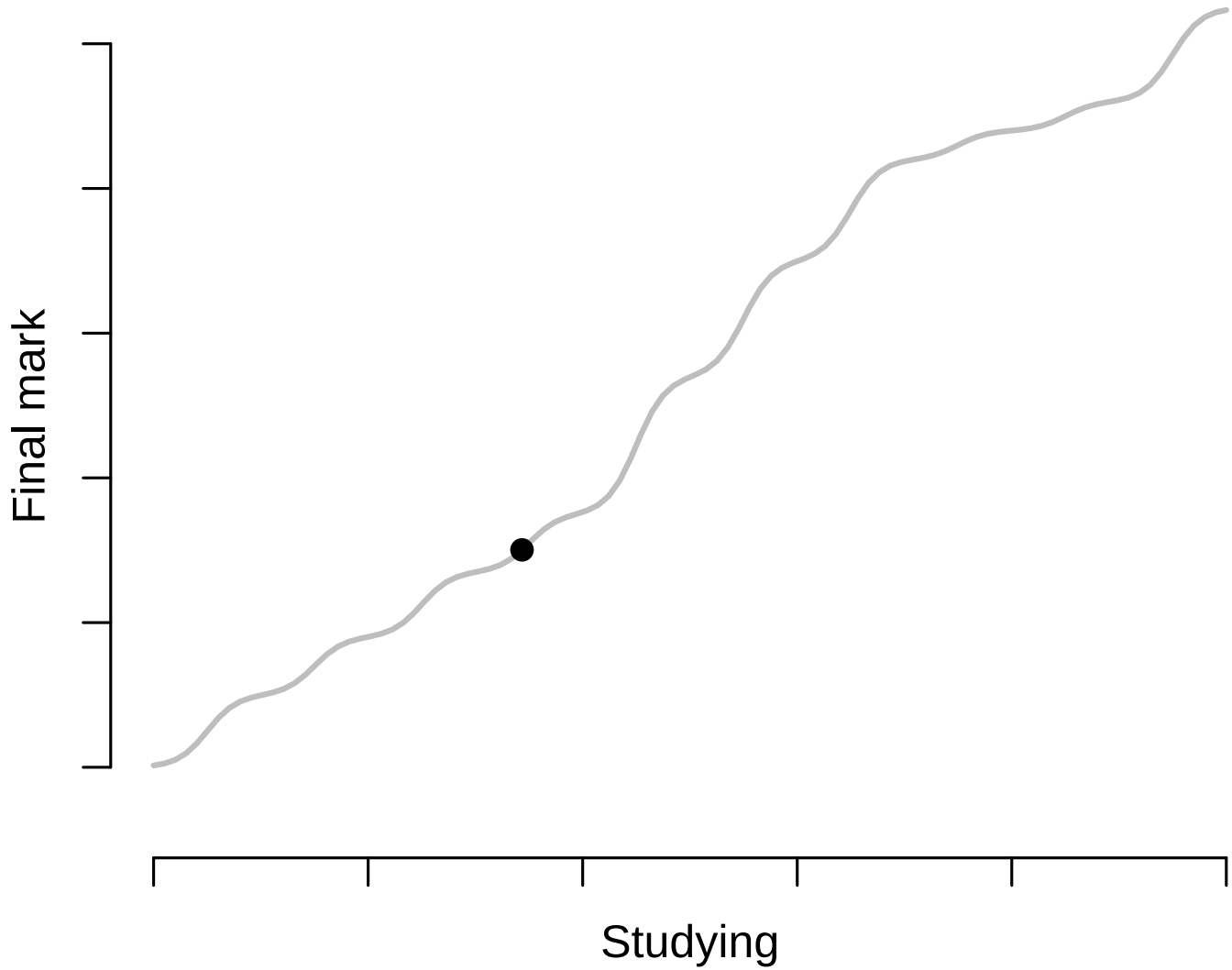

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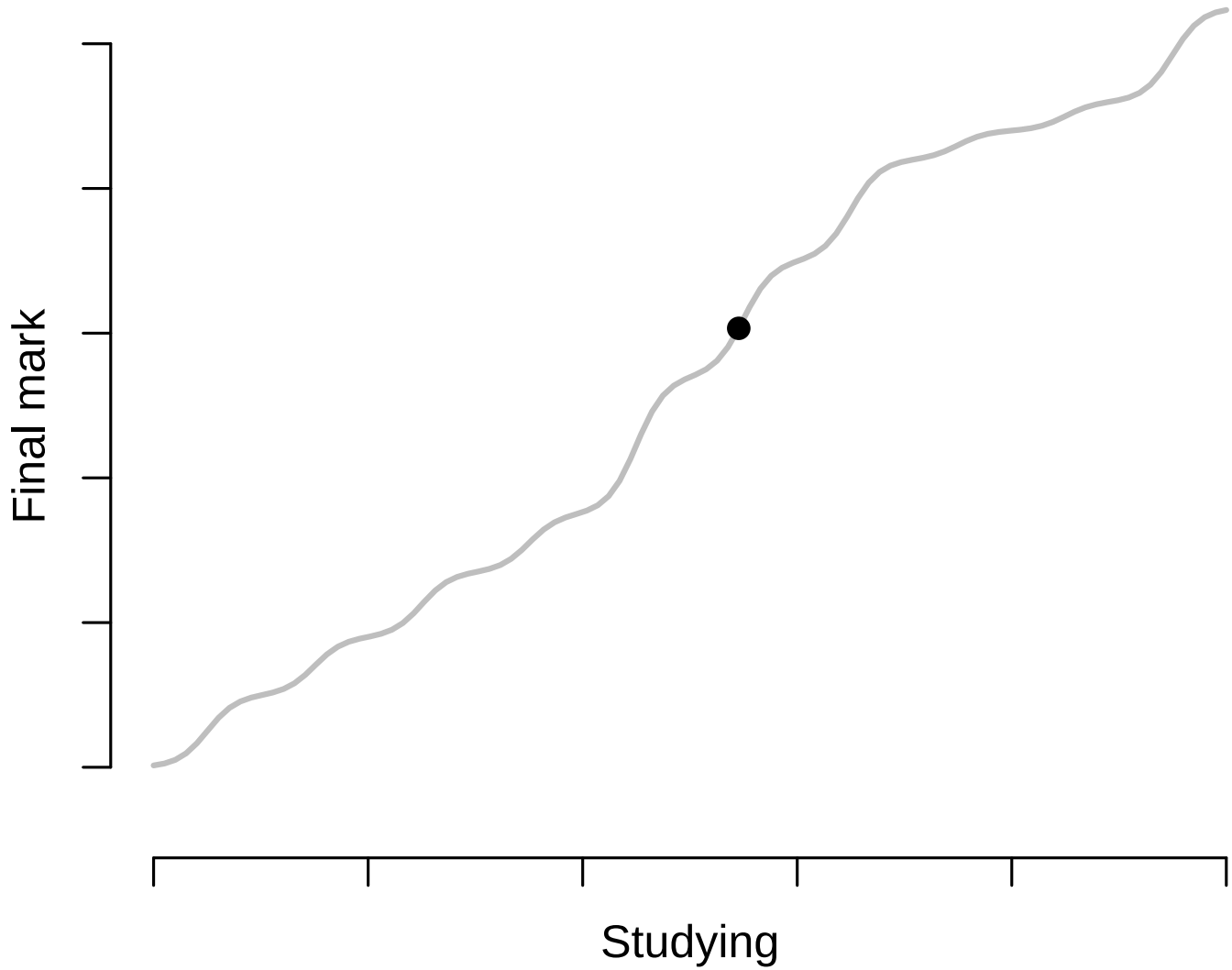

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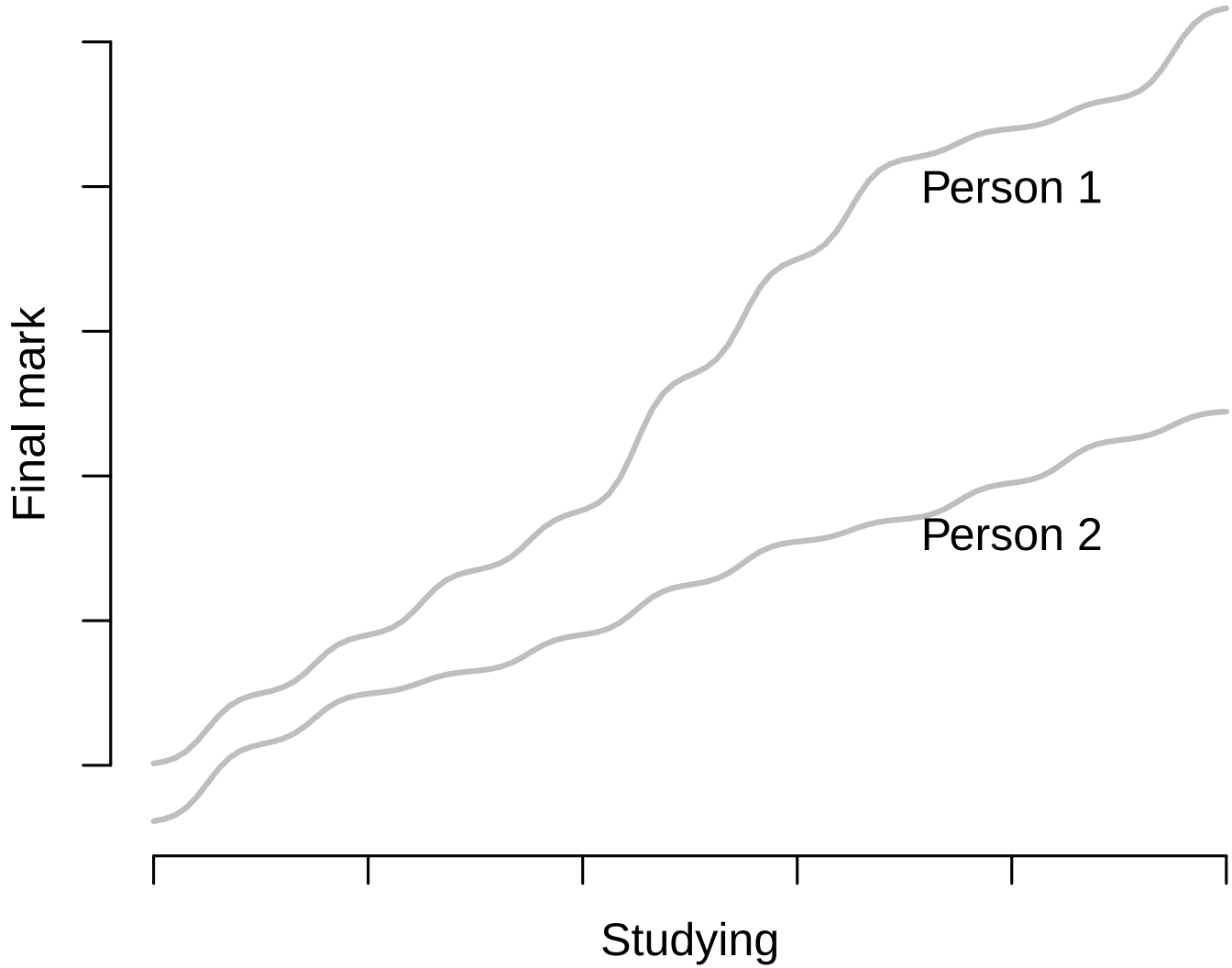
Depends on whether (a) other things change over time and (b) outcomes depend on past treatments

Comparisons across units

The only other option is to compare across units. But be careful!



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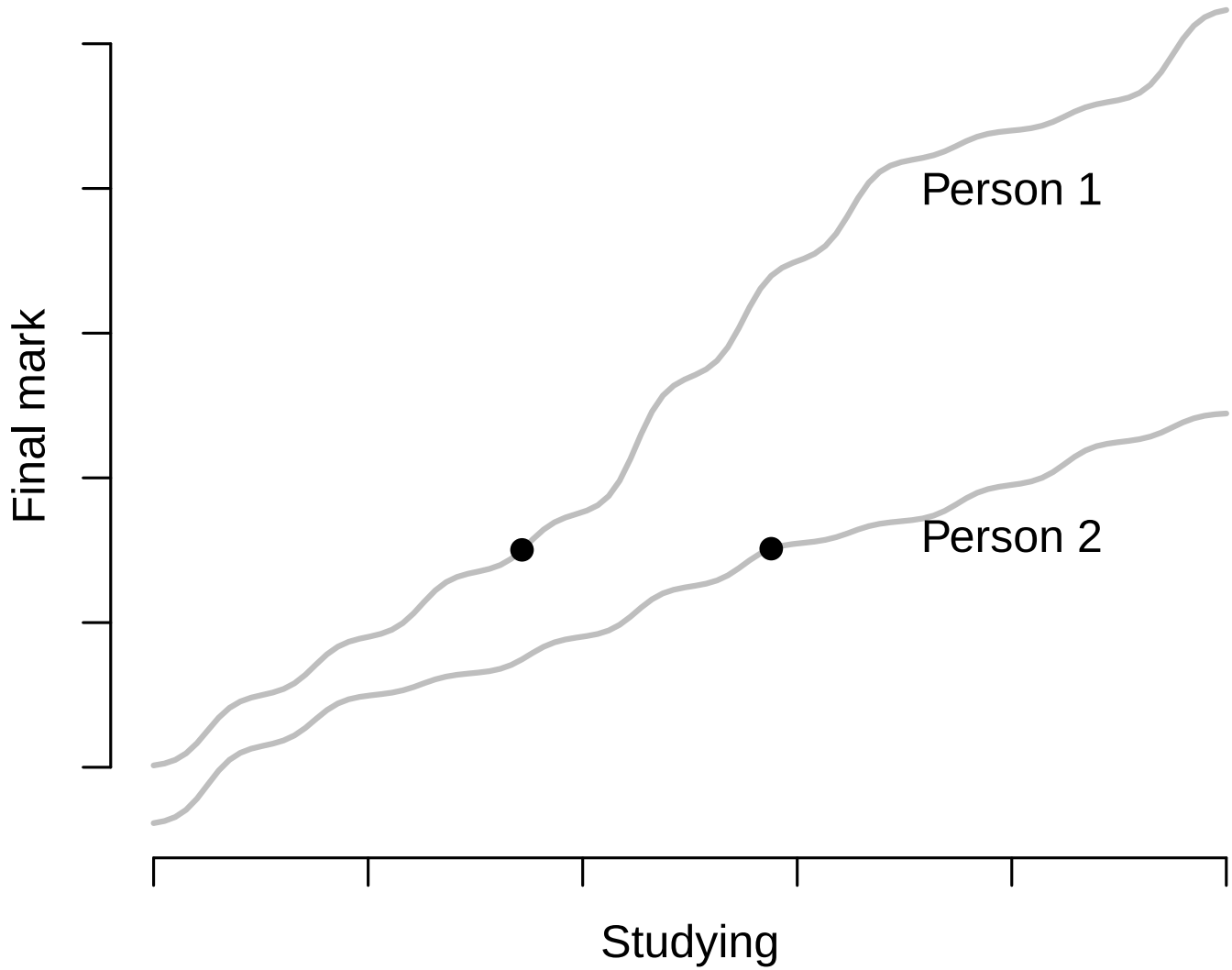

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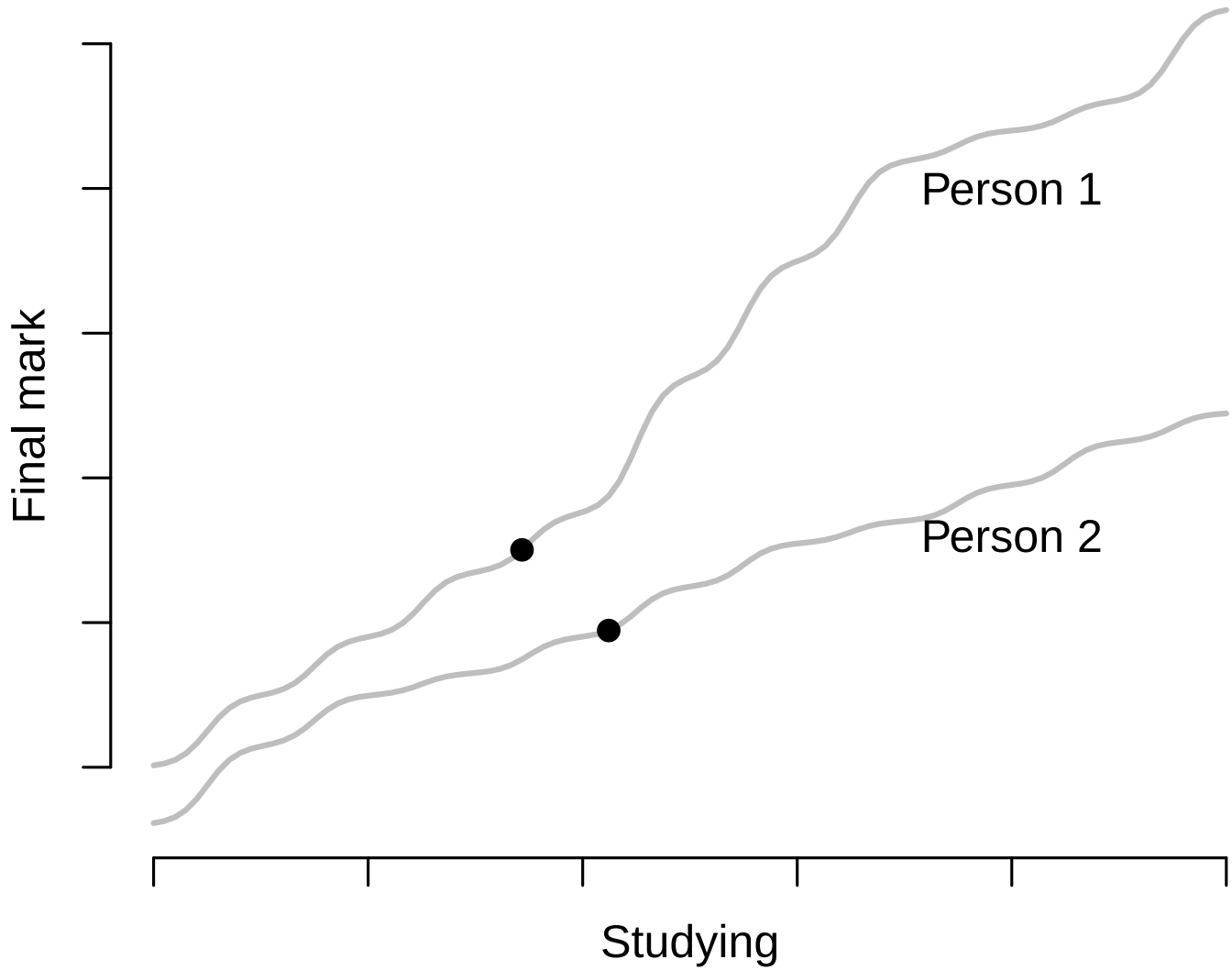

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Causal inference is partly about managing **heterogeneity**, mostly about addressing **confounding**.

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In practice, we are usually doing both: using a sample to draw conclusions about the full set of potential outcomes in the population.

Potential outcomes framework: activity 1!

Think of a treatment and an outcome it might affect for some unit.
(Examples from political science, nutrition, fiction, etc. all welcome.)

1. Make a figure showing how the potential outcomes might depend on the treatment for one unit. Identify the unit.
2. Which potential outcome do we observe for this unit?
3. Repeat (1) and (2) for additional unit(s).

A note on the standard "no interference" assumption

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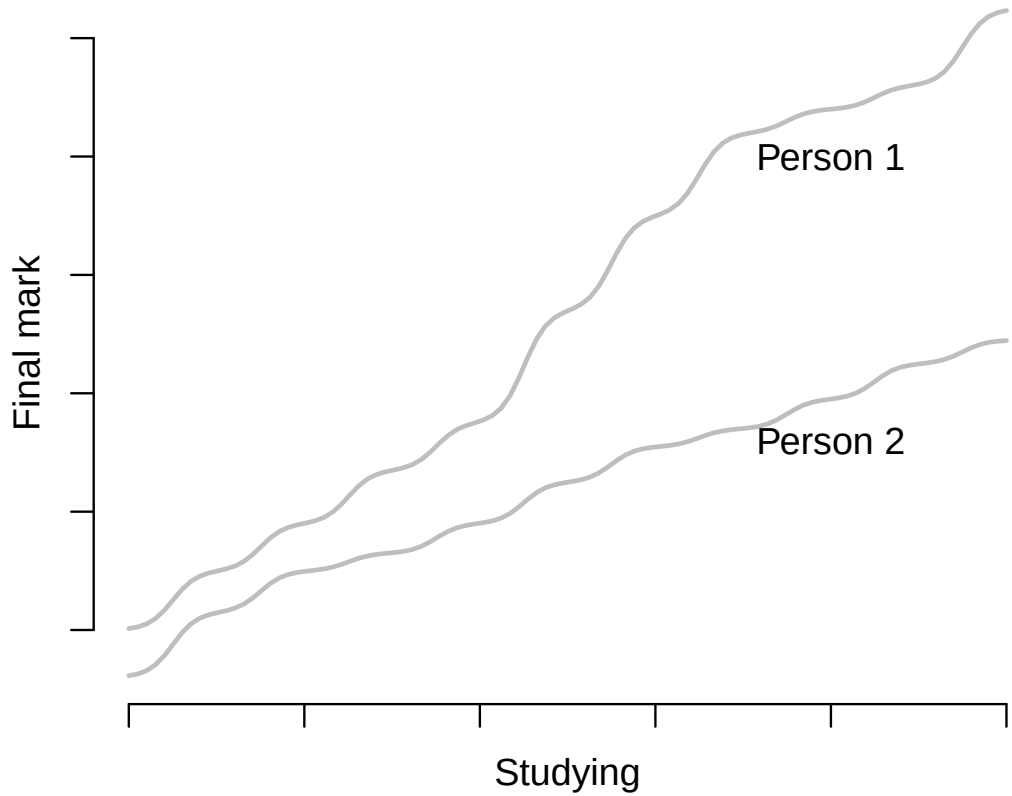

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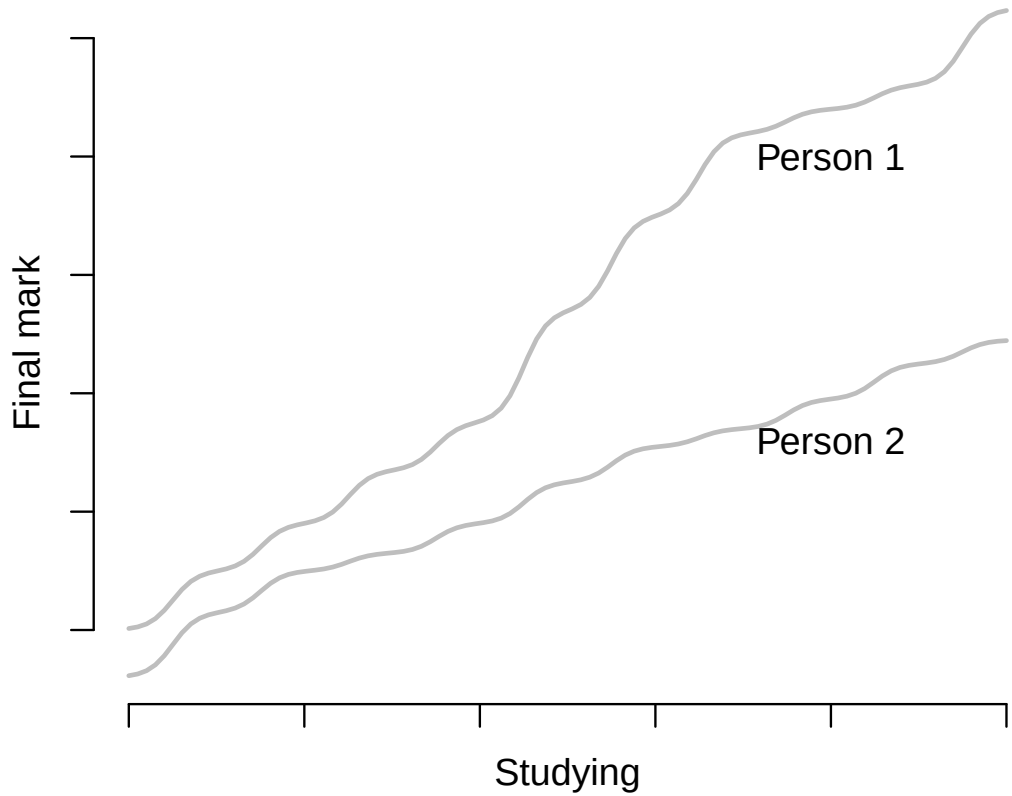

When might this assumption be invalid?

The potential outcomes framework: informal introduction

The potential outcomes framework: more formally

Randomized experiments

Potential outcomes framework: notation

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Effect of D **on** Y for i: $\tau_i = Y_{1i} - Y_{0i}$ (treatment effect, causal effect).

Individual treatment effects: no FPOCI

Imagine a population of 4 units in which both potential outcomes are observed:

i	Y_i	D_i	Y_{1i}	Y_{0i}	τ_i
1	3	1	3	0	
2	1	1	1	1	
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What you actually observe:

i	Y_i	D_i	Y_{1i}	Y_{0i}	τ_i
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Quick check: Name some unobservable potential outcomes.

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- $E[X]$ denotes **expectation** (or **mean**) of a random variable X

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(assuming population of N units).

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i.e. sample mean is unbiased estimator of mean/expectation

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Difference in group means (example)

Recall our toy example:

i	Y_i	D_i	Y_{1i}	Y_{0i}	τ_i
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But let's try to be more general.

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where ATT is the ATE in the treated group (average treatment effect for the treated).

DIGM is an unbiased estimator of ATE if ATT = ATE and if there is no selection bias, i.e. if $E[Y_{0i}]$ is the same in the treatment group and control group.

Potential outcomes framework: activity 2!

Selection bias: the difference in average Y_{0i} between the treatment group and the control group.

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1. Think of a (new) binary treatment and an outcome it might affect.
2. What are the units in your example?
3. What are the potential outcomes?
4. What would selection bias mean in this case?
5. In an observational study seeking to measure the effect of your treatment on your outcome (i.e. not a randomized experiment), do you think selection bias would be positive or negative?

Randomization and selection bias

Recall that

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This is true even for small samples (though the variance across randomizations will be smaller for large samples).

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So DIGM gives an unbiased estimate of ATE.

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But a little controversial: can never say anything about balance in unobserved covariates, so "philosophically unsound, of no practical value, and potentially misleading" (Senn 1994, quoted in Imai, King, and Stuart 2008).

Internal and external validity

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This critique is sometimes simplistic, unfair.

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- I prefer an external validity critique like "Did you answer your question?" rather than "Did you answer my question?" Otherwise, just say you're not interested.

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Activity: Assess the internal and external validity of these types of experiments.

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- Treatments can also be composite of many features: e.g. factorial design, forced-choice conjoint